

以下公式使用 L<sup>A</sup>T<sub>E</sub>X 编写, 感兴趣的同学可以联系助教获得安装及使用指导。

一维深势阱的定态薛定谔方程的解法。

$$\left[ -\frac{\hbar^2}{8m\pi^2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + V(x, y, z) \right] \psi(x, y, z) = E\psi(x, y, z) \quad (1)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (2)$$

$$\hbar = \frac{h}{2\pi} \quad (3)$$

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V \right] \psi(x, y, z) = E\psi(x, y, z) \quad (4)$$

$$\hat{H}\psi = E\psi \quad (5)$$

$m$  粒子的质量

$\psi(x, y, z)$  粒子的波函数

$V$  粒子的势能

$\nabla^2$  拉普拉斯算符

$E$  粒子在波函数下的能量

$\hat{H}$  哈密顿算符

$$x \leq 0 \quad V(x) = \infty$$

$$0 < x < l \quad V(x) = 0$$

$$x \geq l \quad V(x) = \infty$$

$$\left[ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi(x) = E\psi(x) \quad (6)$$

$$x \leq 0, x \geq l \quad V(x) = \infty$$

$$\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = (\infty - E)\psi(x) = \infty\psi(x) \quad (7)$$

$$\psi(x) = \frac{1}{\infty} \frac{d^2\psi(x)}{dx^2} \quad (8)$$

$$\psi(x) = 0 \quad (x \leq 0, x \geq l) \quad (9)$$

$$0 < x < l \quad V(x) = 0$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x) \quad (10)$$

$$\text{Let} \quad k^2 = \frac{2mE}{\hbar^2} \quad (11)$$

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0 \quad (12)$$

$$\psi(x) = A \cos(kx) + B \sin(kx) \quad (13)$$

$$\begin{aligned} \psi(0) &= 0 \implies A \cos 0 + B \sin 0 = 0 \implies A = 0 \\ \psi(l) &= 0 \implies B \sin(kl) = 0 \implies \sin(kl) = 0 \end{aligned}$$

$$\begin{aligned} \sin(kl) &= 0 \\ k &= \frac{n\pi}{l} \quad (n = 0, \pm 1, \pm 2, \dots) \\ \psi &= B \sin\left(\frac{n\pi}{l}x\right) \end{aligned} \quad (14)$$

$$\int_0^l |\psi(x)|^2 dx = \int_0^l B^2 \sin^2 \frac{n\pi x}{l} dx = 1 \quad (15)$$

$$\int \sin^n cx dx = -\frac{1}{nc} \sin^{n-1} cx \cos cx + \frac{n-1}{n} \int \sin^{n-2} cx dx \quad (16)$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin (\alpha + \beta) + \sin (\alpha - \beta)] \quad (17)$$

$$\begin{aligned} B &= \sqrt{\frac{2}{l}} \\ \psi &= \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l} x \end{aligned} \quad (18)$$

$$\begin{aligned} k^2 &= \frac{2mE}{\hbar^2} \implies E = \frac{k^2 \hbar^2}{2m} \\ k &= \frac{n\pi}{l}, \quad \hbar = \frac{h}{2\pi} \implies E = \frac{n^2 h^2}{2ml} \end{aligned}$$

$$\boxed{\psi_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l} x \quad E_n = \frac{n^2 h^2}{8ml^2} \quad n = 1, 2, \dots}$$

$$\Delta E = E_{n+1} - E_n = \frac{(n+1)^2 h^2}{8ml^2} - \frac{n^2 h^2}{8ml^2} = \frac{(2n+1)h^2}{8ml^2} \quad (19)$$