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Social dilemmas in an online social network: The structure and evolution of cooperation

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Abstract

We investigate two paradigms for studying the evolution of cooperation—Prisoner's Dilemma and Snowdrift game in an online friendship network, obtained from a social networking site. By structural analysis, it is revealed that the empirical social network has small-world and scale-free properties. Besides, it exhibits assortative mixing pattern. Then, we study the evolutionary version of the two types of games on it. It is found that cooperation is substantially promoted with small values of game matrix parameters in both games. Whereas the competent cooperators induced by the underlying network of contacts will be dramatically inhibited with increasing values of the game parameters. Further, we explore the role of assortativity in evolution of cooperation by random edge rewiring. We find that increasing amount of assortativity will to a certain extent diminish the cooperation level. We also show that connected large hubs are capable of maintaining cooperation. The evolution of cooperation on empirical networks is influenced by various network effects in a combined manner, compared with that on model networks. Our results can help understand the cooperative behaviors in human groups and society.

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Keywords: Social networks; Prisoner's Dilemma; Snowdrift game; Cooperation; Network effects

1. Introduction

Cooperative behaviors (mutualism and altruism) are ubiquitous in human society as well as in virtual online community. For instance, people unselfishly and collaboratively recommend commodities such as books, songs, CD/DVDs, etc. to each other. Accordingly, this cooperative behavior (collaborative recommendation) promotes the long tail which is the success foundation of Amazon and eBay [1]. And yet, according to Darwinism, natural selection is based on competition. How can natural selection lead to cooperation among selfish individuals? Fortunately, together with classic game theory, evolutionary game theory provides a systematic framework for investigating the emergence and maintenance of cooperative behavior among unrelated and selfish individuals. Two simple games, namely, Prisoner's Dilemma game (PDG) and Snowdrift game (SG), as metaphors for studying the evolution of cooperation have been extensively adopted by researchers from different background [2–7]. In the original PDG, two players simultaneously decide whether to cooperate (C) or to defect (D). They both receive R upon mutual cooperation and P upon mutual defection. A defector exploiting a C player gets T, and the exploited cooperator receives S, such that $T > R > P > S$ and $2R > T + S$. As a result, it is best to defect regardless of the co-player's decision. Thus, in well-mixed infinite populations, defection is the evolutionarily stable strategy (ESS) [8], even though all individuals would be better off if they cooperated. Thereby this creates the social dilemma, because when every-

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body defects, the mean population payoff is lower than that when everybody cooperates. Whereas in the SG, the order of P and S is exchanged, such that $T > R > S > P$. Its essential ingredient is that in contrast to the PDG, cooperation has an advantage when rare, which implies that the replicator dynamics of the SG converges to a mixed stable equilibrium where both C and D strategies are present [8]. It is important to note that in this state the population payoff is smaller than it would be if everyone played C, hence the SG still represents a social dilemma [9]. In addition, the SG is of much applications and interests within biological context. In order to solve these social dilemmas, a variety of suitable extensions on these basic models has been investigated [3,5–7]. Most importantly, it is found that cooperation can be promoted and sustained in the network-structured population substantially [6,10–13]. Indeed, the successful development of network science provides a convenient framework for describing the dynamical interactions of games. The evolution of cooperation on model networks with features such as lattices [14–17], small-world [18–20], scale-free [6], and community structure [21] has been scrutinized. Moreover, the understanding of the effect of network structure on the evolution of cooperation reaches to consensus gradually: the heterogeneity of the network of contacts plays a significant role in the emergence of cooperation. However, the puzzle of cooperation is unanswered yet. What on earth conditions the emergence of cooperation is still a challenging problem [22,23]. Most noteworthy, Nowak summarized five possible rules for the evolution of cooperation corresponding to different situations (see Ref. [24] and references therein). Nevertheless, to our best knowledge, these results are mostly based upon simplified scenario and model. To inspect the evolution of cooperation, further details and characteristics of real world should be considered and integrated.

