

A Sequential Bayesian Partitioning Approach for Online Steady State Detection of Multivariate Systems

Jianguo Wu¹, Honglun Xu, Chen Zhang, Yuan Yuan

Abstract— Steady state detection is critically important in many engineering fields, such as fault detection and diagnosis, process monitoring and control. However, most of the existing methods were designed for univariate signals, and thus are not effective in handling multivariate signals. In this paper, we propose an efficient online steady state detection method for multivariate systems through a sequential Bayesian partitioning approach. The signal is modeled by a Bayesian piecewise constant mean and covariance model, and a recursive updating method is developed to calculate the posterior distributions analytically. The duration of the current segment is utilized for steady state testing. Insightful guidance is also provided for hyperparameter selection. The effectiveness of the proposed method is demonstrated through thorough numerical and real case studies.

Note to Practitioners—This paper addresses the problem of online steady state detection of systems captured by multivariate signals. Existing approaches often monitor each signal independently, and the system is claimed steady when all signals reach steady state. These methods have many shortcomings, such as failing to consider the correlations among signals, and suffering the multiple testing problem. In this paper, we propose a novel joint monitoring approach, where the multivariate signal is sequentially partitioned into segments of constant mean and covariance through an online Bayesian inference scheme, and once the current segment duration is sufficiently large, the signal is considered steady. We also provide several insightful guidelines to select appropriate hyperparameters under different scenarios. The proposed approach is much more accurate and robust than existing ones. However, this method may face prohibitive computational cost and ill-posed covariance inversion problem when there are hundreds or even thousands of variables in the system. In future research, we will develop efficient distributed

monitoring and data fusion techniques to overcome these challenges.

Index Terms—Steady state detection, multivariate system, sequential Bayesian partitioning, hyperparameter selection, change-point detection.

I. INTRODUCTION

DETEECTING whether a system is operating under steady-state condition is essential in process performance assessment and optimization [1, 2], fault detection and diagnosis [3, 4], process automation and control [5-9]. It arises in many engineering fields, such as process industries, chemical engineering, and manufacturing process automation and control. In these applications, a steady state condition is the basic requirement for process modeling, evaluation, monitoring and control. In discrete-event simulations [1, 10], for example, the steady state is not achieved until some time after the system is started or initialized. The initial situation is often referred to as transient state, start-up or warm-up period. Only the steady-state period data (e.g., throughput, work-in-process) represents the true performance of the system and thus needs to be identified for process assessment and optimization. In process or chemical industries, it is often mandatory to use steady-state data (e.g., flow rate, pH value, temperature, pressure, etc.) for process modeling and design, and real time optimization (RTO) [6, 11]. In batch processes manufacturing [12], the operation is often unsteady during the start-up period due to unstabilized material or machine conditions, which cannot guarantee a satisfied product quality. To avoid costly quality inspection and scrap costs, the steady state operation needs to be identified. In process automation and control, the steady state can be used to trigger the next action. For example, in ultrasonic cavitation-based nanoparticle dispersion process, the particles are considered completely dispersed and the process can be stopped when the cavitation noise signal enters into steady state [7-9].

In the past few decades, various types of univariate offline methods have been developed for initial bias truncation in discrete-event simulations [1, 10]. These methods often require a sufficiently large number of steady-state observations for warm-up truncation, and thus are not applicable for online detection. In contrast, there is a relatively small number of online steady-state detection algorithms for univariate signals in the existing literature, most of which can be classified into

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the following categories: (1) regression based approach [13] where a simple linear regression is performed over a moving data window and the fitted slope is monitored. Once the magnitude of the slope is below a predefined threshold, the signal is considered steady; (2) performing a t -test on the difference of the means of two adjacent moving windows. If the difference is significantly small, the signal is claimed steady [14]; (3) performing an F -test (variance ratio test) on the ratio of two variances of a moving window calculated using different methods, namely, the mean squared deviation and the mean squared differences of successive data [15]. In the steady state period, the ratio is expected to be near unity; and (4) monitoring the variance of a moving window [4]. When the variance is below a threshold, the signal is steady.

However, all the aforementioned online methods are developed for univariate signals. In practice, most of the systems or processes are inherently multivariate. With the rapid development of sensing technology, multiple sensor signals have become unprecedentedly available to better capture the system conditions. Therefore, multivariate steady state detection algorithms are highly desirable. To the best of our knowledge, there are very limited methods for multivariate signals. Brown and Rhinehart [16] proposed to monitor each signal separately using an existing univariate steady state detection algorithm. Once all signals are steady, the process is claimed to be steady. However, this strategy inevitably suffers the notorious multiple testing problem [17] with inflated type II error or detection delay. Besides, it is incapable of detecting the change of correlation among different variables. Jiang *et al* [18] proposed to fuse the steady-state indices of all variables into one through the Dempster's rule of combination [19]. This approach is nevertheless a generalization of Brown and Rhinehart's method [16] and thus shares the same shortcomings. Besides, it requires sufficient historical steady-state data to determine the testing threshold, which is often unrealistic in practice due to the data unavailability and run-to-run trajectory variations. Note that if sufficient historical steady-state observations are available and follow the same statistical distribution for all runs, a large number of existing statistical process control (SPC) techniques [20] are readily available for steady state detection, e.g., detecting the first in-control sample. However, in steady-state detection applications, the distribution of steady state data often varies from run to run, due to known or unknown process conditions. Therefore, the existing multivariate SPC techniques cannot be directly used. Most of the other multivariate methods are developed in the chemical batch processes [12, 21]. In these methods, a dimension reduction technique is applied first, such as the multi-way principal component analysis (MPCA) [21], dynamic principal component analysis (DPCA) [12], and then either a univariate method is applied on a combined index, or each extracted feature is monitored individually. These methods also have more or less the aforementioned disadvantages.

To overcome these disadvantages, this paper develops an efficient online multivariate steady state detection method using a sequential Bayesian partitioning approach. In this

method, the multivariate signal is sequentially segmented into phases of constant mean and covariance matrix under the Bayesian framework, and the posterior distribution of the phase duration is used to test the steady state. Once the duration is sufficiently large, the signal is claimed steady. The main challenges of this method are how to sequentially find the change-point regarding the mean and covariance, and how to select appropriate hyperparameters. To overcome these challenges, we develop an efficient recursive method to calculate the posterior distributions analytically, and then provide several insightful guidelines on hyperparameter selection.

The rest of this article is organized as follows. In Section II, the steady state detection problem is formulated into a piecewise constant modeling of multivariate signals. Section III presents the technical details of online change-point detection, computational cost reduction and appropriate hyperparameter selection. The numerical illustration, performance comparison, and real case studies are provided in Section IV. Section V presents our conclusions and discussions.

II. PIECEWISE CONSTANT MODELING OF MULTIVARIATE SIGNALS FOR STEADY STATE DETECTION

To detect whether a system is steady, it is necessary to first define what steady state is. In mathematics or statistics, an alternative term – stationary process, is often used, which is defined as a stochastic process $\{x_t\}$ whose joint probability distribution $p(x_t, x_{t+1}, \dots, x_{t+s})$ does not change over time t (strict or strong stationarity). Consequently, the mean and variance or covariance parameters do not change over time. In this paper, we define steady state as the condition where the mean and covariance of signals capturing the system dynamics are unchanging in a certain period. It is worth noting that in some practical applications, such as the discrete-event simulations and batch processes manufacturing, once steady state occurs, it is expected not to change anymore. Therefore, the online monitoring can be stopped once steady state is detected. However, in many other applications, e.g., monitoring temperature, pressure and pH value in process or chemical industries [6], the transient state and steady state often occur alternatively, due to unexpected system faults, disturbances or closed-loop control actions. Therefore, it requires the monitoring scheme to be able to detect the occurrence of multiple steady states and transient states.

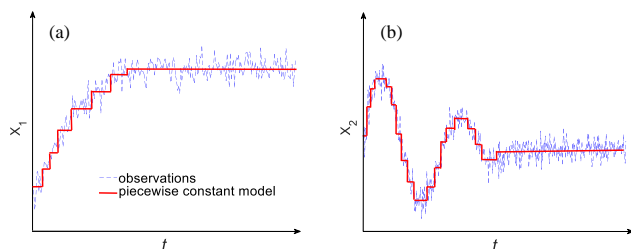


Fig. 1 Illustration of piecewise constant modeling of nonlinear signals with initial transient period: (a) exponential function in the transient period; (b) oscillating function in the transient period.

Based on the definition, we propose to utilize a piecewise constant model to fit the multivariate signals, where each segment is modeled with a unique mean and covariance matrix. Take two univariate signals for example (Fig. 1). The segment duration would be very short in the transient period whereas it is expected to be long in the steady state period. In other words, the change-points between the successive segments occur more frequently and continuously in the transient period due to the rapid change of mean or covariance. When the system is in the steady state period, there would be no change-points. In online steady state detection, the segment duration can be used as a monitoring statistic. Once it is sufficiently large, e.g., larger than a certain threshold, the system is considered steady.

Let the multivariate signal be $\mathbf{X}_t, t = 1, 2, \dots$, where \mathbf{X}_t is a p -dimensional vector. Suppose $\mathbf{X}_t \sim N(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$ and $\mathbf{X}_t, t = 1, 2, \dots$, are independent. Define $\boldsymbol{\Phi}_t = (\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$. Suppose the change-points are at positions $\{c_1, c_2, \dots, c_k, \dots\}$, where $0 < c_1 < c_2 < \dots < c_k < \dots$. Then the piecewise constant model can be mathematically expressed as

$$\boldsymbol{\Phi}_t = \begin{cases} \boldsymbol{\Phi}^{(1)}, & \text{if } c_0 < t \leq c_1 \\ \boldsymbol{\Phi}^{(2)}, & \text{if } c_1 < t \leq c_2 \\ \dots & \dots \\ \boldsymbol{\Phi}^{(k)}, & \text{if } c_{k-1} < t \leq c_k \\ \dots & \dots \end{cases} \quad (1)$$

where $\boldsymbol{\Phi}^{(i)}$ is the distribution parameter of the i^{th} segment. Based on the above discussion, any interval $[c_{k-1}, c_k]$ with length $c_k - c_{k-1}$ greater than a certain value, e.g., L_0 , can be claimed as a steady state period.

In the online steady state detection, the testing is performed each time a new observation is obtained. Therefore, the multivariate signal needs to be fitted sequentially using a piecewise constant model, and the duration of the current segment has to be estimated to decide if the current segment is long enough to claim a steady state. Although the idea is simple, how to efficiently estimate the latest change-point (i.e., the starting time of the current segment) in a real time manner is nevertheless very challenging. Online Bayesian updating is particularly effectively in the dynamic analysis of a sequence of data with change-points [22]. Besides, it provides uncertainty estimates in the number and locations of change-points, which is more realistic in applications. Therefore, in this article, we propose to use a Bayesian approach, where a Bayesian piecewise constant model is formulated, and then the posterior distribution of the latest change-point is calculated for steady state detection. The technical details are provided in Section III.

