Interaction stochasticity supports cooperation in spatial Prisoner's dilemma

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Previous studies mostly assume deterministic interactions among neighboring individuals for games on graphs. In this paper, we relax this assumption by introducing stochastic interactions into the spatial Prisoner's dilemma game, and study the effects of interaction stochasticity on the evolution of cooperation. Interestingly, simulation results show that there exists an optimal region of the intensity of interaction resulting in a maximum cooperation level. Moreover, we find good agreement between simulation results and theoretical predictions obtained from an extended pair-approximation method. We also show some typical snapshots of the system and investigate the mean payoffs for cooperators and defectors. Our results may provide some insight into understanding the emergence of cooperation in the real world where the interactions between individuals take place in an intermittent manner.

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I. INTRODUCTION

How to understand cooperative behavior among selfish individuals in a group is a central problem in social and biological systems, and evolutionary game theory has provided a powerful framework to study this problem. As one of the most famous games, the Prisoner's dilemma game (PDG) can describe the conflict of interests between what is best for the group and what is best for the individual. In the original PDG, two players simultaneously decide whether to cooperate or defect. They will receive R if both cooperate, and P if both defect. While a player receives S when confronted to a defector, which in turn receives T, where $T>R>P>S$. Evidently, for one-shot PDG defection is unbeatable and thus preferred by rational players, although they can realize that mutual cooperation yields higher payoff than mutual defection.

However, the unfavorable equilibrium behavior (defection) obtained in the original PDG is often violated in the real world [1]. To understand the evolution of cooperation in social and biological systems, noteworthy, several mechanisms have been proposed [2–4], including kin selection [5], direct reciprocity [6], indirect reciprocity [7], graph selection (or spatial reciprocity) [8,9], and group selection [10]. Very importantly, the combination of original evolutionary games and graph theory provides an extended framework to study the emergence of cooperation in social systems, as some network models can precisely depict the structures of social networks including small-world and scale-free properties. Generally, for games on graphs, players occupying the vertices of a graph are constrained to interact only with their nearest neighbors for collecting payoffs based on the set of payoff values, then players update their strategies by learning from players in their neighborhoods according to the proposed updating rule. Within this framework, in the last decades the evolutionary PDG has been studied in different network models, and it is found that population structures play a crucial role in the organization and emergence of cooperative behaviors (for example, see [8,11–15] and references therein).

However, herein we would like to point out that, to our knowledge, in most previous studies of games on graphs, a common simplifying assumption is made, that players always fully interact with all their neighbors during the interaction stage. Namely, the deterministic interactions existing among neighboring individuals are fully in action. Actually, in real social systems, not all of the possible interaction relationships are always in effect; instead sometimes these pairwise interactions are activated in an intermittent manner even if there are links among neighboring individuals. In this sense, the actual interaction graph should be only a subset of the full one. In the vain of this spirit, Traulsen *et al.* relaxed the setting that individuals deterministically interact with their neighbors, and first studied the case of finite well-mixed populations in which each pair of individuals interact with a probability, leading to different numbers of interactions per individual [16]. This work shows this introduction of stochastic interaction for neighboring individuals results in heterogeneous payoff evaluation for a well-mixed population, and further enriches the knowledge of evolutionary dynamics in sophisticated yet realistic situations. Naturally, it is worth further studying the effects of stochastic interaction on the evolution of cooperation in structured populations.

In view of the above situations, presently we abandon the assumption of deterministic interactions that has been used in the previous study of networked games, and investigate the effects of stochastic interaction on the evolution of cooperation in spatial PDG. For the sake of simplicity, here, we assume that each pair of directly connected players engages in pairwise interaction with a probability p, where $p \in [0,1]$ measures the intensity of interaction. $p=1$ means full interactions, and our model recovers to the classical spatial game model. For $0<p<1$, the actual number of interaction partners is subject to a binomial distribution. Interest-

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ingly, we find that some certain amount of interaction stochasticity substantially promotes cooperation in the system. We also provide an extended pair-approximation method to theoretically predict the cooperation level. In the rest of the context, we will present in detail our findings and corresponding explanations.

