HW2: Due April 1, Monday.

1. 60 pts (Exercise 5.14 in O'Reilly's book)

Exercise 5.14: This exercise outlines the establishment of the following extension to Kirchhoff's kinetic analogue that was proposed by Joseph Larmor (1857–1942) [197, Section 8]: "That the motion of a gyroscopic pendulum (i.e., a solid rotating about a fixed point with a revolving fly-wheel mounted on an axis fixed in it) is exactly analogous to the form assumed by a wire originally helical, when deformed by any distribution of forces applied to its extremities."[55] Larmor's extension is helpful when analyzing the deformed shapes of plant tendrils and wires that have intrinsic curvatures, geometric torsion, and twist.

(a) Consider a rod which has an intrinsic curvature $\mathbf{v}_0 = v_{0_1}\mathbf{D}_1 + v_{0_2}\mathbf{D}_2 + v_{0_3}\mathbf{D}_3$. Under which conditions does the rod correspond to a twisted rod with a straight centerline, a twisted rod with a circular centerline, and a twisted rod where the centerline is bent into a circular helix?

(b) Show that

$$\mathbf{d}_k' = \left(\sum_{i=1}^{3}\left(v_i + v_{0_i}\right)\mathbf{d}_i\right) \times \mathbf{d}_k. \tag{5.268}$$

(c) Suppose the rod is bent by application of terminal forces $\pm\mathbf{F}_0$ and terminal moments. Using balances of linear and angular momentum, $\mathbf{n}' = \mathbf{0}$ and $\mathbf{m}' + \mathbf{r}' \times \mathbf{n} = \mathbf{0}$, and assuming a constitutive relation

$$2\rho_0\psi = EI_1 v_1^2 + EI_2 v_2^2 + \mathscr{D} v_3^2, \tag{5.269}$$

establish the equations governing $v_k(\xi)$:

$$\begin{aligned} EI_1 v_1' - EI_2 v_2\left(v_3 + v_{0_3}\right) + \mathscr{D}\left(v_2 + v_{0_2}\right) v_3 &= -\mathbf{d}_2 \cdot \mathbf{F}_0, \\ EI_2 v_2' - \mathscr{D}\left(v_1 + v_{0_1}\right) v_3 + EI_1\left(v_3 + v_{0_3}\right) v_1 &= \mathbf{d}_1 \cdot \mathbf{F}_0, \\ \mathscr{D} v_3' - EI_1\left(v_2 + v_{0_2}\right) v_1 + EI_2\left(v_1 + v_{0_1}\right) v_2 &= 0. \end{aligned} \tag{5.270}$$

The constitutive parameters EI_1, EI_2, and $\mathscr{D}$ are assumed to be constant. HINT: the derivation of Eqn. (5.270) closely follows the developments of Eqns. (5.119).

(d) Using the conservations (5.115) and (5.116), show that the following three quantities are conserved along the length of the rod:

$$\mathbf{n} \cdot \mathbf{m}, \qquad \mathsf{C}, \qquad \mathbf{n} \cdot \mathbf{n}. \tag{5.271}$$

For the conservation of C, it is necessary to assume that v_{k_0} are constant. This assumption is adopted for the remainder of this exercise.

2. 40 pts

(a) Read the reference: **Yu, Tian, and J. A. Hanna. "Bifurcations of buckled, clamped anisotropic rods and thin bands under lateral end translations."** ***Journal of the Mechanics and Physics of Solids*** **122 (2019): 657-685.**

(b) Follow Appendix B and reproduce one of the configurations in Fig. 2 using the 'anisotropic rod' method. You may use Mathematica, MATLAB or any other software to solve the ODE system developed in Appendix B.