III. ONLINE BAYESIAN PIECEWISE CONSTANT MODEL FITTING AND STEADY-STATE DETECTION

A. Bayesian Formation and Prior Specification

For a Bayesian piecewise constant model, appropriate priors for the change-points, the mean and covariance matrix of each segment need to be assigned. For a time series of fixed length, a joint prior can be specified for both the change-point number k and positions [23], e.g., $\pi(k, \{\delta_i\}_{i=1}^{i=k+1}) = \pi(k)\pi(\{\delta_i\}_{i=1}^{i=k+1}|k)$ where δ_i is the duration of the i^{th} segment. However, this

approach is not appropriate or straightforward for dynamic sequences with an increasing length. Instead, the prior is often specified by modeling the occurrence of change-points through a Markov process, where the next change-point only depends on the duration of the current segment [22]. For example, a Poisson point process can be assumed for the occurrence of the change-points, or equivalently an exponential distribution is assumed for the segment durations. Another simple prior is the geometric prior applied to the segment duration, which corresponds to a constant Markov transition probability for the latest change-point at each time step. Actually, the stochastic process approach indirectly specifies a joint prior distribution for the number of change-points and their positions [24]. The advantage of this approach is that the prior transition probability of the latest change-point can be easily calculated, which is convenient for online change-point detection. Suppose the latest change-point (the time index for the last observation of the previous segment) at time step t is τ_t , then

$$P(\tau_t = j | \tau_{t-1} = j') = \begin{cases} \frac{1 - G(t - j')}{1 - G(t - 1 - j')}, & \text{if } j = j' \\ \frac{G(t - j') - G(t - 1 - j')}{1 - G(t - 1 - j')}, & \text{if } j = t - 1 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where $0 \leq j \leq t - 1$, $G(\cdot)$ is the cumulative distribution function for the segment duration. The geometric distribution is the most popular and simplest one for the segment duration in sequential change-point inferences [22, 25]. Other priors include Poisson distribution and gamma distribution, which are often used in speech segmentation [26]. As observed in our study, the detection is not very sensitive to the distribution type. Therefore, we select the geometric distribution for the segment duration for the purpose of simplicity. It is easy to show that when a geometric distribution with parameter p_0 is assumed for the segment duration δ , i.e., $P(\delta = l) = (1 - p_0)^{l-1}p_0$, the prior transition probability is simply

$$P(\tau_t = j | \tau_{t-1} = j') = \begin{cases} 1 - p_0, & \text{if } j = j' \\ p_0, & \text{if } j = t - 1 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

As we can see, it specifies a constant prior Markov transition probability for the latest change-point.

For the changing parameters $\boldsymbol{\Phi}_t = (\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$, a conjugate prior is specified as follows for all segments:

$$p(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t) = p(\boldsymbol{\Sigma}_t)p(\boldsymbol{\mu}_t | \boldsymbol{\Sigma}_t) = \text{InvWish}_p(\boldsymbol{\Psi}_0, v_0)N\left(\boldsymbol{\mu}_0, \frac{1}{v_0}\boldsymbol{\Sigma}_t\right), \quad (4)$$

where $\boldsymbol{\Sigma}_t \sim \text{InvWish}_p(\boldsymbol{\Psi}_0, v_0)$ is a p -dimensional Inverse-Wishart distribution with degrees of freedom v_0 and scale matrix $\boldsymbol{\Psi}_0$, and $\boldsymbol{\mu}_t | \boldsymbol{\Sigma}_t \sim N\left(\boldsymbol{\mu}_0, \frac{1}{v_0}\boldsymbol{\Sigma}_t\right)$ is a p -dimensional normal distribution. Besides, to facilitate online Bayesian updating, we assume that the changing parameters are independent across different segments. In the following section, we will show that such prior is a conjugate prior and the posterior distribution of both the latest change-point and

changing parameters, i.e., $p(\tau_t|\mathbf{X}_{1:t})$ and $p(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t|\mathbf{X}_{1:t}, \tau_t)$, where $\mathbf{X}_{1:t} = \{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_t\}$, are analytically tractable.

B. Sequential Bayesian Change-point Detection and Steady State Detection

As mentioned in Section II, the duration of the current segment is a critical parameter to determine if the system is steady. Therefore, it is essential to calculate the posterior distribution of the segmental duration, or equivalently the latest change-point sequentially. In this subsection, the exact posterior probability mass function (PMF) of the latest change-points and the posterior density functions of the changing parameters will be derived.

The posterior distribution can be expressed as

$$P(\tau_{t+1} = j|\mathbf{X}_{1:t+1}) \propto p(\tau_{t+1} = j, \mathbf{X}_{t+1}|\mathbf{X}_{1:t}) = P(\tau_{t+1} = j|\mathbf{X}_{1:t})p(\mathbf{X}_{t+1}|\mathbf{X}_{1:t}, \tau_{t+1} = j), \quad (5)$$

where $j = 0, 1, \dots, t$ is the observation index. The predictive probability mass function of the latest change-point in Eq. (5) can be calculated by

$$P(\tau_{t+1} = j|\mathbf{X}_{1:t}) = \sum_{i=0}^{\min(j, t-1)} P(\tau_{t+1} = j|\tau_i = i)P(\tau_t = i|\mathbf{X}_{1:t}), \quad (6)$$

Based on Eq. (3), Eq. (6) can be further simplified as

$$P(\tau_{t+1} = j|\mathbf{X}_{1:t}) = \begin{cases} p_0, & j = t \\ P(\tau_t = j|\mathbf{X}_{1:t})(1 - p_0), & j \leq t - 1 \end{cases} \quad (7)$$

The predictive density function $p(\mathbf{X}_{t+1}|\mathbf{X}_{1:t}, \tau_{t+1} = j)$ in Eq. (5) can be expressed as

$$p(\mathbf{X}_{t+1}|\mathbf{X}_{1:t}, \tau_{t+1} = j) = \begin{cases} p(\mathbf{X}_{t+1}|\mathbf{X}_{j+1:t}, \tau_{t+1} = j), & j \leq t - 1 \\ p(\mathbf{X}_{t+1}), & j = t \end{cases} \quad (8)$$

Therefore Eq. (5), (7) and (8) can be summarized as

$$P(\tau_{t+1} = j|\mathbf{X}_{1:t+1}) \propto \begin{cases} p_0 p(\mathbf{X}_{t+1}), & j = t \\ (1 - p_0)P(\tau_t = j|\mathbf{X}_{1:t})p(\mathbf{X}_{t+1}|\mathbf{X}_{j+1:t}, \tau_{t+1} = j), & j \leq t - 1 \end{cases} \quad (9)$$

Let $P_j^{(t+1)} = P(\tau_{t+1} = j|\mathbf{X}_{1:t+1})$ and $p_{j+1,t} = p(\mathbf{X}_{t+1}|\mathbf{X}_{j+1:t}, \tau_{t+1} = j)$ then Eq. (5), (7) and (8) can be summarized using a state transition equation as follows

$$\begin{bmatrix} P_0^{(t+1)}, P_1^{(t+1)}, \dots, P_t^{(t+1)} \end{bmatrix} \propto \begin{bmatrix} P_0^{(t)}, P_1^{(t)}, \dots, P_{t-1}^{(t)} \end{bmatrix} \times \begin{bmatrix} (1 - p_0)p_{1,t} & 0 & \dots & 0 & p_0 \cdot p(\mathbf{X}_{t+1}) \\ 0 & (1 - p_0)p_{2,t} & \dots & 0 & p_0 \cdot p(\mathbf{X}_{t+1}) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & (1 - p_0)p_{t,t} & p_0 \cdot p(\mathbf{X}_{t+1}) \end{bmatrix} \quad (10)$$

The matrix in Eq. (10) can be considered as a posterior transition matrix. It is the only term involving the newest observation X_{t+1} , and thus is essential in updating the posterior PMF of the latest change-point. From Eq. (10) we can see that if Eq. (8), or equivalently the posterior transition matrix, is tractable, the posterior PMF can be recursively calculated based on the posterior PMF obtained at the previous time step.

Therefore the calculation of Eq. (8) is critical for sequential change-point detection.

To get the analytical form of $p(X_{t+1}|\mathbf{X}_{1:t}, \tau_{t+1} = j)$, we first derive the posterior $p(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t|\mathbf{X}_{1:t}, \tau_t)$, which is provided in Theorem 1.

Theorem 1 Suppose the joint prior for $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$ is specified as $\boldsymbol{\Sigma}_t \sim \text{InvWish}_p(\boldsymbol{\Psi}_0, \nu_0)$ and $\boldsymbol{\mu}_t|\boldsymbol{\Sigma}_t \sim N(\boldsymbol{\mu}_0, \frac{1}{\gamma_0}\boldsymbol{\Sigma}_t)$, then the posterior $p(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t|\mathbf{X}_{1:t}, \tau_t)$ can be derived as

$$\begin{aligned} (\boldsymbol{\Sigma}_t|\mathbf{X}_{1:t}, \tau_t) &\sim \text{InvWish}_p(\boldsymbol{\Psi}_{\tau_t+1,t}^*, \nu_{\tau_t+1,t}^*) \\ (\boldsymbol{\mu}_t|\boldsymbol{\Sigma}_t, \mathbf{X}_{1:t}, \tau_t) &\sim N(\boldsymbol{\mu}_{\tau_t+1,t}^*, \boldsymbol{\Sigma}_t/\gamma_{\tau_t+1,t}^*) \end{aligned} \quad (11)$$

where

$$\begin{aligned} \nu_{\tau_t+1,t}^* &= (t - \tau_t) + \nu_0, \quad \gamma_{\tau_t+1,t}^* = \gamma_0 + (t - \tau_t), \\ \boldsymbol{\mu}_{\tau_t+1,t}^* &= \frac{(t - \tau_t)\bar{\mathbf{X}}_{\tau_t+1,t} + \gamma_0\boldsymbol{\mu}_0}{\gamma_0 + (t - \tau_t)}, \\ \boldsymbol{\Psi}_{\tau_t+1,t}^* &= \boldsymbol{\Psi}_0 + (t - \tau_t)S_{\tau_t+1,t} \\ &+ \frac{(t - \tau_t)\gamma_0}{(t - \tau_t) + \gamma_0} (\bar{\mathbf{X}}_{\tau_t+1,t} - \boldsymbol{\mu}_0)(\bar{\mathbf{X}}_{\tau_t+1,t} - \boldsymbol{\mu}_0)' \end{aligned} \quad (12)$$

Here $\bar{\mathbf{X}}_{\tau_t+1,t}$ and $S_{\tau_t+1,t}$ are the mean and variance of the observations $\mathbf{X}_{\tau_t+1:t}$ calculated as

$$\begin{aligned} \bar{\mathbf{X}}_{\tau_t+1,t} &= \frac{1}{t - \tau_t} \sum_{i=\tau_t+1}^t \mathbf{X}_i, \\ S_{\tau_t+1,t} &= \frac{1}{t - \tau_t} \sum_{i=\tau_t+1}^t (\mathbf{X}_i - \bar{\mathbf{X}}_{\tau_t+1,t})(\mathbf{X}_i - \bar{\mathbf{X}}_{\tau_t+1,t})' \end{aligned}$$

The proof is given in Appendix A. Based on Theorem 1, the predictive density $p(\mathbf{X}_{t+1}|\mathbf{X}_{1:t}, \tau_{t+1} = j)$ can be derived as follows.