II. MODEL

In this study, we consider the evolutionary PDG on a square lattice with periodic boundary conditions. Each player engages in pairwise interactions within its von Neumann neighborhood. The reason that we concentrate on the simple case of square lattices is twofold: First, it is a well-studied model of population structures in the last decades, and therefore we can easily compare our results with those previously investigated [8,11,17,18]; second, for square lattices, the pair-approximation (PA) technique can be used to estimate the equilibrium frequency of cooperators [15,19–21]. Following common practice [8,11], we adopt the rescaled payoff matrix depending on one single parameter b: $T=b>1$, $R=1$, and $P=S=0$. During the evolutionary process, each player who occupies one site of the graph can either cooperate or defect, and in each generation each pair of directly connected players plays the PDG with a probability p independently. Here, the parameter p $(0\leq p\leq 1)$ measures the intensity of interaction. For $p\rightarrow 1$, our model recovers to the classical spatial game. For $p\rightarrow 0$, all interactions between players are "frozen," and thus no game (interaction) happens. For $0<p<1$, the actual number of interactions with neighbors satisfies a binomial distribution, determining each individual's total payoff. After playing games, each player is allowed to learn from one of its adjacent neighbors and update its strategy. As reported in Ref. [17] individuals with better performance in collecting payoffs may have stronger attractiveness than others in society, here we also consider this realistic scenario for individuals and incorporate individuals' inhomogeneous attractiveness in choosing neighbors for strategy updating. Similarly, we define the selection probability of a player x selecting one of its neighbors y as

$$P_{x\rightarrow y}=\frac{P_y}{\sum_{z\in\Omega_x} P_z}, \tag{1}$$

where the sum runs over all neighbors of player x, and we denote by P_x the payoff of player x. Moreover, we would like to point out that in our study there are instances where $\Sigma_{z\in\Omega_x} P_z$ is equal to zero. Where this occurs, player x will randomly select one player y from its adjacent neighbors. Whereafter, player x adopts the neighbor's strategy with a probability depending on the payoff difference by the Fermi function

$$f(P_y-P_x)=\frac{1}{1+\exp[-(P_y-P_x)/K]}, \tag{2}$$

where K characterizes the noise effects in the strategy adoption process [11]. Herein, we only consider the simple situation for individuals' selection probability and simply set $K=0.1$ in this work, and mainly focus on how the interaction stochasticity affects the evolution of cooperation in spatial PDG.

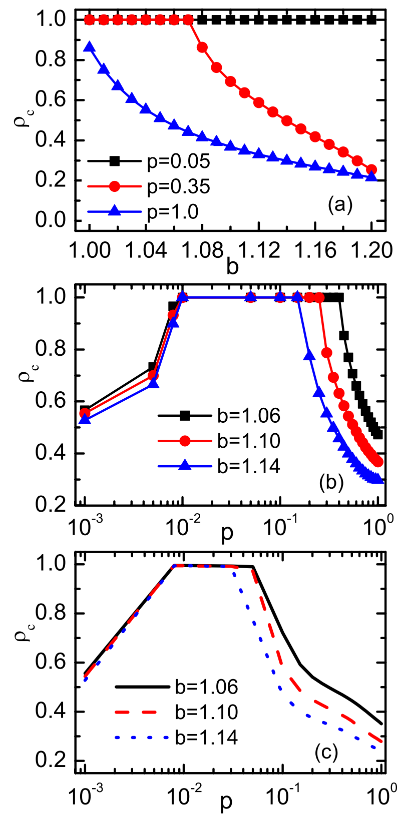

FIG. 1. (Color online) The cooperation level as a function of b for different values of p in (a). Simulation results (b) and theoretical analysis by pair approximation (c) for the cooperation level as a function of p for different values of the temptation to defect b.

