HW5: Due June 17, Monday.

1. (20 points)
Prove a n-dimensional spherical surface is a manifold.

2. (20 points)
Use the variational principle to derive the Euler-Lagrange equation in a classic mechanical system.

3. (20 points)

PROBLEM. Let $\mathbf{x} = (x_1, \ldots, x_m)$ be coordinates in a neighborhood of $\mathbf{x} \in M$, and $\mathbf{y} = (y_1, \ldots, y_n)$ be coordinates in a neighborhood of $\mathbf{y} \in N$. Let $\boldsymbol{\xi}$ be the set of components of the vector $\mathbf{v}$, and $\boldsymbol{\eta}$ the set of components of the vector $f_{*\mathbf{x}}\mathbf{v}$. Show that

$$\boldsymbol{\eta} = \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \boldsymbol{\xi}, \quad \text{i.e.,} \quad \eta_i = \sum_j \frac{\partial y_i}{\partial x_j} \xi_j.$$

Taking the union of the mappings $f_{*\mathbf{x}}$ for all $\mathbf{x}$, we get a mapping of the whole tangent bundle

$$f_*: TM \to TN \qquad f_*\mathbf{v} = f_{*\mathbf{x}}\mathbf{v} \quad \text{for } \mathbf{v} \in TM_{\mathbf{x}}.$$

PROBLEM. Show that f_* is a differentiable map.

4. (20 points)

PROBLEM 1. Suppose that a particle moves in the field of the uniform helical line $x = \cos\varphi$, $y = \sin\varphi$, $z = c\varphi$. Find the law of conservation corresponding to this helical symmetry.

ANSWER. In any system which admits helical motions leaving our helical line fixed, the quantity $I = cP_3 + M_3$ is conserved.

5. (20 points)

PROBLEM. Investigate the characteristic oscillations of a planar double pendulum (Figure 88).

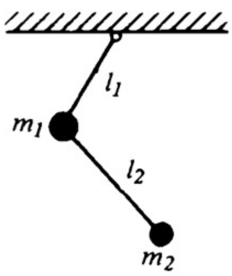

Figure 88 Double pendulum