Herein, we consider the two aforementioned social dilemmas over an online friendship network, obtained from a Chinese social networking site (SNS)—Xiaonei [25]. In the age of so-called Web 2.0, SNS as well as blogs provides an extraordinary online laboratory to study dynamic pattern of socio-economic systems. Evidently cooperative (altruistic) behaviors are ubiquitous and robust in natural systems. Actually, people, especially college students, take advantage of online social network services for messaging, sharing information, and keeping in touch with each other. Mutual cooperation thus consolidates the existent basis of this virtual online community. Consequently, it is meaningful and interesting to study the evolution of cooperation on such social network, of which who-meets-whom relationships are abstracted from Xiaonei. In what follows, two metaphors—PDG and SG in the empirical social network will be scrutinized.

In this Letter, we first present a detailed structural analysis of Xiaonei network. It is demonstrated that the network has small-world and scale-free properties. Noticeably, the average connectivity is relatively high. Thus it is shown that cooperation is significantly suppressed in this highly connected network when the values of the game parameters increase, even though the heterogeneity of the underlying network to a certain extent promotes cooperation. In addition, it exhibits assortative mixing pattern. However, we find that increasing degree of assortativity diminishes the cooperation level by random edge rewiring. Furthermore, the appearance of direct links between hubs in a way contributes to the enhancement and sustainment of cooperation. Therefore it is suggested that the evolution of cooperation on realistic social networks is influenced by a variety of network effects in a combined manner, compared to that on model networks. In the rest of this Letter, first, we will analyze the structure of the online social network, then investigate the two social dilemmas (PDG and SG) on this social network by the method analogous to replicator dynamics, exploring the influence of network topological features on cooperation. After that, we discuss the simulation results and make explanations. Finally, we draw the conclusion remarks.

2. The structure of Xiaonei network

In this section, we present our observations into a Chinese social networking site Xiaonei, which is open to college students. It began in late 2005 in select universities, but grew quickly to encompass a very large number of universities. Each registered user has a web page hosted within Xiaonei domain. The user's web page can be visited through a pointer specified by the user id. Users can add others as friends in their own web pages. The friendship is constructed by bilateral agreement. Thus Xiaonei network is bidirectional one (we viewed this network as undirected one). The original Xiaonei network has hundreds of thousands of nodes and millions of edges,[1] which seems too huge to be handled in studying the evolution of cooperation on it due to the limited capacity of our computer resources. Instead, for practical purpose, we focus our eyesight in a connected sub-community of the original large-scale Xiaonei network, and this subnet consists of 9590 vertices and 89873 edges (we view this sub-community as a close world, hence the edges out of this community are omitted. It is referred to as Xiaonei network thereafter). Noteworthy, although this sub-community could not be a good representative of the original one, its size and topological features are still sufficient to study the cooperative dynamics on it.

We perform statistical analysis of the structure of this social network, i.e., the above sub-community network. The quantities including degree distribution, clustering coefficient, average shortest path length, etc. are calculated to capture the topological features. In Fig. 1, we report the cumulative degree distribution $P(> k)$, which gives the probability that randomly selected node has more than k edges. Clearly, except that $P(> k)$ has a flat head, it obeys a power-law form as $\sim k^{-\tau}$ with $\tau = 2.21 \pm 0.01$ for large degrees. One can immediately get the degree distribution $p(k) \sim k^{-\gamma}$ with $\gamma = \tau + 1 = 3.21$. This small-scale Xiaonei network has a degree exponent similar to the web of human sexual contacts, whereas most social networks have the degree exponent falling into the range $2 < \gamma < 3$ [26]. It is apparent that this subnet is deviated from the large-scale one in power law exponent. Actually, the original Xiaonei

[1] The structural analysis of the original one will be presented elsewhere.