III. RESULTS

In what follows, we will show the simulation results carried out on a square lattice of size 100×100. Initially, the two strategies of C and D are randomly distributed among the population with an equal probability 0.5. The key quantity for characterizing the cooperative behavior of the system is the density of cooperators, ρ_c, which is defined as the fraction of cooperators in the whole population. In our study, we implement this simulation model with synchronous update. After a suitable transient time (this transient time varies if we choose different parameters for the system), the system reaches a dynamical equilibrium, and the density ρ_c reaches its asymptotic value and remains there within small fluctuations (the fluctuations are smaller than 0.01) [22–24]. This asymptotic value is taken to describe the cooperation level, and all the simulation results reported below are averaged over 100 different realizations of initial conditions.

First we present the cooperation level as a function of the temptation to defect b for different values of p, as shown in Fig. 1(a). For $p=0.05$, full cooperation is achieved irrespective of the value of b. While for larger p the region of full cooperation decreases or vanishes, that is, full cooperation

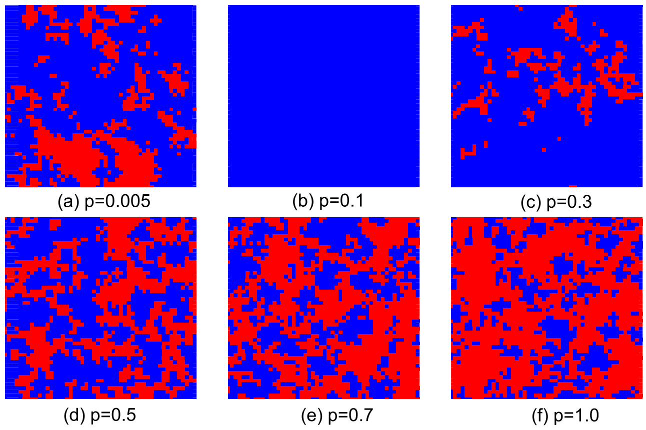

FIG. 2. (Color online) Snapshots of typical distributions of cooperators (blue) and defectors (red) on a square lattice obtained by $b=1.10$ and different values of p. These snapshots, as well as subsequent ones, are a 50×50 portion of the full 100×100 lattices. (a) $p=0.005$, (b) $p=0.1$, (c) $p=0.3$, (d) $p=0.5$, (e) $p=0.7$, and (f) $p=1.0$.

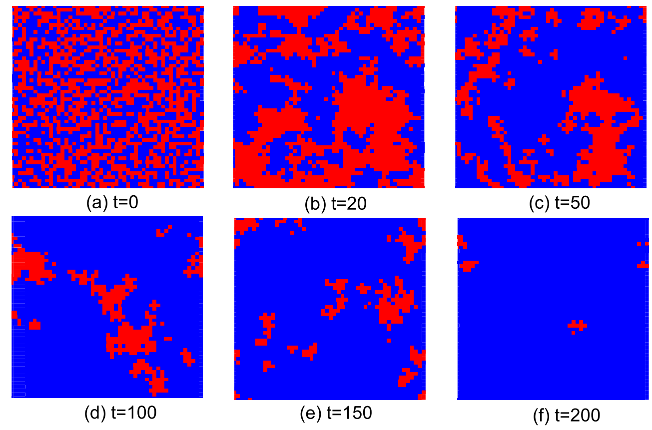

FIG. 3. (Color online) Snapshots of typical distributions of cooperators (blue) and defectors (red) on a square lattice for $b=1.10$ and $p=0.05$ at different time step t. (a) $t=0$ [$\rho_c(0)=0.50$], (b) $t=20$ [$\rho_c(20)=0.5108$], (c) $t=50$ [$\rho_c(50)=0.7192$], (d) $t=100$ [$\rho_c(100)=0.8584$], (e) $t=150$ [$\rho_c(150)=0.886$], and (f) $t=200$ [$\rho_c(200)=0.9864$].