Theorem 2 For $\tau_{t+1} < t$

$$(\mathbf{X}_{t+1}|\mathbf{X}_{1:t}, \tau_{t+1}) \sim t\left(d_{\tau_{t+1}+1,t}^*, \boldsymbol{\mu}_{\tau_{t+1}+1,t}^*, \frac{(\gamma_{\tau_{t+1}+1,t}^* + 1)\boldsymbol{\Psi}_{\tau_{t+1}+1,t}^*}{\gamma_{\tau_{t+1}+1,t}^* d_{\tau_{t+1}+1,t}^*}\right). \quad (13)$$

For $\tau_{t+1} = t$

$$(\mathbf{X}_{t+1}|\mathbf{X}_{1:t}, \tau_{t+1} = t) \sim t\left(\nu_0 - p + 1, \boldsymbol{\mu}_0, \frac{(\gamma_0 + 1)\boldsymbol{\Psi}_0}{\gamma_0(\nu_0 - p + 1)}\right), \quad (14)$$

where $d_{\tau_{t+1}+1,t}^* = \nu_{\tau_{t+1}+1,t}^* - p + 1$ is the degree of freedom, and the other two arguments are the mean and shape matrix of the p -dimensional multivariate t distribution, respectively. The proof is provided in Appendix B.

After the posterior PMF of the latest change-point is updated, the distribution of the duration of the current segment can be easily obtained to test if the multivariate signal is steady. Specifically, we define a probability index P_t , which is the posterior probability of the current segment being longer than a threshold L_0 :

$$P_t = P(t - \tau_t \geq L_0|\mathbf{X}_{1:t}) = P(\tau_t \leq t - L_0|\mathbf{X}_{1:t}) = \sum_{i=0}^{t-L_0} P_i^{(t)} \quad (15)$$

Once P_t is larger than a threshold α , the signal is claimed to be steady. It is intuitive that L_0 directly influences the detection timeliness and false alarm rate or misclassification rate, i.e., the probability of signaling a steady state alarm in transient state. It may be determined based on engineering knowledge or process requirement. However, in most of the practical applications, the steady state needs to be detected as early as possible, yet without causing the false alarm rate to exceed a certain level. Therefore L_0 can be treated as a tuning parameter to make trade-off between the detection delay and false alarm rate. For α , since the probability index often increases rapidly to a value close to 1 (see Section IV.A for details), we simply set it to 0.9 and do not treat it as a tuning parameter.

C. Controlling the Computational Cost

From Section II we know that the posterior distribution of the latest change-point can be calculated analytically. However, the computational and memory cost of each time step increase almost linearly with time t , as can be seen from Eq. (10). At time t , we need to calculate the posterior PMF $P(\tau_t = j | \mathbf{X}_{1:t})$ at t positions, i.e., $j = 0, 1, \dots, t - 1$. For a long multivariate signal, the computational cost may become very prohibitive for online applications and thus needs to be controlled.

As observed in applications, the posterior PMF often concentrates around a small region and is almost zero at all other positions, especially those far before the latest change-point. Therefore, a natural way to control the computational cost is to approximate the posterior PMF using a fixed-support-size strategy, where a fixed number of positions with high probabilities are selected to calculate the posterior PMF and set the posterior PMF at other locations to zero. Specifically, suppose the support size is m , then at time $t \geq m + 1$, $P(\tau_t = j | \mathbf{X}_{1:t})$ is calculated at the m positions selected at the previous time step along with the position $j = t - 1$. Therefore there are in total $m + 1$ positions to update the posterior PMF $P(\tau_t = j | \mathbf{X}_{1:t})$. After the $m + 1$ probabilities are calculated, we randomly select m positions using weighted sampling without replacement to approximate $P(\tau_t = j | \mathbf{X}_{1:t})$. The weight for each location in the random sampling is simply its posterior PMF. Note that from Eq. (9) we can see that if $P(\tau_t = j | \mathbf{X}_{1:t}) = 0$, then $P(\tau_{t+1} = j | \mathbf{X}_{1:t+1}) = 0$, therefore we only need to calculate the posterior PMF at the m positions selected at the previous time step and the new position $j = t - 1$. Using this fixed-support-size strategy, the computational cost can be effectively controlled and balanced without influencing much of the detection accuracy.

D. Hyperparameter Selection

The choice of hyperparameters is often crucial in Bayesian data analysis when the sample size is limited. In our online application, the change-point needs to be detected in a timely manner, e.g., detecting the occurrence of a new change-point with only a few observations in the new segment, yet without resulting in overfitting or excessive change-points. Therefore, the hyperparameters need to be selected appropriately. In Bayesian inference, if a sufficient amount of historical data is available, informative priors are more preferable and could be

estimated through these data. However, in many applications, historical data is very limited. Besides, in our case, to simplify the problem, we assume that all segments of different characteristics (in terms of duration, noise, amplitude) are independent and share the same hyperparameters. As a result, it may be unrealistic to obtain a set of hyperparameters that is informative for all segments. In this section, we provide some guidelines and heuristics for hyperparameter selection.

Recall that the priors are $\Sigma_t \sim \text{InvWish}_p(\Psi_0, v_0)$, $\mu_t | \Sigma_t \sim N(\mu_0, \Sigma_t / \gamma_0)$, and the prior transition probability given in Eq. (3). Therefore the hyperparameters include p_0 , v_0 , Ψ_0 , γ_0 and μ_0 . Similar to the proof of Theorem 1 and 2, we can get the prior distribution of μ_t by integrating out Σ_t as

$$\mu_t \sim t(v_0 - p + 1, \mu_0, \Psi_0 / [(v_0 - p + 1)\gamma_0]). \quad (16)$$

As observed in the numerical and real case studies, the detection results are not sensitive to the prior transition probability p_0 . Any values in the interval [0.05, 0.2] works quite well. For the covariance prior $\text{InvWish}_p(\Psi_0, v_0)$, the mean value is $E(\Sigma_t) = \Psi_0 / (v_0 - p - 1)$ for $v_0 > p + 1$. Based on the mean and covariance of Σ_t [27], we can see that v_0 directly controls the noise level. The larger the value is, the smaller the noise level of the prior, and thus the more sensitive the change-point detection will be or the more change-points it will result in. In other words, if the prior noise level is much larger than the actual one, the algorithm may not be able to detect the mean-shift timely, as the shift is masked by the large noise specified by the prior. On the other hand, if the prior noise level is too low, an overfitting issue may occur, i.e., too many change-points are produced. To select Ψ_0 and v_0 appropriately, several scenarios are considered:

(1) The noise covariance is constant in the whole process and some historical data is available. We could calculate the sample covariance matrix \mathbf{S} using the steady state data, and then select a very large v_0 and set $\Psi_0 = v_0 \mathbf{S}$. Based on Theorem 1, as $v_0 \rightarrow \infty$, $(\Sigma_t | \mathbf{X}_{1:t}, \tau_t) \rightarrow \mathbf{S}$. Therefore, the problem degenerates to a piecewise constant model with fixed covariance Σ_t (only mean-shift), which could significantly reduce the uncertainty and thus improve the detection accuracy.

(2) No prior information is available but the noise level is roughly known (e.g., within certain range). In this case, for simplicity, we can roughly set $\Psi_0 = \mathbf{I}_p$ and select v_0 accordingly to match the noise level.

(3) The prior information is not available, yet we want our algorithm to be robust enough to handle signals with significantly different noise levels. In such case, selecting an appropriate set of hyperparameters is not easy. Besides, using a single set of hyperparameters may be too restrictive and cannot handle all signals. To solve this problem, we propose to use an adaptive prior approach, where the covariance prior is dynamically updated or learned from the data in the monitoring process. More specifically, based on Eq. (11) we can get

$$\begin{aligned}
 E(\boldsymbol{\Sigma}_t | \mathbf{X}_{1:t}) &= \sum_{j=0}^{t-1} E(\boldsymbol{\Sigma}_t | \mathbf{X}_{1:t}, \tau_t = j) P(\tau_t = j | \mathbf{X}_{1:t}) \\
 &= \sum_{j=0}^{t-1} \frac{\boldsymbol{\Psi}_{\tau_t+1,t}^*}{v_{\tau_t+1,t}^* - p + 1} P(\tau_t = j | \mathbf{X}_{1:t}).
 \end{aligned} \quad (17)$$

At time step $t + 1$, we set $\boldsymbol{\Sigma}_{t+1} \sim \text{InvWish}_p(\boldsymbol{\Psi}_0^{(t+1)}, v_0)$ where

$$\boldsymbol{\Psi}_0^{(t+1)} = (v_0 - p - 1)E(\boldsymbol{\Sigma}_t | \mathbf{X}_{1:t}). \quad (18)$$

It is easy to show that $E(\boldsymbol{\Sigma}_{t+1}) = E(\boldsymbol{\Sigma}_t | \mathbf{X}_{1:t})$, which is often more informative than arbitrarily specified priors. This strategy is particularly effective for signals with only mean shift.

For the mean prior $\boldsymbol{\mu}_t | \boldsymbol{\Sigma}_t \sim N(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_t / \gamma_0)$ or Eq. (16), since different segments may have different means along the multivariate trajectories, a non-informative prior or a “flat” prior is recommended to reduce the influence of priors and let the data “speak” for themselves. To make the prior non-informative, we could roughly select a $\boldsymbol{\mu}_0$ (e.g., 0) based on the order of the signal magnitude and then select a very small positive value for γ_0 . From Eq. (11) we can see that as $\gamma_0 \rightarrow 0$, $\boldsymbol{\mu}_{\tau_t+1,t}^* \rightarrow \bar{\mathbf{X}}_{\tau_t+1,t}$ and $(\boldsymbol{\mu}_t | \boldsymbol{\Sigma}_t, \mathbf{X}_{1:t}, \tau_t) \sim N(\bar{\mathbf{X}}_{\tau_t+1,t}, \boldsymbol{\Sigma}_t / (t - \tau_t))$, which does not involve $\boldsymbol{\mu}_0$.