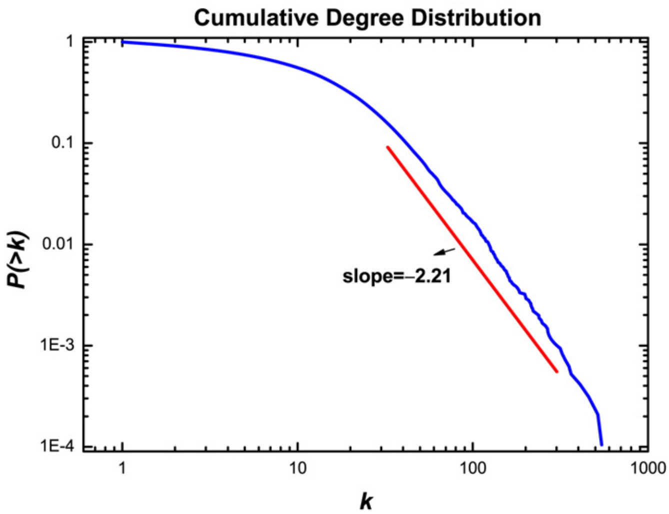

Fig. 1. The cumulative degree distribution $P(>k)$ of the online social network, which has a power-law tail as $P(>k) \sim k^{-\tau}$ with $\tau = 2.21$. The straight line has slope -2.21.

Table 1
Percentage of nodes with 1, 2, 3, 4, and 5 degrees. Note that a large fraction of nodes have only small degrees

k	1	2	3	4	5
	8.4%	6.7%	5.5%	5.0%	4.2%

network has a degree exponent about 2.12. Considerable fraction of nodes have only low connectivity (see Table 1). About 70% nodes' degrees are not more than 20, while the average degree $\langle k \rangle$ is 18.74.

The length of average shortest path $\langle l \rangle$ is computed, which is the mean of geodesic distance between any pairs that have at least a path connecting them. In this case, $\langle l \rangle = 3.48$. And the diameter D of this social network which is defined as the maximum of the shortest path length, is 9. We also present the shortest path length distribution in Fig. 2, where most of distances are 3 or 4. The clustering coefficient of node i is defined as $C_i = 2E_i/[k_i(k_i - 1)]$, that is the ratio between the number E_i of edges that actually exit between these k_i neighbor nodes of node i and the total number $k_i(k_i - 1)/2$ (for $k_i = 1$, $C_i = 0$). The clustering coefficient of the whole network is the average of all individual C_i's. We find the clustering coefficient $C = 0.20$, order of magnitude much higher than that of a corresponding random graph of the same size $C_{\text{rand}} = 18.74/9590 = 0.001954$. Besides, the degree-dependent local clustering coefficient $C(k)$ is averaging C_i over vertices of degree k. Fig. 3 plots the distribution of $C(k)$ vs. k. For clarity, we add the dashed line with slope -1 in the log–log scale. However, it is hard to declare a clear power law in our case. Nevertheless, the nonflat clustering coefficient distributions shown in the figure suggests that the dependency of $C(k)$ on k is nontrivial, and thus points to some degree of hierarchy in the networks. In many networks, the average clustering coefficient $C(k)$ exhibits a highly nontrivial behavior with a power-law decay as a function of k [27], indicating that low-degree nodes generally belong to well-interconnected communities (corresponding

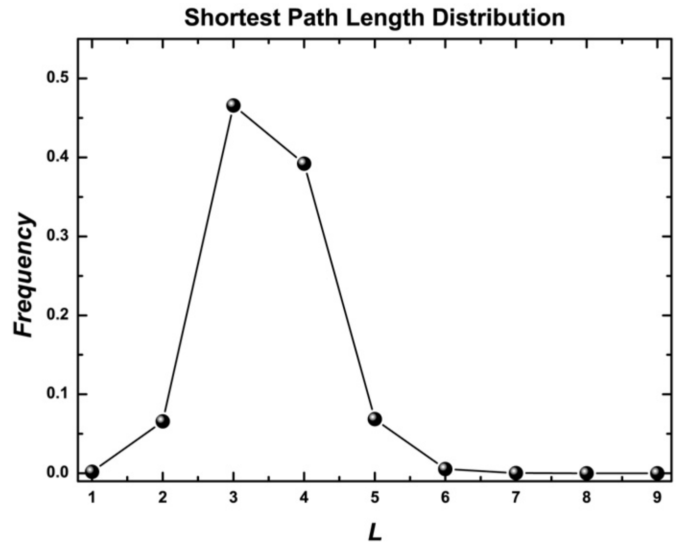

Fig. 2. The plot of shortest path length distribution.