cannot be maintained and cooperation decreases with increasing b. It is shown that interaction stochasticity can promote cooperation and there may exist some values of p, resulting in a plateau of high cooperation level. Further, to quantify the role of stochastic interaction in promoting cooperation more precisely, we then study the dependence of the cooperation level on p for different values of b, as shown in Fig. 1(b). Clearly, for fixed b, there exists an optimal region of p, leading to full cooperation. When p becomes larger and goes beyond this region, the cooperation level monotonically decreases with p. For infinitesimal $p\to 0$, ρ_c converges to 0.5; with increasing $p>0$, ρ_c first increases sharply until a plateau of full cooperation is achieved [see Fig. 1(b)]. Notably, the length of this plateau decreases with increasing b, which means the positive effect of interaction stochasticity (i.e., p) on the enhancement of cooperation is diminished, to a certain extent, by increasing the amount of temptation to defect. Here, we only show the results for $b\leqslant 1.20$ in our study. Actually, for b larger than 1.20 this result of optimal cooperation is significantly weakened or vanishes, and the cooperation level approaches zero for large $p\to 1$ [17]. Besides, we provide an extended pair-approximation (PA) method (see the Appendix) to predict the cooperation level as a function of p [see Fig. 1(c)]. In accordance with our simulation results, the PA method qualitatively reflects the role of interaction stochasticity in cooperation, but underestimates the resulting cooperation level in general. Note that the reason of the deviations between these two approaches is that our extended pair approximation does not fully take into account the effects of the spatial structures, especially spatial clusters. Herein, our simulation results as well as PA predictions explicitly show that cooperation can be promoted substantially by this sort of interaction stochasticity.

In order to intuitively understand the effect of interaction stochasticity on cooperation, we plot some typical snapshots of the system at equilibrium for fixed $b=1.10$ with respect to different p values (see Fig. 2). One can find that cooperators can survive or even thrive by means of forming tight compact clusters. Generally, the formations of cooperator clusters arise from the spatial effect on cooperation in the PDG [8]; nevertheless, the introduction of interaction stochasticity significantly influences the cluster formation process as well. From Figs. 2(a)–2(f), we can find that varying the amount of interaction stochasticity leads to different cluster patterns of cooperators and defectors. In particular, when p is small, e.g., $p=0.005$ as shown in Fig. 2(a), cooperators can survive by forming some compact clusters during the process of fighting with defectors; whereas when p is in the optimal region, e.g., $p=0.1$ as shown in Fig. 2(b), cooperators can form a single large contiguous cluster leading to the extinction of defectors finally; in contrast, when p is large, e.g., $p=0.7$ as shown in Fig. 2(e), only small and isolated patches for cooperators can be formed to minimize exploitation by defectors such that cooperators and defectors can coexist on a square lattice in this situation. It is shown that cooperators are more likely to benefit from such stochastic interactions than defectors, and therefore able to resist the invasion of periphery defectors and prevail ultimately. Noticeably, for large p exceeding the optimal region, the typical cluster size of cooperators decreases with increasing p, resulting in a gradual drop of cooperation level. Furthermore, we investigate the cluster formation process at different time step t for $b=1.10$ and $p=0.05$, as reported in Fig. 3. Initially, cooperators and defectors are randomly distributed with the same probability on the square lattice [see Fig. 3(a)]. Interestingly, from Fig. 3(b) we can see that cooperators and defectors are quickly clustered with themselves, respectively ($t=20$), although the density of cooperators at this moment is not much higher than the initial one. As time step t increases, the size of defector clusters deceases gradually. Finally, the defector clusters disappear and cooperators take over the population. It can be indicated that cooperators are strongly favored under this mechanism of interaction stochasticity: They not only succeed in resisting the invasion of boundary defectors, but also expand themselves by turning defectors into cooperators through strategy imitation.