IV. NUMERICAL STUDIES FOR ILLUSTRATION AND COMPARISON

In this section, we use numerical studies to illustrate the sequential change-point and steady-state detection process, and compare our method with several existing approaches. To simulate signals with initial bias in the comparison, we use four types of bias functions as signal means, namely, the linear, quadratic, exponential and oscillating functions, which are commonly used in testing initial bias elimination heuristics [10]. The bias functions and their shapes are shown in TABLE I.

TABLE I
BIAS FUNCTIONS AND THEIR SHAPES

Bias Type	Function Form	Shape
Linear	$ x(i) = \begin{cases} \frac{i-1}{T_0} H, & i = 1, \dots, T_0 \\ H, & i = T_0 + 1, \dots, T \end{cases} $	
Quad.	$ x(i) = \begin{cases} H \left[1 - \frac{(i-T_0-1)^2}{T_0^2} \right], & i = 1, \dots, T_0 \\ H, & i = T_0 + 1, \dots, T \end{cases} $	
Exp.	$ x(i) = \begin{cases} H \left[1 - \frac{(i-T_0-1)^2}{T_0^2} \right], & i = 1, \dots, T_0 \\ H, & i = T_0 + 1, \dots, T \end{cases} $	
Osc.	$ x(i) = \begin{cases} \frac{T_0 - i + 1}{T_0} \sin\left(\frac{\pi i}{f}\right), & i = 1, \dots, T_0 \\ 0, & i = T_0 + 1, \dots, T \end{cases} $	

A. Illustration

To illustrate the detection process and also show its robustness, we use four types of signals with different characteristics in terms of the change of mean and covariance matrix: (1) continuous mean and constant covariance; (2) abrupt mean shift and constant covariance; (3) constant mean

and abrupt variance change; and (4) constant mean and abrupt correlation change, as shown in Fig. 2.

For display convenience, we only consider bivariate signals (i.e., $p = 2$) in the illustration. For the signal with continuous mean and constant covariance (Fig. 2a), the first dimension x_1 is an exponential signal while the second dimension x_2 is an oscillating signal. The signal parameter is set as $H = 1, T_0 = 200, f = 30$, and $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}_2$ where $\sigma = 0.1$. For the signal with abrupt mean shift and constant covariance (Fig. 2b), the covariance is $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}_2$, the mean for x_1 is set to 0, 0.8, 0.5 and 0.2 within the time intervals (0, 100), (100, 200), (200, 300) and (300, 400), respectively, and the mean for x_2 is set to 0.2, 0.5 and 0 within (0, 150), (150, 300) and (300, 400), respectively. For the signal with constant mean and abrupt variance change (Fig. 2c), $\boldsymbol{\mu} = \mathbf{0}$ and $\boldsymbol{\Sigma} = \text{diag}(\sigma_1^2, \sigma_2^2)$, where $\sigma_1 = 0.3, 1$ and 0.3 in the time intervals (0, 100), (100, 300) and (300, 400) respectively, and $\sigma_2 = 0.3, 1$ and 0.3 in the time intervals (0, 150), (150, 300) and (300, 400) respectively. For the case with constant mean and abrupt correlation change (Fig. 2d), $\boldsymbol{\mu} = \mathbf{0}$, and $\boldsymbol{\Sigma} = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, where $\sigma^2 = 1, \rho = 0.9$ for $t \in (0, 200)$ and $\rho = 0.6$ for $t \in (200, 400)$. The hyperparameters are set to $\boldsymbol{\mu}_0 = \mathbf{0}, \boldsymbol{\Psi}_0 = \mathbf{I}_2, p_0 = 0.1, \gamma_0 = 0.01$ for all cases. For the hyperparameter v_0 , which is critical to control the prior noise level, we set it to 100 for Fig. 2 (a) and (b), and 2 for Fig. 2 (c) and (d). The duration threshold $L_0 = 30$ and probability threshold $\alpha = 0.9$.

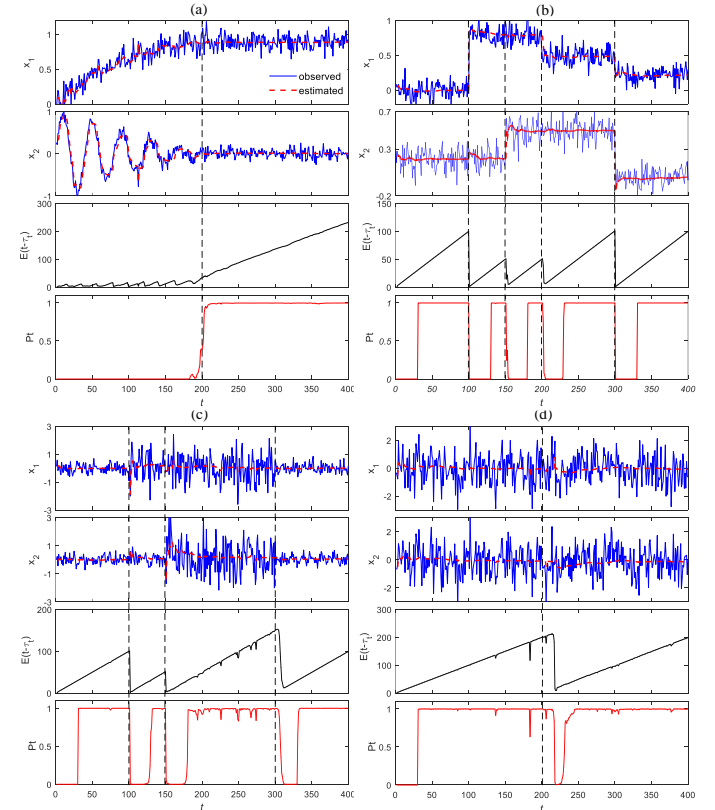


Fig. 2 Illustration of the sequential change-point detection and steady state detection process. (a) continuous mean and constant covariance; (b) abrupt mean shift and constant covariance; (c) constant mean and abrupt variance change; and (d) constant mean and abrupt correlation change.

In Fig. 2, the vertical dashed lines denote the true steady state time. The dashed lines among the observations are the estimated means $E(\boldsymbol{\mu}_t|\mathbf{X}_{1:t})$ for the posterior distribution $\boldsymbol{\mu}_t|\mathbf{X}_{1:t}$. Similar to Eq. (16), we can prove that

$$(\boldsymbol{\mu}_t|\mathbf{X}_{1:t}, \tau_t) \sim t \left(d_{\tau_t+1,t}^*, \boldsymbol{\mu}_{\tau_t+1,t}^*, \frac{\boldsymbol{\Psi}_{\tau_t+1,t}}{\gamma_{\tau_t+1,t}^* d_{\tau_t+1,t}^*} \right).$$

Therefore

$$E(\boldsymbol{\mu}_t|\mathbf{X}_{1:t}) = \sum_{\tau_t=0}^{\tau_t=t-1} \boldsymbol{\mu}_{\tau_t+1,t}^* P(\tau_t|\mathbf{X}_{1:t}).$$

Clearly, the estimated means are very close to true values, indicating that the proposed method can effectively fit the signal sequentially through Bayesian inference. The third row of each subfigure is the expected duration of the current segment, i.e., $E(t - \tau_t)$, which is used to demonstrate the change-point detection. The sharp decrease of the duration indicates a newly detected change-point. As we can see, the detection is very accurate and timely. The last row of each subfigure is the probability index for steady state detection.

Recall that the probability index is defined as the probability of the duration of current segment larger than the threshold L_0 . We can see that the index often increases rapidly from a near-zero value to a value close to 1. Therefore, we simply select the probability threshold $\alpha = 0.9$ and do not treat it as a tuning parameter.

To show the effectiveness of the adaptive prior with dynamic updating strategy, we choose a signal with the same parameters as Fig. 2 (a). As shown in Fig. 3, three cases are considered: (a) a covariance prior with an appropriate noise level and without dynamic updating; (b) a covariance prior with an excessively large noise level, and without dynamic updating; and (c) a covariance prior with an excessively large noise level, but with dynamic updating. From Fig. 3 (b) we can clearly see that if the prior noise level is too high, the algorithm is not able to timely detect the change-points, resulting in poor model fitting and steady state detection. However, as shown in Fig. 3 (c), if we use dynamic updating strategy to “correct” the prior, the model fitting and steady state detection become much more accurate, even if the initial prior is specified inappropriately.

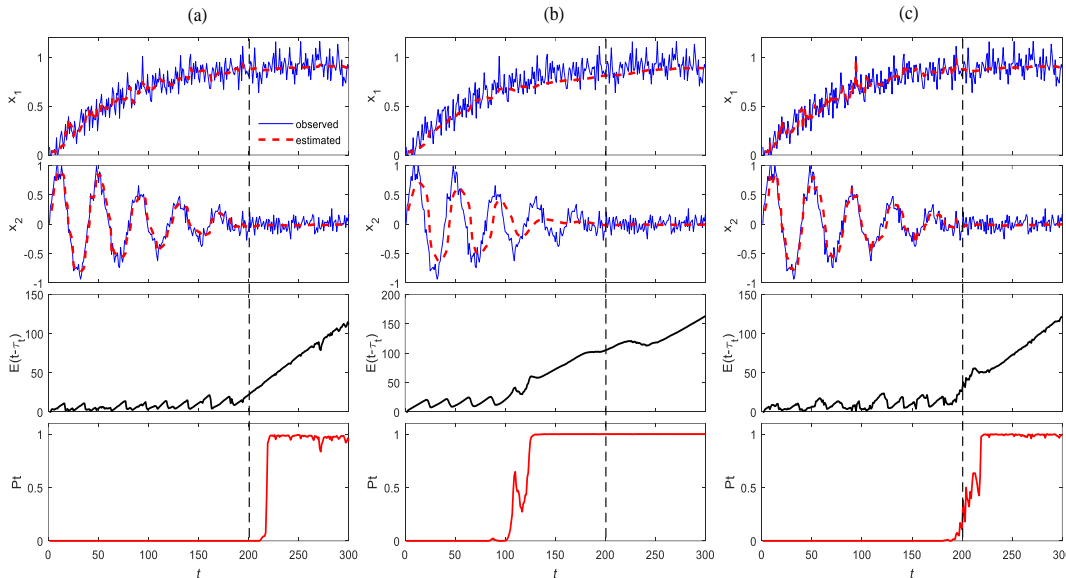


Fig. 3 The performance of adaptive prior in steady-state detection process: (a) $\boldsymbol{\Psi}_0 = \mathbf{I}_2, v_0 = 100$, no dynamic updating; (b) $\boldsymbol{\Psi}_0 = \mathbf{I}_2, v_0 = 4$, no dynamic updating; and (c) $\boldsymbol{\Psi}_0 = \mathbf{I}_2, v_0 = 4$, dynamic updating.