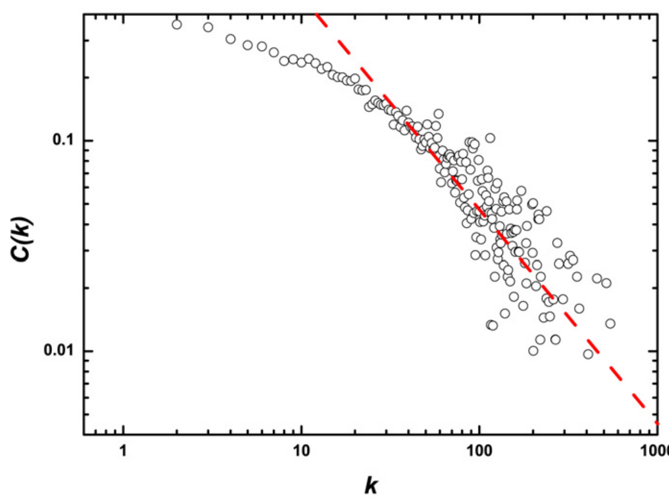

Fig. 3. The plot of degree-dependent clustering coefficient $C(k)$ vs. degree k. A clear power-law is absent but $C(k)$'s dependence on k is nontrivial. The dashed line has slope -1.

to high clustering coefficient of the low-connectivity nodes), while high-degree sites are linked to many nodes that may belong to different groups (resulting in small clustering coefficient of the large-degree nodes). This is generally the feature of a nontrivial architecture in which small-degree vertices are well-clustered around the hubs (high degree vertices), and organized in a hierarchical manner into increasingly large groups. Thus, our empirical social network has such fundamental characteristic of hierarchy.

Another important element characterizing the local organization of complex networks is the degree correlation of node i and its neighbor. Following Newman [28], assortativity coefficient r is measured by the Pearson correlation coefficient of the degrees at either ends of an edge, which can be written as

$$r = \frac{M^{-1}\sum_i j_i k_i - [M^{-1}\sum_i \frac{1}{2}(j_i + k_i)]^2}{M^{-1}\sum_i \frac{1}{2}(j_i^2 + k_i^2) - [M^{-1}\sum_i \frac{1}{2}(j_i + k_i)]^2}, \tag{1}$$

where j_i, k_i are the degrees of the vertices at the ends of the ith edge, with $i = 1, \dots, M$ (M is the total number of edges in the observed graph or network). We calculate the degree assortativity coefficient (or degree–degree correlation) r of the online social network. In our case, $r = 0.0170374$, which means the social network shows "assortative mixing" on its degrees. Networks with assortative mixing pattern are those in which nodes with large degree tend to be connected to other nodes with many connections and vice versa. Social networks are often assortatively mixed as demonstrated by the study on scientific collaboration networks [28].

We have presented the structural analysis of our online social network. The observed network has small-world property, that is, high clustering coefficient and short average shortest path length. Moreover, it is a heterogeneous one, namely, the tail of degree distribution obeys a power law. Additionally, it has nontrivial hierarchical organizations—low-degree nodes generally belong to well-interconnected clusters, while high-degree vertices are linked to many nodes that may belong to different groups. Besides, it exhibits assortative mixing pattern. Therefore, Xiaonei network, as a kind of social network embedded in a technical one, well represents the interaction relationships between online users, reflecting features of social networks in general. People communicate and cooperate with each others through Xiaonei network, while mutual defection will be an unfavorable result for bilateral users. Accordingly, further study on the cooperative dynamics of this system is necessary and important, offering an insight into the emergence of cooperation among selfish individuals linked in realistic social networks. In the successive section, we will investigate the evolution of cooperation in Xiaonei network, revealing the cooperation level is affected by the topological organizations.