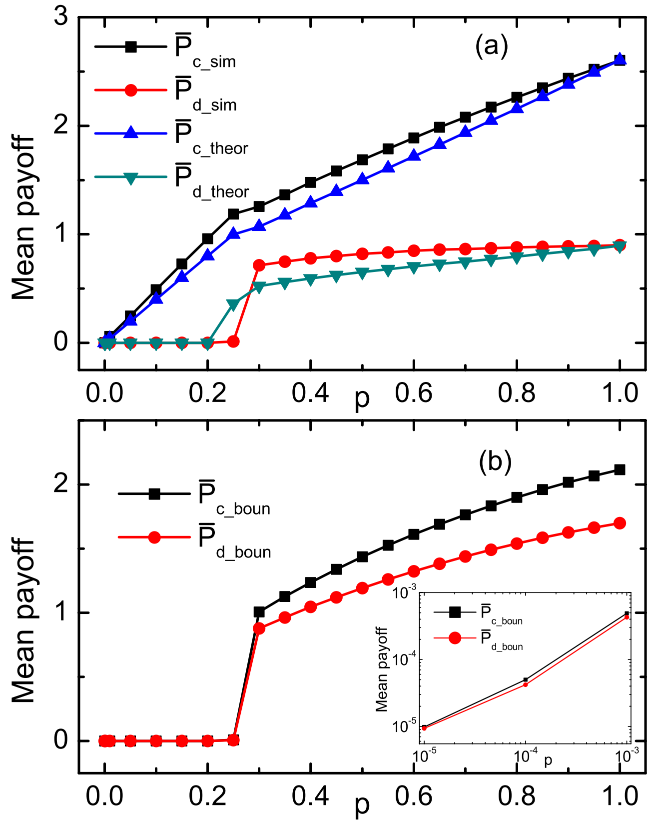

FIG. 4. (Color online) (a) The mean payoffs of cooperators and defectors in the whole population, obtained respectively by simulations ($\bar{P}_{c\_sim}$ and $\bar{P}_{d\_sim}$) and theoretical analysis ($\bar{P}_{c\_theor}$ and $\bar{P}_{d\_theor}$), as a function of p for $b=1.10$. (b) The mean payoffs of cooperators and defectors along the boundary ($\bar{P}_{c\_boun}$ and $\bar{P}_{d\_boun}$) as a function of p for $b=1.10$. The inset in (b) shows the results for small values of p in detail.

Notice that cooperators tend to form cluster patterns where cooperators assist each other to avoid defectors' exploitation in spatial games during the evolutionary process [17,19,20,25]. Generally, this pattern formation for cooperators influences the interactions between cooperators and defectors, and thus affects the collecting payoffs of cooperators and defectors. This naturally results in the changes of cooperation level according to the updating rule. Now, let us study the mean payoffs of cooperators and defectors in the population as well as the ones of cooperators and defectors lying around the boundary, in order to qualitatively explain these nontrivial results. Interestingly, one can find that the mean payoff of cooperators is much higher than that of defectors for all p, as plotted in Fig. 4(a) [26]. This result indicates that cooperators are at a greater advantage in collecting payoffs over defectors under our proposed mechanism, which favors the evolution of cooperation. To confirm our simulation results, we also provide theoretical predictions for the mean payoffs of cooperators and defectors in the whole population (see the Appendix). We can find that the predictions are in accordance with the trends (i.e., the changes of mean payoffs for p) in comparison with numerical simulations, although there are little deviations between them. Further, we study the mean payoffs of cooperators and defectors along the boundary, respectively [see Fig. 4(b)]. Different from the results shown in Fig. 4(a), both the mean payoffs of cooperators and defectors along the boundary ($\bar{P}_{c\_boun}$ and $\bar{P}_{d\_boun}$) are zero when p is in the optimal region, since defectors become extinct in the population and there is no boundary between cooperators and defectors in this situation. We also plot the results of $\bar{P}_{c\_boun}$ and $\bar{P}_{d\_boun}$ for very small p as shown in the inset of Fig. 4(b), and find that $\bar{P}_{c\_boun}$ is not less than $\bar{P}_{d\_boun}$ for various values of p. In fact, the introduction of stochastic interaction between connected players can effectively inhibit interactions between cooperators and defectors along the boundary. For defectors, this induces a negative feedback mechanism that can reduce their payoff. As a result, such a mechanism of stochastic interaction can promote the cooperation level. Specifically, for very small p, interactions between all of the paired individuals are remarkably inhibited simultaneously, thus the mean payoffs of cooperators and defectors in the system are both small. But in this situation the mean payoff of cooperators is still higher than that of defectors. As a consequence, cooperators have a weak advantage over defectors in strategy updating; hence during the strategy updating, the probability for defectors along the boundary changing into cooperators is slightly higher than vice versa for fixed value of K [see the inset of Fig. 4(b)]. Therefore, cooperation can be maintained in this case (in particular, the cooperation level is around 0.5 when p approaches zero since the strategy updating probability for all of the players is near 0.5). While in the case of intermediate p, from the beginning of evolution until defectors die out in the system, the mean payoff of defectors is smaller than that of cooperators, and the difference between them is comparable with K. Therefore, concerning players' attractiveness, cooperators are more likely to be imitated as role model than defectors; meanwhile, the transition probability of defectors changing into cooperator, $f(P_C-P_D)$, is considerably large, whereas, $f(P_D-P_C)$ is relatively small so that eventually cooperators tend to replace defectors and take over the whole population. Whereas for larger $p\to 1$, both the mean payoffs of cooperators and defectors become higher. However, the clustering advantage in providing higher payoffs for cooperators than defectors is insufficient for large p, especially for large values of b, since there are little limitations of interactions between connected defectors and cooperators. Generally, the strategy updating probability for both defectors and cooperators along the boundary is not very high for fixed K. In this sense, cooperators and defectors may coexist for an exceedingly long time. In combination with these different cases of p, there should exist an appropriate region of intermediate interaction intensity, which can induce the most favorable cooperation.