B. Performance Comparison with Other Methods

In this subsection the performance of the proposed method is evaluated and compared with existing methods. In statistical process control area, two types of performance measures are often used, the α -error and the β -error (or detection delay). Usually, the α -error is specified at a desired level (e.g., 0.05) and the corresponding β -error is used as an evaluation metric to compare different control charts. However, this comparison scheme is not appropriate for steady state detection, as the α -error does not make any sense for non-i.i.d. samples in the transient period. Instead, another evaluation metric, namely, the false alarm rate (FAR) may be used, which is defined in our case as the probability of signaling a steady-state alarm in the

transient period. Nevertheless, this metric still has shortcoming, in that it does not capture the closeness of the false alarm time to the true steady state time. In fact, the closeness of the alarm time in the transient period to the true steady state is very important as it directly reflects the amount of initial bias undetected or the remaining time needed to reach steady state. Naturally, we could use the closeness measure to evaluate the performance. Considering the fact that the detection delay is often better than false alarm with the same closeness, we use another metric, the weighted standard error (WSE) [8], defined as

$$WSE = \sqrt{\frac{1}{N} \sum_{i=1}^N w(\hat{T}_i) (\hat{T}_i - T_{i0})^2}, \quad (19)$$

where \hat{T}_i is the detected time, T_{i0} is the true steady state time, N is the total number of multivariate signals, and $w(\cdot)$ is the penalty weight ratio of detection delay over false alarm given as

$$w(\hat{T}_i) = \begin{cases} w \in (0,1], & \text{if } \hat{T}_i \geq T_{i0} \\ 1, & \text{otherwise} \end{cases} \quad (20)$$

Note that if $w = 1$, only the closeness is considered in performance assessment. In the comparison, $p = 4$ are selected and each dimension is simulated by a bias function, which is randomly selected from TABLE I to cover various initial bias. To further diversify the initial bias severity, different H , T_0 and noise levels are specified. Specifically, $H = 1$ and 2 , $T_0 = 200, 300$. The length of the signal is set as $T = 500$. To test the robustness of the algorithm under different noise types, four scenarios are considered: (1) no auto-correlation and no correlation among variables, denoted by AR(0); (2) first-order auto-correlation and no correlation, denoted by AR(1); (3) second-order auto-correlation and no correlation, denoted by AR(2); and (4) no auto-correlation and with correlation among variables, denoted by CR. The noise types and their parameters are shown in TABLE II.

TABLE II
FOUR NOISE TYPES

Type	Equation	Parameter
AR(0)	$\psi_t = \epsilon_t$	$\epsilon_t \sim N(0, \sigma^2 I_p)$
AR(1)	$\psi_t = \phi_1 \psi_{t-1} + \epsilon_t$	$\epsilon_t \sim N(0, \sigma^2 I_p)$, $\phi_1 = 0.4$
AR(2)	$\psi_t = \phi_2 \psi_{t-1} + \phi_3 \psi_{t-2} + \epsilon_t$	$\epsilon_t \sim N(0, \sigma^2 I_p)$, $\phi_2 = -0.25, \phi_3 = 0.5$
CR	$\psi_t = \epsilon_t$	$\epsilon_t \sim N(0, \sigma^2 C_r)$

For each type of signal, three noise levels are considered. For AR(0) and CR, $\sigma = 0.06, 0.1, 0.14$. For CR, the correlation matrix C_r is randomly generated through vines method [28]. In the simulation, each signal is replicated 100 times, so that a total of 1200 signals ($2H \times 2T_0 \times 3\sigma \times 100$) are generated for each of the four noise types.

The proposed method (SBP) is compared with three existing methods. The first method is the exponentially weighted moving average based variance ratio test (VRT) [16], where each dimension is monitored separately using the well-known method by Cao [6], and the steady state is claimed once all dimensions reach steady state. The second method is the SSD algorithm [29], which employs a moving window and tests if there is any non-stationary drift within that window. The third one is a wavelet transform (WT) based method [18], where the status index for each dimension is combined using the Dempster's combination rule to form a global detection index.

The hyperparameters for SBP are set and fixed as $\mu_0 = 0$, $\Psi_0 = I_2$, $v_0 = 100$, $p_0 = 0.1$, $\gamma_0 = 0.01$ for all cases. The duration threshold L_0 is selected by optimizing the overall WSE under each noise type and weight w . For all the other three methods, the detection parameters are chosen by optimizing the overall WSE under each noise type and weight w . Note that in practical applications, the true steady state times of the training data may be unknown, or there may even not be sufficient training data. For the former case, some offline method could be used as a benchmark to estimate the steady state times, and then the estimated values can be used to evaluate WSE. For the latter case, we could use Monte Carlo simulation to generate a training database covering various initial bias conditions of different characteristics, e.g., noise level and changing rate to select an optimal L_0 .

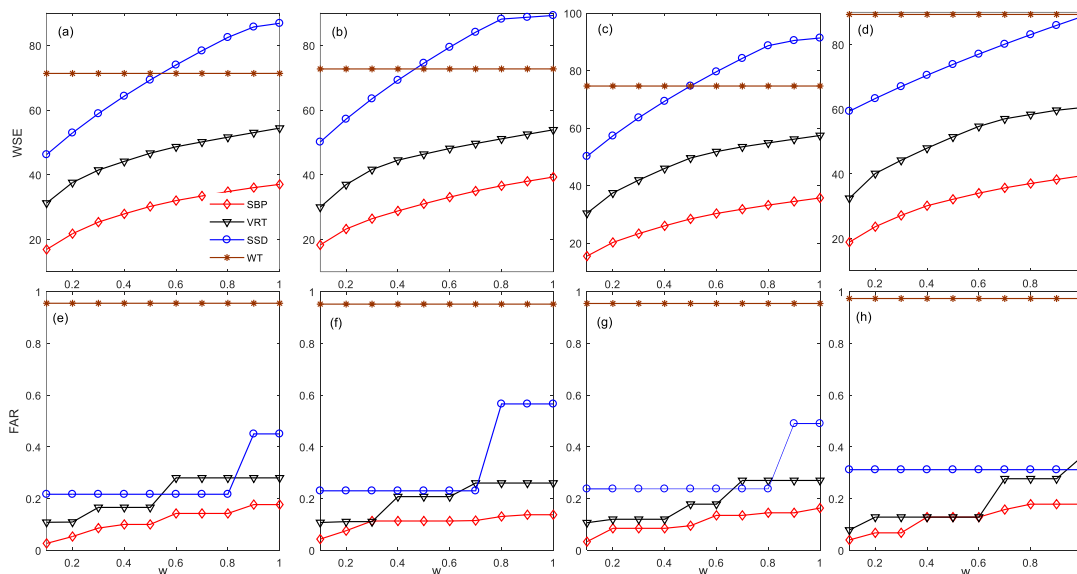


Fig. 4 The WSE and FAR of SBP, VAR, SSD and WT as functions of penalty weight ratio for different noise types: (a-e) AR(0), (b-f) AR(1), (c-g) AR(2) and (d-h) CR.

Fig. 4 shows the WSE and FAR of the four detection methods as functions of w under different noise types. It is worth noting that here the FAR is used only as an auxiliary

metric to show the detection details. Clearly, the proposed SBP outperforms VRT, SSD and WT methods significantly in terms of WSE. The FAR of SBP is also much lower than other three

methods, indicating that if we reduce the FAR of all other methods to the same level as SBP, the WSE will become worse. For WT, the WSE does not change when w varies. The reason is that FAR is above 0.9 for all cases. Based on Eq. (20), the WSE will not change much when w varies. Note that the hyperparameters of the SBP are selected using only several trials under the guidelines of the hyperparameter selection in Section III.D. The performance could be further improved if these parameters are optimized.

TABLE III

COMPARISON OF SBP, VRT, SSD AND WT FOR $w = 1$. THE DETECTION PARAMETERS ARE (1) SBP, $L_0 = 50$; (2) VRT, $\lambda_1 = 0.05, \lambda_2 = \lambda_3 = 0.2$, THRESHOLD=0.8; (3) SSD, WINDOW SIZE=24 AND (4) WT, $\Delta t = 20$.

Signal		WSE($w = 1$)				FAR				
H	T_0	SBP	VRT	SSD	WT	SBP	VRT	SSD	WT	
1	0.06	27.3	47.8	66.3	38.2	0.03	0.01	0.24	0.91	
	200	0.10	28.1	30.0	65.8	49.3	0.14	0.35	0.55	0.91
		0.14	26.6	35.1	70.1	59.8	0.15	0.69	0.68	0.94
	0.06	39.2	36.0	83.0	73.8	0.42	0.17	0.57	0.97	
	300	0.10	35.9	74.8	127.5	105.9	0.37	0.75	0.78	1.00
		0.14	69.7	97.9	155.4	135.3	0.64	0.78	0.84	1.00
2	0.06	36.3	66.8	60.4	31.6	0	0.00	0.00	0.97	
	200	0.10	36.9	52.4	63.3	36.6	0	0.00	0.11	0.92
		0.14	32.6	39.6	57.3	39.0	0.02	0.02	0.29	0.94
	0.06	28.3	51.8	70.2	61.1	0.1	0.00	0.13	0.99	
	300	0.10	32.9	40.0	73.2	72.0	0.05	0.12	0.51	0.94
		0.14	29.8	39.8	91.2	76.6	0.2	0.47	0.70	0.97
Overall		37.0	54.4	86.8	71.4	0.18	0.28	0.45	0.96	

TABLE III shows the detailed detection for each type of signal with noise AR(0) and penalty weight ratio $w \equiv 1$ (only

consider closeness). Due to space limitation, the detailed results for other noise types and penalty weight ratios are not provided here. We can see that SBP is much more robust in handling signals of various noise levels and initial bias severity. Bear in mind that SSD and WT are moving window based methods, and thus are not robust. Too long a moving window may delay the detection while too short a moving window may result in large FAR. As shown in TABLE III, all the three methods could not uniformly perform well across all types of signals. The proposed SBP method incorporate the sequential Bayesian inference scheme and thus could online “learn” the monitoring signal, which could significantly improve its robustness.