3. Social dilemmas on Xiaonei network

We consider the evolutionary PDG and SG on Xiaonei network which retains the essential of realistic interaction relationships. Each vertex represents an individual and the edges denote links between players in terms of game dynamical interaction. The individuals are pure strategists, following two simple strategies: cooperate (C) and defect (D). The spatial distribution of strategies is described by a two-dimensional unit vector for each player x, namely,

$$s = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2}$$

for cooperators and defectors, respectively. Each individual plays the PDG/SG with its immediate "neighbors" defined by their who-meets-whom relationships and the incomes are accumulated. The total income of the player at the site x can be expressed as

$$P_x = \sum_{y \in \Omega_x} s_x^T M s_y, \tag{3}$$

where the Ω_x denotes the neighboring sites of x, and the sum runs over neighbor set Ω_x of the site x. Following common practice [3,9,14,29], the payoff matrices have rescaled forms for PDG and SG respectively,

$$M = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix}, \tag{4}$$

where $1 < b < 2$ and $0 < r < 1$.

In evolutionary games the players are allowed to adopt the strategies of their neighbors after each round. Then, the individual x randomly selects a neighbor y for possibly updating its strategy. The site x will adopt y's strategy with probability determined by the total payoff difference between them [14]:

$$W_{s_x \leftarrow s_y} = \frac{1}{1 + \exp[(P_x - P_y)/T]}, \tag{5}$$

where the parameter T characterizes the noise effects, including fluctuations in payoffs, errors in decision, individual trials, etc., $T = 0$ denotes the complete rationality, in which the individual always adopts the better strategy determinately. Whereas $T \to \infty$ denotes the complete randomness of decision. For finite value of T, it introduces bounded rationality to individual's decision making.

In what follows, we present our investigations into the two social dilemmas played by individuals occupying the vertices of the small-scale Xiaonei network. The evolution of the frequency of cooperators as a function of the parameter b for PDG and r for SG is obtained. Initially, an equal percentage of cooperators and defectors is randomly distributed among the elements of the population. Here, we adopt the synchronous updating rule. Each individual will adapt its strategy according to Eq. (5) after each round game. Equilibrium frequencies of cooperators are obtained by averaging over 2000 generations after a transient time of 10 000 generations. Each data point results from averaging over 100 runs. In the following simulations, $T = 10$ is kept invariant [30].

In Fig. 4, we report the results of simulations carried out for both the PDG and the SG in Xiaonei network, as well as in the randomized versions of the same network, which have different levels of assortativity. To this end, the links of original network were randomly reshuffled by the edge-swapping mechanism used in Refs. [31–33] such that the rewiring process does not change the degree of nodes involved and thus the overall degree distribution in the network, but does generate different degrees of asssortativity. Since most of social networks are demonstrated to be assortative, we thus are interested in the effect of different levels of assortative correlation on evolution of cooperation. As is shown in Fig. 4, frequency of cooperators nontrivially decreases with increasing value of the parameters in both games. In particular, cooperation dominates for small value of the parameters in both cases, while for large value of the parameters, cooperation is significantly inhibited in a way that it asymptotically approaches that in the scenario of well-mixed populations. To be concrete, it is found that when $1 < b < 1.35$ for the PDG and $0 < r < 0.45$ for the SG, cooperation is dramatically promoted by the underlying network of interactions, that is, the frequency of cooperators is above 80%. Howbeit, in the PDG cooperation becomes extremely poor with $1.35 < b < 2$, i.e., for $b \to 2$, cooperation level drops sharply from high to very low. For the SG, when $r \to 1$, the coopera-

