微分几何与力学 – 2024 春季

Differential Geometry and Mechanics – 2024 Spring

开课教师：冯帆　　**时间**：周一 15:10-18:00　　**地点**：三教 401

课程号：08611660　　**学分**：3 学分　　**邮箱**：fanfeng@pku.edu.cn

Course Page: http://www2.coe.pku.edu.cn/faculty/fengfan/courses.html

DISCLAIMER: The syllabus is not necessarily what we will exactly learn in this class. Some flexibility will be essential for our journey. I reserve the right to make the final decision.

Course description: Differential geometry is a mathematical subject that studies the geometry of curves, surfaces and manifolds (higher-dimensional analogues of surfaces). This old subject and its modern development answer the following questions: How does one measure the geometric quantity of a curve and a surface? How to describe the shape evolution? What is the shortest path between two points on a curved surface? What if the surfaces and curves are in a curved space? Clearly, its connection to our physical world, in particular mechanics, is straightforward. We see rods, strips, plates and membranes everywhere in our daily life. Differential geometry can help us understand these objects in a more abstract and fundamental way – this is so-called 'science'.

This course will emphasize the connection between differential geometry and mechanics, without overpaying attention to the rigorous foundation of differential geometry itself. We will start by learning curves and surfaces in classical differential geometry. Then we will move on to modern differential geometry, Riemannian geometry and symplectic geometry. We will see their connection with Newtonian mechanics, Lagrangian mechanics and Hamiltonian mechanics. The course is designed to be accessible to senior undergraduates.

Prerequisities: Calculus and linear algebra.

Contents:

Week 1 – Week 6: Classical differential geometry

- Curves
  - Euclidean space, metric, parametric description
  - Frenet frame, curvature, torsion
  - Mechanics of rods
- Surfaces
  - Parametric description, tangent, normal, first and second fundamental forms
  - Isometric mapping, Gauss theorem, Equation of Compatibility
  - Geodesics, curvature, Gauss-Bonnet theorem
  - Mechanics of curved surfaces
- Newtonian mechanics (brief)

Week 7 – Week 14: Modern differential geometry

- Introduction to Riemannian geometry
  - Topology, differentiable manifolds
  - Concepts of bundles, connections, metrics and curvature

    - Levi-Civita connection
- Lagrangian mechanics
    - Calculus of variations, Liouville's theorem
    - Noether's theorem, D'Alembert's principle
- Introduction to symplectic geometry
    - Differential forms, Lie algebra
- Hamiltonian mechanics

Week 15 – Week 16: Special topics

- Rational treatment of the Kirchhoff's plate theory
- Geometric compatibility and martensitic phase transformation
- Non-Euclidean mechanics

Grading Policy: 10 HW x 10 pts/HW = 100 pts. You are encouraged to discuss with classmates and me.

Texts:

Classical differential geometry

- **Differential Geometry of Curves and Surfaces, by Manfredo Perdigao do Carmo.**
- 《微分几何》（第二版），陈维恒 编著， 北京大学出版社

Modern differential geometry

- **Mathematical Methods of Classical Mechanics, by V. I. Arnold**
- 《微分几何入门与广义相对论》，梁灿彬
- Riemannian Geometry, by M.P. do Carmo, (Birkhause, 1992).

References:

- Elasticity and Geometry, by B. Audoly and Y. Pomeau, Oxford University Press, 2010 (many interesting mechanics problems from the geometric point of view)
- What is differential geometry: curves and surfaces, by Anton Petrunin and Sergio Zamora Barrera, https://arxiv.org/abs/2012.11814v2
- An Introduction to Differentiable Manifolds and Riemannian Geometry, by W. Boothby, 2nd edition, Academic Press, 1986