Finally, we would like to point out that the results reported here are robust with respect to the detailed updating mechanism (the Fermi function, finite analog of replicator dynamics), to the updating fashion (synchronous, asynchronous), to different initial frequencies of cooperators, and to a different payoff matrix [$P=\varepsilon$ $(0<\varepsilon\ll 1)$] [8,27]. We have confirmed that the qualitative results do not change if we made the above-mentioned variations to our model.

IV. DISCUSSION AND CONCLUSION

So far, we have presented the main results of our model. In our study, individuals are allowed to intermittently interact with their social partners. That is to say, among neighboring individuals, pairwise partnerships uniformly occur with a probability p controlling the intensity of interaction. In this sense, the actual partnership networks are dynamical in nature, i.e., time-varying ones. Besides, considering the effects of players' performance in collecting payoffs on choosing neighbors for strategy updating, players are able to preferentially choose a role model from their neighbors. Interestingly, we find there exists an intermediate region of p resulting in optimal cooperation (massive cooperation). In fact, note that payoff-based preferential imitation [17], or equivalently, heterogeneity in teaching activity [18,28], has already been recognized as an important mechanism of promoting cooperation in spatial games, recently. However, this sort of preferential imitation does not directly link with the optimal phenomena observed in our study. To verify this, we thus conduct a comparative study in which we relax this condition of preferential imitation, i.e., individuals randomly imitate one of their neighbors. Both our Monte Carlo simulations and pair-approximated results exactly show that there still exists optimal cooperation for intermediate p. Therefore, the existence of optimal cooperation is essentially attributed to the interaction stochasticity. The largest differences are that the optimal region is much narrowed to a single point and the critical value of b, below which optimal cooperation exists, is much lower. Here, we introduce this preferential imitation in our proposed model in order to mainly consider the realistic situation, and this introduction of preferential imitation also contributes to the sustainment of massive cooperation. Not surprisingly, in the case without preferential imitation, the cooperation level is correspondingly lowered and approaches zero just for very small temptation to defect [17].

It is worth noting that, in Ref. [16], the authors found that the resulting "payoff stochasticity" reduces the intensity of selection in finite well-mixed populations. In their model, similarly, individuals randomly interact with others, leading to a heterogeneity in the number of interaction partners. From this perspective, roughly speaking, our model is a spatially extended version of theirs. Our work further complements their finding: Interaction stochasticity provides a favorable environment for cooperators to thrive. In addition to the source of payoff stochasticity discussed here, Perc first introduced payoff variations, generated from a chaotic attractor, into spatial PDG, and found that cooperation is promoted as well as a dynamical "coherence resonance" behavior in such a system [29]. In Perc's work, chaotic variations, modeled by a spatially extended Lorenz system, are introduced to individuals' payoffs collected from interactions with all their neighbors. Thus, payoff stochasticity results from such environmental disturbances other than interaction randomness. In line with these previous works, the present study further highlights the positive effect of payoff stochasticity on the evolution of cooperation.