V. REAL CASE STUDIES

In this section we apply the proposed method to two real cases to demonstrate its effectiveness: the Tennessee Eastman (TE) process [30] and a serial production line with perishable products [31].

A. Tennessee Eastman Process

The TE process is based on a simulation of a realistic chemical plant. It has been widely used as a benchmark process in the process monitoring community to test various fault detection, identification, diagnosis and closed-loop control methodologies [32]. As shown in Fig. 5, the process consists of five major units: a reactor, condenser, compressor, separator, and stripper, and it contains eight chemical components: A, B, C, D, E, F, G, and H, where A, C, D and E are reactants, B is inert gas, G and H are products and F is byproduct. For the detailed process description, please refer to [30] and [32].

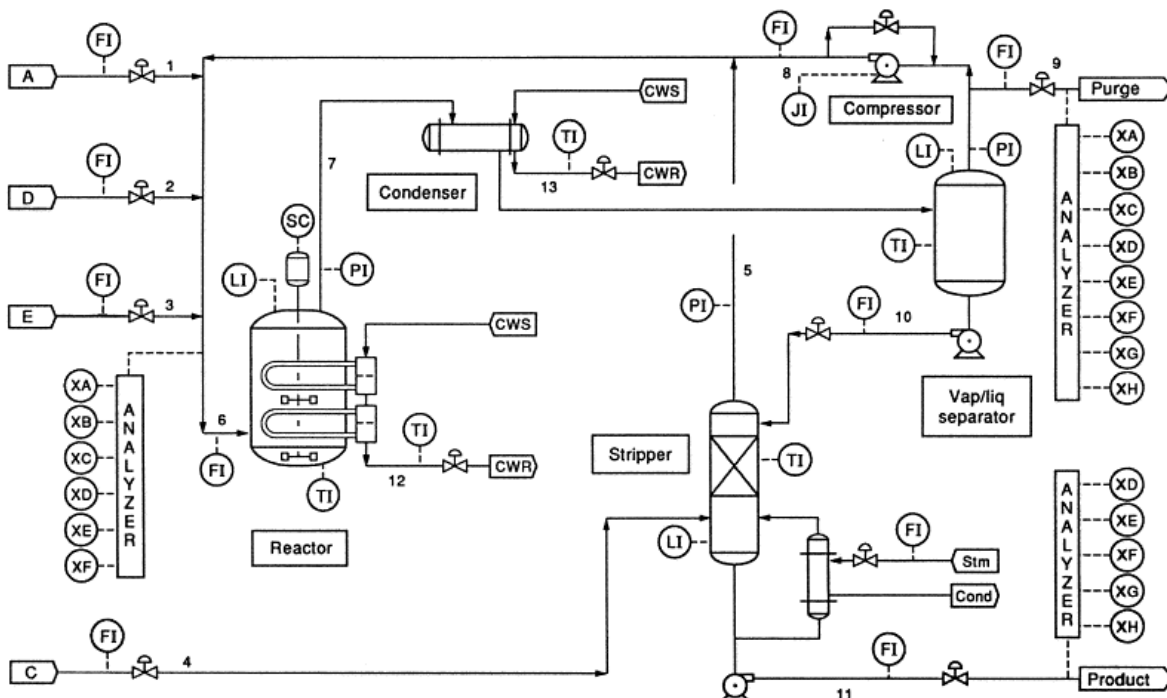


Fig. 5 The Tennessee Eastman process.

The process contains in total 53 measurement variables (see [30, 32] for details), out of which 41 are process variables, i.e., XMEAS(1) to XMEAS(41), and 12 manipulated variables, i.e., XMV(1) to XMV(12). A total of 21 process faults are preprogrammed, i.e., IDV(1) to IDV(21), including A/C feed ratio step change, B composition step change, D feed temperature step change, etc. Here we only consider Fault 1. When Fault 1 occurs at time step 160 (8 hours), a step change is induced for the A/C feed ratio, which results in a decrease in A feed in Stream 5 and control loop reacts to increase the A feed in Stream 1. After a certain amount of time, the A feed becomes steady in Stream 6. Fig. 6 shows the dynamic change of A Feed in Stream 1 and Composition of A in Stream 6 once Fault 1 occurs.

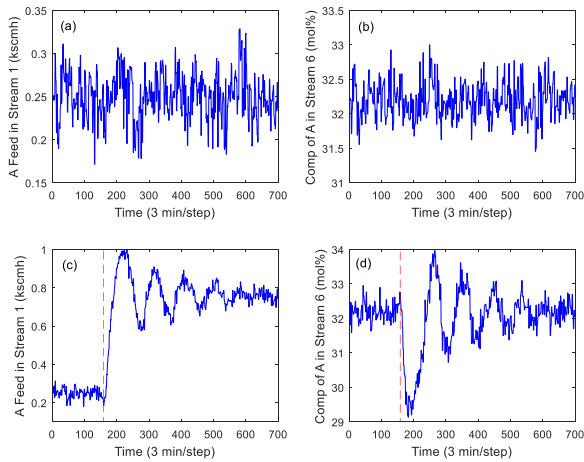


Fig. 6 A Feed in Stream 1 and Composition of A in Stream 6 under normal operating condition and Fault 1: (a) A Feed under normal operating condition (XMEAS(1)); (b) A composition under normal operating condition (XMEAS(23)); (c) A Feed under Fault 1 (XMEAS(1)) and (d) A composition under Fault 1 (XMEAS(23)). The vertical dashed lines denote the onset of Fault 1.

To monitor the steady state of the whole system, we utilize all the 41 measurement variables. Since the historical data of all these variables under the normal operating condition is available or can be easily generated, we use these data to roughly estimate the hyperparameter μ_0 and Ψ_0 . Suppose the mean and sample covariance of these normal data are \bar{X} and S respectively. Then we select $\mu_0 = \bar{X}$, $v_0 = 1 \times 10^6$, and $\Psi_0 = v_0 S$. Other parameters are set as $p = 0.1$, $\gamma_0 = 1 \times 10^{-4}$ and $L_0 = 60$.

Fig. 7 shows the signal segmentation and steady state detection results. Note that for space limitation, here we only show the first 6 process variables, from XMEAS(1) to XMEAS(6). Fig. 7 (g) is the histogram of the simulated change-points. They are simulated in this way: (1) randomly draw a sample τ_T from $P(\tau_T|X_{1:T})$ and then randomly draw τ_{TT} from $P(\tau_{TT}|X_{1:\tau_T})$, continue this process until we reach the beginning of the signal; (2) repeat the whole process 1000 times.

As we can see, the proposed method can accurately detect the onset of Fault 1 and can effectively partition the whole

multivariate system into transient period and steady state period. The detected onset time of Fault 1 is 166, which is very close to the true onset time 160. Since we know the true onset time, we use it to compare the detection accuracy between the proposed method and SSD, VT and WT methods. The optimal detection parameters that minimize the detection error are (1) VRT: $\lambda_1 = 0.1, \lambda_2 = \lambda_3 = 0.2$, Threshold = 1.7; (2) SSD: window size = 15; and (3) WT: $\Delta t = 7$. The detection results are 139, 135, and 237 for VRT, SSD and WT respectively. We can see that the proposed SBP method is much more accurate. It is worth noting that in the other three methods, we need to build up to 41 monitoring charts, which is very time consuming, let alone the detection accuracy after fusing all detection results.

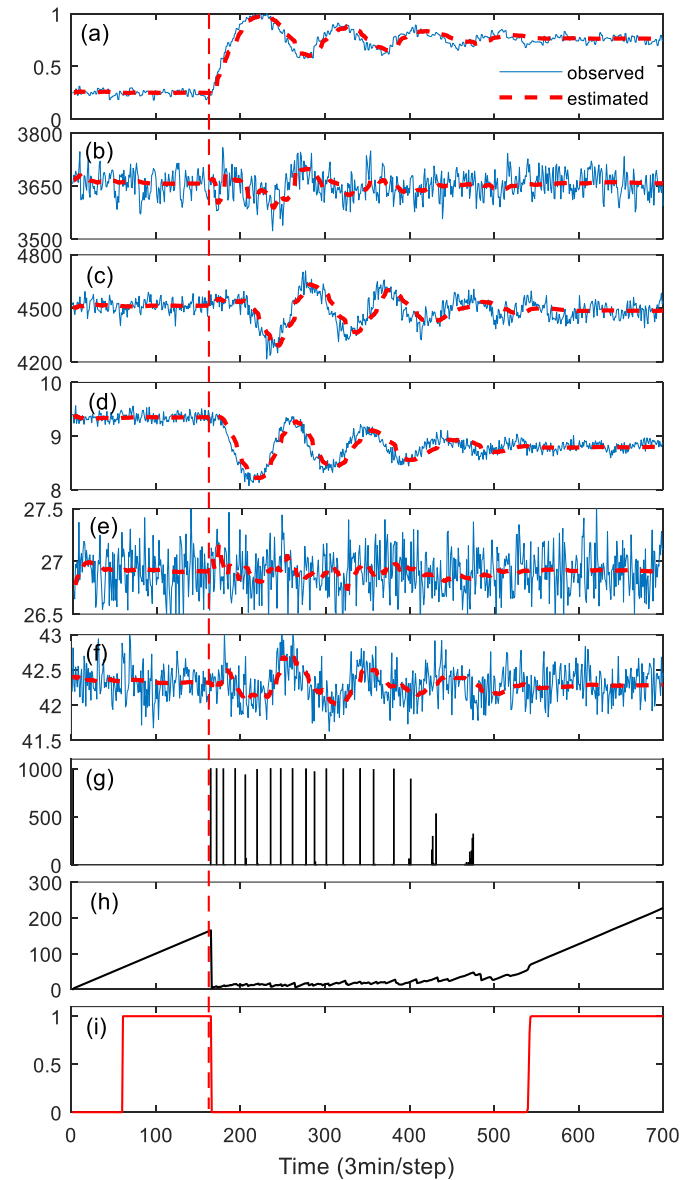


Fig. 7 Steady state detection of the TE process under Fault 1 with 41 measurements (only the first 6 measures are shown here). (a)-(f): XMEAS(1) to XMEAS(6); (g) the histogram of change-points; (h): the mean duration of the current segment $E(t - \tau_t)$; and (i): the probability index P_t . The vertical dashed lines denote the onset of Fault 1.

