HW4: Due June 3, Monday.

1. (60 points)

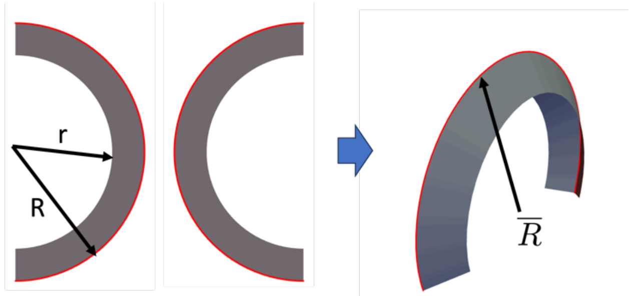

Figure 1

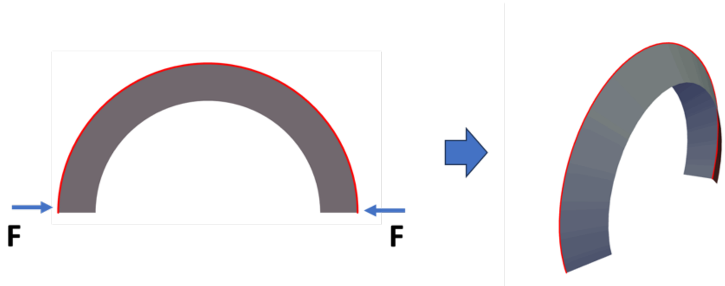

Figure 2

Consider a curved fold origami made of two identical half circular strips with outer radius R and inner radius r (see above).

(1) Assume the deformed crease of the curved fold origami is part of a circle with radius $\overline{R}$ and the deformation is isometric. Compute the shape of the two flanks (curved surfaces on the two sides of the crease).

(2) Compute the bending energy.

(3) Given the reference strips (two identical half circular strips) and the deformed crease (an arc with radius $\overline{R}$), how many configurations does the curved fold origami have? (Hint: 3) What are they?

(4) Under a large enough uniaxial force at the end points, the initially closed origami (i.e. the opening angle is zero) will 'buckle' and open. Which configuration in (3) will the origami end up with? What are the critical forces? (Hint: Euler's buckling)

2. (20 points) Prove that the extremals (驻定曲线) of length of a curve are straight lines in both Cartesian and polar coordinates.

3. (20 points) Prove that an n-dimensional spherical surface embedded in (n+1)-dimensional space is a manifold.