On the other hand, the introduction of stochastic interaction reduces the effective neighborhood size, since the expected number of interaction partners monotonically decreases with decreasing p values. In other words, interaction randomness essentially decreases the population viscosity (or average number of neighbors), which plays a decisive role in the evolution of cooperation on graphs, according to the simple rule found in Ref. [9]. In general, the effects of network topologies, such as degree heterogeneity [12,14], topological randomness [30], and average degree [31], significantly affect the outcome of games on networks. Specially, a dynamical feature of optimal cooperation can emerge as a result of the randomness in topologies [30]. However, in our study, the randomized interaction networks are continuously evolving during the evolutionary process; while the investigations in Ref. [30] focus on static networks, that is, the interaction neighbor set for every player is fixed, but the amount of randomness in topologies can be tuned by a parameter. Furthermore, the dual graph model, in which the interaction graph and replacement graph are separated, has been studied in Refs. [21,32–34]. Actually, although our model also involves the essence of dual graph, previous explorations mainly concentrate on the effect of dual graph on the evolution of cooperation; differently, we focus on the role of interaction stochasticity, one of the possible stochastic effects, in the cooperative dynamics. Generally speaking, the dynamical randomness stemming from diverse factors of stochasticity constitutes an important mechanism to maintain cooperation, as already demonstrated by previous studies [29,30,35,36].

In summary, we have studied the effect of interaction stochasticity on the evolution of cooperation in spatial PDG. Interestingly, by means of Monte Carlo simulations, we showed that there exists an optimal region of the amount of interaction stochasticity resulting in massive cooperation. Our simulation results are in good agreement with theoretical predictions obtained from an extended pair-approximation method. Furthermore, in order to give an intuitive account of the maintenance of cooperation in our model, we provided some typical snapshots of the system and compared mean payoffs of defectors and cooperators along the boundary. Our work may evidence stochastic interaction as an alternative mechanism to enhance cooperation in self-interested individuals.

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APPENDIX: EXTENDED PAIR-APPROXIMATION METHOD

We consider the randomized interaction for connected players on the square lattice. For the sake of clarity, we plot

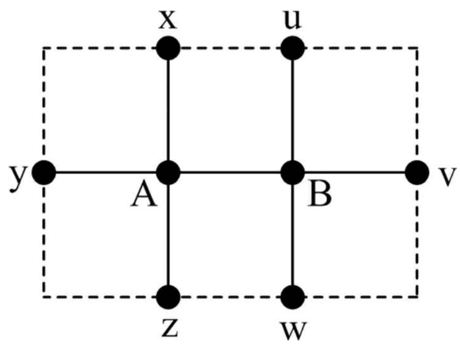

FIG. 5. Small part of the square lattice indicating the relevant configuration for the pair approximation with sites A and B. This scheme is used to determine changes in the pair configuration probabilities $p_{A,B\to B,B}$.

a small part of the square lattice as shown in Fig. 5. Here, we denote by i_c (i_d) the actual number of an individual's interactions with other cooperators (defectors) among its neighborhood. Based on the statement mentioned in the above context, we take site B, for example, and can compute the collecting payoff of a cooperator occupying the site of B plus a defector occupying the site of A for different configurations,

$$P_c(u,v,w,i_c,i_d) = \binom{n_c(u,v,w)}{i_c}\binom{4-n_c(u,v,w)}{i_d} \times p^{i_c+i_d}(1-p)^{4-i_c-i_d} i_c, \quad \text{(A1)}$$

where $n_c(u,v,w)$ is the number of cooperators among the neighbors u, v, w. Similarly, we obtain the collecting payoff of a defector occupying the site of B plus a cooperator occupying the site of A for different configurations as

$$P_d(u,v,w,i_c,i_d) = \binom{n_c(u,v,w)+1}{i_c}\binom{3-n_c(u,v,w)}{i_d} \times p^{i_c+i_d}(1-p)^{4-i_c-i_d} b i_c. \quad \text{(A2)}$$