B. Serial Production Lines with Perishable Products

The perishable products refer to those having maximum allowable waiting time, exceeding which the item will be scrapped due to quality deterioration. Typical examples including yogurt and battery production, etc. [31]. Due to dynamic changes and frequent disruptions in the manufacturing process, and product perishability, the production system often operates partially or even entirely in the transient regime. After the production system is initiated, it needs some time (warm-up period) for the production to reach steady state, e.g., production rate, scrap rate, etc. To facilitate process monitoring and real time control policy optimization, it is essential to detect when the production system reaches steady state.

Suppose there is a serial production with two Bernoulli reliability machines, m_1 and m_2 , a finite buffer B_1 and perishable products, as shown in Fig. 8.

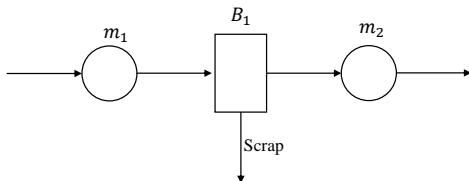


Fig. 8 Bernoulli line with perishable products

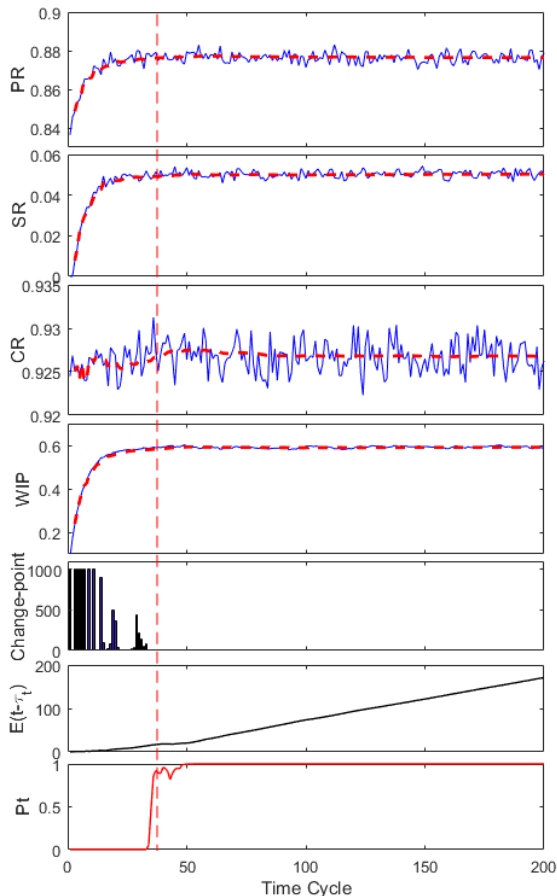


Fig. 9 Steady state detection of the Bernoulli line of two machines. The bottom three subfigures are the histogram of the change-points, the mean duration of the current segment, and the probability index P_t , respectively. The vertical dashed line denotes the detected steady state time.

The performance measures that are of interest and are used to describe the system state include: (1) the production rate $PR(t)$, which is the average number of parts produced by machine m_2 in the t^{th} cycle; (2) the consumption rate $CR(t)$, which is the average number of parts consumed by machine m_1 in the t^{th} cycle; (3) scrap rate $SR(t)$, i.e., the expected number of scrapped parts in the t^{th} cycle; and (4) the work-in-process $WIP(t)$, which is the average number of parts in buffer B_1 at the end of the t^{th} cycle. For the detailed description of the dataset, please refer to [31, 33].

The four-dimensional signal and detection results are shown in Fig. 9. All the other detection parameters are set using the same way as Fig. 7, except that $\mu_0 = 0$ and $L_0 = 20$ are set here. To evaluate the detection accuracy, we use an offline method, namely, the adaptive minimal confidence region rule (AMCR), as a benchmark. AMCR determines the steady state starting time by minimizing the confidence region of the mean estimate using all the observations since that time [33]. The detected steady state starting time using the proposed online method is 37, which is very close to the AMCR detected time 35. In comparison, the VRT, SSD and WT detected times are 38, 40 and 30 respectively. The corresponding optimal detection parameters are $\lambda_1 = \lambda_2 = \lambda_3 = 0.5$, Threshold = 1 for VRT, window size = 5 for SSD, and $\Delta t = 12$ for WT. We can see that the proposed SBP approach is still better than the other three methods. Note that the advantage of the proposed method is that it is much more robust in handling various signals of different characteristics using only one set of detection parameters. For one multivariate signal, the advantage may not be obvious, as other methods can always find a set of detection parameters that works well on that specific signal.

VI. CONCLUSION AND DISCUSSION

In this paper, an efficient online steady state detection method has been developed for multivariate systems through a sequential Bayesian partitioning approach. In this approach, multivariate signals are modeled by piecewise constant models, where the mean and covariance are constant in each segment, and then the duration of each segment is utilized to determine if the signal is steady. To facilitate online inference, a Bayesian formulation of the piecewise constant model is proposed. By using conjugate priors, it is found that the posterior distributions of the latest change-points can be calculated analytically through a recursive updating approach. Once the posterior probability of the duration larger than a predefined threshold, the signal is considered steady. To control and balance the computational cost, a fixed-support-size strategy is proposed to approximate the posterior probability mass function of the latest change-points. The role and sensitivity of hyperparameters are discussed and several guidelines are provided to help select hyperparameters appropriately. Thorough simulation and real case studies have demonstrated that the proposed method can timely detect the change-points and effectively partition the multivariate signal sequentially.

The comparison results show that the proposed method is much more accurate and robust than existing methods in tackling signals of various characteristics.

On the other hand, we need to point out that although our approach can handle signals with mild autoregressive noise, due to its Gaussian noise assumption, it may not be able to detect the steady state accurately when the autocorrelation is very severe. Besides, in the current work, we assume that all the dimensions of the multivariate signal reach steady state at the same time. In practice, however, some dimensions may reach steady state earlier than the other ones, which will influence the detection performance. We will leave these problems in our future investigation. Last but not least, with the recent advances in sensing and information technology, we are more and more faced with the problem of monitoring up to hundreds or even thousands of process variables. In such cases, the proposed method may face severe challenges, e.g., prohibitive computational cost for online application and ill-posed covariance inversion. To overcome these challenges, data fusion and distributed monitoring techniques may be needed, which will be investigated in near future.

APPENDIX A PROOF OF THEOREM 1

Based on the definition of change-point, $\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} = \dots = \boldsymbol{\mu}_{\tau_t+1}$, $\boldsymbol{\Sigma}_t = \boldsymbol{\Sigma}_{t-1} = \dots = \boldsymbol{\Sigma}_{\tau_t+1}$.

$$\begin{aligned} f(\boldsymbol{\mu}_t | \boldsymbol{\Sigma}_t, \mathbf{X}_{1:t}, \tau_t) &\propto f(\boldsymbol{\mu}_t | \boldsymbol{\Sigma}_t) f(\mathbf{X}_{\tau_t+1:t} | \boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t, \tau_t) \\ &\propto \exp \left[\frac{-\gamma_0 (\boldsymbol{\mu}_t - \boldsymbol{\mu}_0)' \boldsymbol{\Sigma}_t^{-1} (\boldsymbol{\mu}_t - \boldsymbol{\mu}_0)}{2} \right] \exp \left[-\frac{\sum_{i=\tau_t+1}^t (\mathbf{X}_i - \boldsymbol{\mu}_t)' \boldsymbol{\Sigma}_t^{-1} (\mathbf{X}_i - \boldsymbol{\mu}_t)}{2} \right] \\ &\propto \exp \left[\frac{-\gamma_{\tau_t+1,t}^* (\boldsymbol{\mu}_t - \boldsymbol{\mu}_{\tau_t+1,t}^*)' \boldsymbol{\Sigma}_t^{-1} (\boldsymbol{\mu}_t - \boldsymbol{\mu}_{\tau_t+1,t}^*)}{2} \right] \end{aligned}$$

where

$$\begin{aligned} \gamma_{\tau_t+1,t}^* &= \gamma_0 + (t - \tau_t) \\ \boldsymbol{\mu}_{\tau_t+1,t}^* &= \frac{(t - \tau_t) \bar{\mathbf{X}}_{\tau_t+1,t} + \gamma_0 \boldsymbol{\mu}_0}{\gamma_0 + (t - \tau_t)} \end{aligned}$$

Therefore

$$(\boldsymbol{\mu}_t | \boldsymbol{\Sigma}_t, \mathbf{X}_{1:t}, \tau_t) \sim N \left(\boldsymbol{\mu}_{\tau_t+1,t}^*, \frac{\boldsymbol{\Sigma}_t}{\gamma_{\tau_t+1,t}^*} \right)$$

For $(\boldsymbol{\Sigma}_t | \mathbf{X}_{1:t}, \tau_t)$,

$$\begin{aligned} f(\boldsymbol{\Sigma}_t | \mathbf{X}_{1:t}, \tau_t) &= \int f(\boldsymbol{\Sigma}_t, \boldsymbol{\mu}_t | \mathbf{X}_{\tau_t+1:t}, \tau_t) d\boldsymbol{\mu}_t \\ &\propto \int f(\boldsymbol{\Sigma}_t, \boldsymbol{\mu}_t) f(\mathbf{X}_{\tau_t+1:t} | \boldsymbol{\Sigma}_t, \boldsymbol{\mu}_t, \tau_t) d\boldsymbol{\mu}_t \end{aligned}$$

where

$$\begin{aligned} f(\boldsymbol{\Sigma}_t, \boldsymbol{\mu}_t) &\propto |\boldsymbol{\Sigma}_t|^{-\frac{v_0+p+1}{2}} \exp \left[-\frac{1}{2} \text{tr}(\boldsymbol{\Psi}_0 \boldsymbol{\Sigma}_t^{-1}) \right] \left| \frac{1}{\gamma_0} \boldsymbol{\Sigma}_t \right|^{-\frac{1}{2}} \\ &\exp \left[\frac{-\gamma_0 (\boldsymbol{\mu}_t - \boldsymbol{\mu}_0)' \boldsymbol{\Sigma}_t^{-1} (\boldsymbol{\mu}_t - \boldsymbol{\mu}_0)}{2} \right] \end{aligned}$$

and

$$\begin{aligned} f(\mathbf{X}_{\tau_t+1:t} | \boldsymbol{\Sigma}_t, \boldsymbol{\mu}_t, \tau_t) \\ \propto |\boldsymbol{\Sigma}_t|^{-\frac{t-\tau_t}{2}} \exp \left[-\frac{\sum_{i=\tau_t+1}^t (\mathbf{X}_i - \boldsymbol{\mu}_t)' \boldsymbol{\Sigma}_t^{-1} (\mathbf{X}_i - \boldsymbol{\mu}_t)}{2} \right] \end{aligned}$$