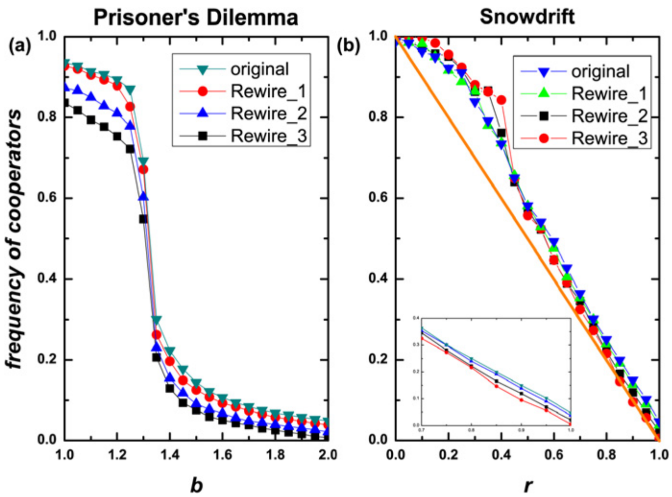

Fig. 4. Evolution of cooperation on Xiaonei network, in comparison with different levels of assortativity generated by random edge rewiring mechanism. Left (right) panel shows frequency of cooperators as a function of temptation to defect b for the PDG (cost-to-benefit ratio r of mutual cooperation for the SG). The inset in right panel indicates the detail around large values of r. Assortativitiy: Original: 0.017; Rewire_1: 0.025; Rewire_2: 0.060; and Rewire_3: 0.104.

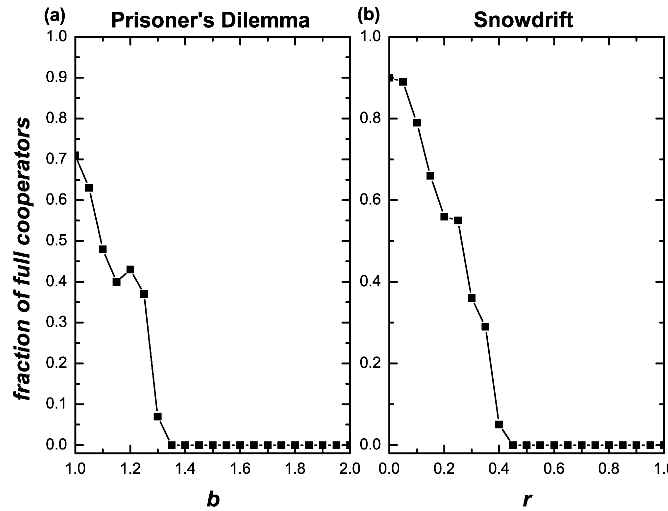

Fig. 5. Fraction of runs which ended up with full cooperators vs. b (r) for PDG (SG). For each value of b (r), we ran 100 simulations, starting from 50% cooperators.

tion level also falls and approaches $1 - r$. Replicator dynamics in well-mixed populations predicts that defection is the only evolutionarily stable strategy (ESS) in PDG, hence cooperators will be wiped out by natural selection, while for the SG the system converges to an equilibrium frequency for cooperators given by $1 - r$, corresponding to the straight line as shown in right panel of Fig. 4. In short, deviations from the well-mixed population limits are fairly prominent for small value of the parameters for both games. Nevertheless, the well-mixed limit is asymptotically recovered for increasing values of parameters.

Let us explain in detail the observed evolution of cooperation in Fig. 4. In order to understand the mutually competitive roles between the underlying network structure and the value of game parameters played in the emergence of cooperation, we look into how the fraction of runs ending up with full cooperators changes along with different values of game parameters, starting from an equal percentage of cooperators and defectors. Relevant results are shown in Fig. 5. It is found that the frequency that Xiaonei network evolves into an absorbing state (full C) decreases with increasing values of b and r. That is, for the PDG (SG), the frequency of converging into full C falls from 70% at $b = 1$ (90% at $r = 0$) to zero with $b > 1.35$ ($r > 0.45$). Additionally, in both games, except for some individual runs ending up with full C, others ended up with low frequency of cooperator on occasion as a result of the different initial distributions of cooperators and defectors among the population. Moreover, when $b > 1.35$ for PDG and $r > 0.45$ for SG, most of the runs ended up with massive defectors, resulting in quite low average frequency of cooperators. Actually, interactions in real-world are heterogeneous, in the sense that different individuals have different numbers of average neighbors with whom they interact with, a feature associated with a power-law dependence of the degree distribution. Previous study on model BA scale-free network, which captures the real-world heterogeneity, found that scale-free networks provide a unifying framework for emergence of cooperation [6,10–12]. Here, our empirical study also provides a convincing evidence that degree heterogeneity is one of the factors promoting cooperation in realistic social networks. As a result, cooperators benefit from the heterogeneity of Xiaonei network in a way. However, the positive role of heterogeneity in evolution of cooperation is to some extent diminished by the increasing parameters b and r. Noticeably, especially for large values of b and r in both games, the cooperation level in Xiaonei network is not as remarkable as that in model scale-free network [Barabási–Albert (BA)], where the cooperation level is mainly affected by heterogeneity. Generally speaking, because our social network incorporates a myriad of topological features, including heterogeneity, high average connectivity, etc, of which effects on evolution of cooperation are additive. Moreover, due to high average degree of our Xiaonei network (about 18.74), the dominance of cooperators is affected in this highly connected network. The role of average connectivity in cooperation has been examined by several previous works. Santos and Pacheco have elegantly shown that increasing average connectivity destroys cooperation in Refs. [6,12]. Besides, as pointed out in Ref. [34], maximum cooperation level occurs at intermediate average degree. Ohtsuki et al. derived an analytic formula for the emergence of cooperation in networked population playing the PDG, namely, the cost-to-benefit ratio has to exceed the average degree. Thus it is extremely difficult for cooperators surviving in dense networks which recover to the well-mixed limit (the limit situation is fully-connected network). Nevertheless, it is totally different when coevolution of network structure and game dynamics happens—cooperators become evolutionarily competitive, even in highly connected networks [35,36]. Overall, compared with well-mixed situation, cooperation is partially enhanced by the heterogeneity of underlying network, but the heterogeneity's role is inhibited substantially with high values of the game parameters. Besides, such nontrivial evolution of cooperation is also affected by the high average connectivity.