Further, by assuming that there are n_c cooperators in the neighborhood of an individual, according to Eq. (A1) the mean payoff of cooperators in the system is given by

$$\bar{P}_c = \sum_{i_c=0}^{n_c}\sum_{i_d=0}^{4-n_c}\binom{n_c}{i_c}\binom{4-n_c}{i_d} p^{i_c+i_d}(1-p)^{4-i_c-i_d} i_c = n_c\left(\frac{1}{1-p}\right)^4 (1-p)^4 p = n_c p = 4\frac{p_{c,c}}{\rho_c}p. \quad \text{(A3)}$$

Similarly, the mean payoff of defector is given by

$$\bar{P}_d = 4\frac{p_{c,d}}{\rho_d}pb. \quad \text{(A4)}$$

Subsequently, following previous works [15,19–21], we extend the pair-approximation method and describe, respectively, the motion of fractions of CC ($p_{c,c}$) and CD ($p_{c,d}$) links as

$$\dot{p}_{c,c} = \sum_{x,y,z}[n_c(x,y,z)+1]p_{d,x}p_{d,y}p_{d,z} \times \sum_{u,v,w} p_{c,u}p_{c,v}p_{c,w} f[P_c(u,v,w) - P_d(x,y,z)] - \sum_{x,y,z} n_c(x,y,z) p_{c,x}p_{c,y}p_{c,z}\sum_{u,v,w} p_{d,u}p_{d,v}p_{d,w} f \times [P_d(u,v,w) - P_c(x,y,z)], \quad \text{(A5)}$$

$$\dot{p}_{c,d} = \sum_{x,y,z}[1-n_c(x,y,z)]p_{d,x}p_{d,y}p_{d,z} \times \sum_{u,v,w} p_{c,u}p_{c,v}p_{c,w} f[P_c(u,v,w) - P_d(x,y,z)] - \sum_{x,y,z}[2-n_c(x,y,z)]p_{c,x}p_{c,y}p_{c,z}\sum_{u,v,w} p_{d,u}p_{d,v}p_{d,w} f \times [P_d(u,v,w) - P_c(x,y,z)]. \quad \text{(A6)}$$

Here, based on the description of the strategy updating rule, $f[P_c(u,v,w)-P_d(x,y,z)]$ and $f[P_d(u,v,w)-P_c(x,y,z)]$ can be described, respectively, as

$$f[P_c(u,v,w) - P_d(x,y,z)] = \sum_{i_c}\sum_{i_d}\sum_{i'_c}\sum_{i'_d}\frac{P_c(u,v,w,i_c,i_d)}{\sum P_B} \times f[P_c(u,v,w,i_c,i_d) - P_d(x,y,z,i'_c,i'_d)], \quad \text{(A7)}$$

$$f[P_d(u,v,w) - P_c(x,y,z)] = \sum_{i_c}\sum_{i_d}\sum_{i'_c}\sum_{i'_d}\frac{P_d(u,v,w,i_c,i_d)}{\sum P_B} \times f[P_d(u,v,w,i_c,i_d) - P_c(x,y,z,i'_c,i'_d)], \quad \text{(A8)}$$

where $\sum P_B$ means the sum of payoffs of A's neighbors for all the possible configurations. In combination with the symmetry condition $p_{c,d}=p_{d,c}$ and the constraint $p_{c,c}+p_{c,d}+p_{d,c}+p_{d,d}=1$, we can obtain that $\rho_c=p_{c,c}+p_{c,d}$. Here, in order to solve for $p_{c,c}$ and $p_{c,d}$, we treat Eqs. (A5) and (A6) by numerical integration, and accordingly get the cooperation level.

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