Therefore

$$\begin{aligned} &f(\boldsymbol{\Sigma}_t | \mathbf{X}_{1:t}, \tau_t) \\ &\propto \int \int |\boldsymbol{\Sigma}_t|^{-\frac{v_0+p+1}{2}} \exp \left[-\frac{1}{2} \text{tr}(\boldsymbol{\Psi}_0 \boldsymbol{\Sigma}_t^{-1}) \right] |\boldsymbol{\Sigma}_t|^{-\frac{t-\tau_t+1}{2}} \exp \left[-\frac{1}{2} \left((\boldsymbol{\mu}_t - \boldsymbol{\mu}_{\tau_t+1,t}^*)' \left(\frac{\boldsymbol{\Sigma}_t}{\gamma_0 + t - \tau_t} \right)^{-1} (\boldsymbol{\mu}_t - \boldsymbol{\mu}_{\tau_t+1,t}^*) \right. \right. \\ &\quad \left. \left. - (\gamma_0 + t - \tau_t) (\boldsymbol{\mu}_{\tau_t+1,t}^*)' \boldsymbol{\Sigma}_t^{-1} \boldsymbol{\mu}_{\tau_t+1,t} + \gamma_0 \boldsymbol{\mu}_0' \boldsymbol{\Sigma}_t \boldsymbol{\mu}_0 + \sum_{i=\tau_t+1}^t \mathbf{X}_i' \boldsymbol{\Sigma}_t^{-1} \mathbf{X}_i \right) \right] d\boldsymbol{\mu}_t \\ &\propto |\boldsymbol{\Sigma}_t|^{-\frac{v_0+p+t-\tau_t+1}{2}} \exp \left[-\frac{1}{2} \text{tr} \left(\boldsymbol{\Psi}_0 + (t - \tau_t) \mathcal{S}_{\tau_t+1,t} \right. \right. \\ &\quad \left. \left. + \frac{(t - \tau_t) \gamma_0}{t - \tau_t + \gamma_0} (\bar{\mathbf{X}}_{\tau_t+1,t} - \boldsymbol{\mu}_0) (\bar{\mathbf{X}}_{\tau_t+1,t} - \boldsymbol{\mu}_0)' \right) \boldsymbol{\Sigma}_t^{-1} \right] \end{aligned}$$

where

$$\begin{aligned} \bar{\mathbf{X}}_{\tau_t+1,t} &= \frac{1}{t - \tau_t} \sum_{i=\tau_t+1}^t \mathbf{X}_i, \mathcal{S}_{\tau_t+1,t} \\ &= \frac{1}{t - \tau_t} \sum_{i=\tau_t+1}^t (\mathbf{X}_i - \bar{\mathbf{X}}_{\tau_t+1,t}) (\mathbf{X}_i - \bar{\mathbf{X}}_{\tau_t+1,t})' \end{aligned}$$

Therefore

$$(\boldsymbol{\Sigma}_t | \mathbf{X}_{1:t}, \tau_t) \sim \text{InvWish}_p(\boldsymbol{\Psi}_{\tau_t+1,t}^*, \nu_{\tau_t+1,t}^*)$$

where

$$\begin{aligned} \boldsymbol{\Psi}_{\tau_t+1,t}^* &= \boldsymbol{\Psi}_0 + (t - \tau_t) \mathcal{S}_{\tau_t+1,t} \\ &\quad + \frac{(t - \tau_t) \gamma_0}{t - \tau_t + \gamma_0} (\bar{\mathbf{X}}_{\tau_t+1,t} - \boldsymbol{\mu}_0) (\bar{\mathbf{X}}_{\tau_t+1,t} - \boldsymbol{\mu}_0)' \\ \nu_{\tau_t+1,t}^* &= (t - \tau_t) + \nu_0 \end{aligned}$$

APPENDIX B PROOF OF THEOREM 2

If $\tau_{t+1} < t$, then $\tau_{t+1} = \tau_t$, $\boldsymbol{\mu}_{t+1} = \boldsymbol{\mu}_t = \dots = \boldsymbol{\mu}_{\tau_{t+1}+1}$, $\boldsymbol{\Sigma}_{t+1} = \boldsymbol{\Sigma}_t = \dots = \boldsymbol{\Sigma}_{\tau_{t+1}+1}$. For notational convenience, let $\boldsymbol{\mu} = \boldsymbol{\mu}_{t+1} = \dots = \boldsymbol{\mu}_{\tau_{t+1}+1}$, $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_{t+1} = \boldsymbol{\Sigma}_t = \dots = \boldsymbol{\Sigma}_{\tau_{t+1}+1}$.

Based on Theorem 1,

$$\begin{aligned} (\boldsymbol{\mu} | \boldsymbol{\Sigma}, \mathbf{X}_{1:t}, \tau_{t+1}) &\sim N \left(\boldsymbol{\mu}_{\tau_{t+1},t}^*, \frac{\boldsymbol{\Sigma}}{\gamma_{\tau_{t+1},t}^*} \right) \\ (\boldsymbol{\Sigma} | \mathbf{X}_{1:t}, \tau_{t+1}) &\sim \text{InvWish}_p(\boldsymbol{\Psi}_{\tau_{t+1},t}^*, \nu_{\tau_{t+1},t}^*) \end{aligned}$$

Since

$$(\mathbf{X}_{t+1} | \mathbf{X}_{1:t}, \tau_{t+1}, \boldsymbol{\Sigma}) = (\boldsymbol{\mu} | \boldsymbol{\Sigma}, \mathbf{X}_{1:t}, \tau_{t+1}) + N(0, \boldsymbol{\Sigma}),$$

it is easy to show that

$$(\mathbf{X}_{t+1} | \mathbf{X}_{1:t}, \tau_{t+1}, \boldsymbol{\Sigma}) \sim N \left(\boldsymbol{\mu}_{\tau_{t+1},t}^*, \left(\frac{1}{\gamma_{\tau_{t+1},t}^*} + 1 \right) \boldsymbol{\Sigma} \right)$$

Therefore

$$\begin{aligned} f(\mathbf{X}_{t+1} | \mathbf{X}_{1:t}, \tau_{t+1}) &= \int f(\mathbf{X}_{t+1} | \mathbf{X}_{1:t}, \tau_{t+1}, \boldsymbol{\Sigma}) f(\boldsymbol{\Sigma} | \mathbf{X}_{1:t}, \tau_{t+1}) d\boldsymbol{\Sigma} \\ &\propto \int \left| \left(\frac{1}{\gamma_{\tau_{t+1},t}^*} + 1 \right) \boldsymbol{\Sigma} \right|^{-\frac{1}{2}} \times \\ &\exp \left[-\frac{(\mathbf{X}_{t+1} - \boldsymbol{\mu}_{\tau_{t+1},t}^*)' \left[\left(\frac{1}{\gamma_{\tau_{t+1},t}^*} + 1 \right) \boldsymbol{\Sigma} \right]^{-1} (\mathbf{X}_{t+1} - \boldsymbol{\mu}_{\tau_{t+1},t}^*)}{2} \right] \times \\ &|\boldsymbol{\Psi}_{\tau_{t+1},t}^*|^{-\frac{\nu_{\tau_{t+1},t}^*}{2}} |\boldsymbol{\Sigma}|^{-\frac{\nu_{\tau_{t+1},t}^*+p+1}{2}} \exp \left[-\frac{1}{2} \text{tr}(\boldsymbol{\Psi}_{\tau_{t+1},t}^* \boldsymbol{\Sigma}^{-1}) \right] d\boldsymbol{\Sigma} \\ &\propto \int |\boldsymbol{\Sigma}|^{-\frac{\nu_{\tau_{t+1},t}^*+p+2}{2}} \exp \left[-\frac{1}{2} \text{tr} \left(\left(\boldsymbol{\Psi}_{\tau_{t+1},t}^* \right. \right. \right. \\ &\quad \left. \left. + \frac{\gamma_{\tau_{t+1},t}^*}{\gamma_{\tau_{t+1},t}^* + 1} (\mathbf{X}_{t+1} - \boldsymbol{\mu}_{\tau_{t+1},t}^*) (\mathbf{X}_{t+1} - \boldsymbol{\mu}_{\tau_{t+1},t}^*)' \right) \boldsymbol{\Sigma}^{-1} \right) \right] d\boldsymbol{\Sigma} \end{aligned}$$

$$\propto \left| \Psi_{\tau_{t+1},t}^* + \frac{\gamma_{\tau_{t+1},t}^*}{\gamma_{\tau_{t+1},t}^* + 1} (X_{t+1} - \mu_{\tau_{t+1},t}^*)(X_{t+1} - \mu_{\tau_{t+1},t}^*)' \right|^{\frac{v_{\tau_{t+1},t}^*+1}{2}}$$

According to the generalized matrix determinant lemma,

$$\propto \left| 1 + \frac{(X_{t+1} - \mu_{\tau_{t+1},t}^*)'}{(\gamma_{\tau_{t+1},t}^* - p + 1)} \left(\frac{(\gamma_{\tau_{t+1},t}^* + 1) \Psi_{\tau_{t+1},t}^*}{\gamma_{\tau_{t+1},t}^* (\gamma_{\tau_{t+1},t}^* - p + 1)} \right)^{-1} (X_{t+1} - \mu_{\tau_{t+1},t}^*) \right|^{\frac{(v_{\tau_{t+1},t}^* - p + 1) + p}{2}}$$

Therefore

$$(X_{t+1} | X_{1:t}, \tau_{t+1}) \sim t \left(d_{\tau_{t+1},t}^*, \mu_{\tau_{t+1},t}^*, \frac{(\gamma_{\tau_{t+1},t}^* + 1) \Psi_{\tau_{t+1},t}^*}{\gamma_{\tau_{t+1},t}^* d_{\tau_{t+1},t}^*} \right)$$

Similarly, for $\tau_{t+1} = t$, we can get

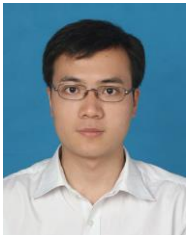
$$(X_{t+1} | X_{1:t}, \tau_{t+1} = t) \sim t \left(v_0 - p + 1, \mu_0, \frac{(\gamma_0 + 1) \Psi_0}{\gamma_0 (v_0 - p + 1)} \right),$$

which is just a special case of Eq. (13) by setting $\tau_{t+1} = t$.

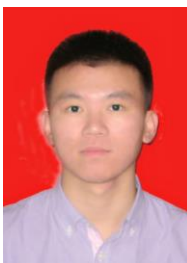
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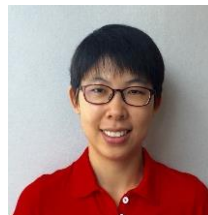


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