In parallel, the influence of increasing degree of assortative correlation on evolution of cooperation is also presented in Fig. 4. It is found that, in both games, increasing amount of assortativity to a certain extent diminishes the cooperation level. Note that in the SG, the effect of different levels of assortativity is not so clear for small value of r, in contrast to the situation of large value of r. Actually, the role of assortative correlation in evolution of cooperation is not so much pronounced in both games, but it is quite recognizable. Furthermore, the qualitative behavior of cooperation is similar with respect to the shape of the curves for both games in the original Xiaonei network and the randomized versions. As a consequence, our case study suggests that the degree of assortative correlation plays a role in the evolution of cooperation. It is thought that the cooperation level is optimum in uncorrelated networks (where the assortativity coefficient is zero) [37]. The Xiaonei network is an assortative one, thus the frequency of cooperators is diminished in a way by such mixing pattern. It is known that assortative correlation means that large degree nodes tend to be connected with nodes with many links and vice versa. Namely, increasing amount of assortative correlation leads to the limit configuration where almost all nodes of the same connectivity are linked only between themselves. When this extreme case occurs, these nearly separated clusters of nodes with the same degree are fully-connected, thus weaken the cooperation extensively. Moreover, the role played by hubs in the evolution of cooperation is highly suppressed in this extreme situation, since C-players occupying the hubs could not directly influence the strategy choice of the low-degree nodes (as they are not directly linked). On the other hand, networks with the same degree distribution can result in different levels of cooperation due to different connection patterns of topological organizations. Hence our investigation to the effect of assortative mixing pattern highlights this point and sheds light on the emergence of cooperation in networked populations.

Further, we study the effect of adding (removing) direct edges between large hubs on the evolution of cooperation. The corresponding results are reported in Fig. 6. Apparently, by removing (adding) direct links between hubs, the equilibrium frequency of cooperators is inhibited (enhanced) for both games. It is found in Refs. [6,10,12] that the appearance of direct links between hubs constitutes sufficient conditions to sustain cooperation in BA scale-free networks. Our results in realistic social network are consistent with the finding of Refs. [6,10,12] in model networks, further validating the mechanism that connected hubs are capable of maintaining cooperation.

Finally, together with heterogeneity, other network effects including average connectivity [34], small-world effect [19], degree-degree correlation [37], randomness in topology [38], etc., play crucial roles in the evolution of cooperation. Rather than investigations on model networks where only one or few features of real-world relationships are present, the evolution of cooperation on empirical social networks, which possess a variety of features in topological organizations, should be understood from a *synthesis* view. In our case, besides the scale-free feature, the online social network has small-world property, hierarchical organizations and assortative mixing pattern. Concerning small-world property of the underlying network, the short average distance promotes the propagation process of cooperators from one node to another. Furthermore, taking account for the hierarchical organizations [due to $C(k)$'s dependence on k], i.e., local well-clustered low-degree nodes, such common cluster structure induces the clustering of cooperators "breeding" each other, leading to the survival and enhancement of cooperation [9]. The mixing pattern also influences the cooperation level substantially. Our Xiaonei network is an assortative one, thus the cooperation is demonstrated to be diminished in a way by such mixing pattern. In our case, the average connectivity of the online network is about 20. To a certain extent, this quantity of average connectivity affects the evolution of cooperation. Consequently, the evolution of cooperation on empirical network is simultaneously affected by these additive factors as the underlying network possesses various characteristics of real-world social interactions. Actually, the combined network effects of these factors facilitate and maintain cooperation among unrelated individuals. Our results may shed some light on the evolution of cooperation in social and economic systems.

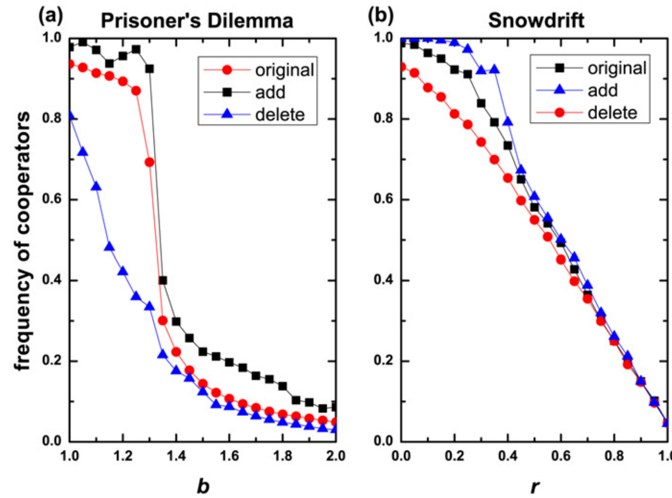

Fig. 6. Frequency of cooperators vs. b in PDG (r in SG), corresponding to add, unchange, and delete direct edges between hubs. Based on the original Xiaonei network, (1) add: we add 29 edges between 12 nodes with connectivity larger than 274; (2) delete: we delete 20 links between the above hubs. (All operations are ensured to keep the changed network connected.) Each data point results from averaging over 100 independent runs.

4. Concluding remarks

In conclusion, we have studied two social dilemmas—PDG and SG in an online social network, Xiaonei. We carried out a detailed topological analysis of the small-scale Xiaonei network, demonstrating that it has small-world and scale-free properties. In addition, it was shown that the underlying network has hierarchical organizations in which low-degree vertices are well-connected in different communities, while large-degree nodes are linked to many nodes that may belong to different groups. We also found that the social network shows assortative mixing pattern. Then we investigated the evolution of cooperation on such empirical social network. We found that cooperation nontrivially decreases with increasing value of the parameters in both games. Namely, for small value of game parameters, cooperation is in part promoted by the heterogeneity of Xiaonei network; while for large value of the parameters, cooperation level asymptotically approaches that of well-mixed scenario. This unfavorable cooperation at large value of parameters is also attributable to high average connectivity of this network. Furthermore, we explored the role of assortativity in evolution of cooperation, based on comparison between results obtained from the original Xiaonei network and randomized versions generated by edge-swapping mechanism, which have different degrees of assortativity. We found that increasing degree of assortative correlation to a certain extent diminishes the cooperation. Finally, we also demonstrated that the appearance of direct links between hubs contributes to the observed sustainment and enhancement of cooperation in our social network.

In general, different from games on model networks, understanding the evolution of cooperation on empirical network should be conducted from a synthesis view because real-world relationship networks incorporate various topological characteristics while model networks generally focus on some specified features. Thus, we conclude that the evolution of cooperation on empirical network is jointly affected by additive network effects, including average connectivity, small-world property, degree heterogeneity (scale-free), degree-degree correlation, hierarchical organizations, etc. Our results may help understand the cooperative behaviors in human societies.

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