Nodal analysis

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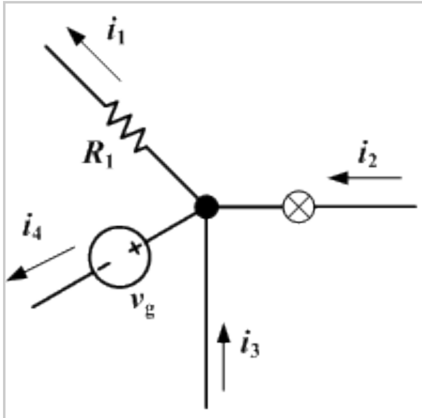


Kirchoff's current law is the basis of nodal analysis.

In electric circuits analysis, **nodal analysis**, **node-voltage analysis**, or the **branch current method** is a method of determining the voltage (potential difference) between "nodes" (points where elements or branches connect) in an electrical circuit in terms of the branch currents.

In analyzing a circuit using Kirchoff's circuit laws, one can either do nodal analysis using Kirchoff's current law (KCL) or mesh analysis using Kirchoff's voltage law (KVL). Nodal analysis writes an equation at each electrical node, requiring that the branch currents incident at a node must sum to zero. The branch currents are written in terms of the circuit node voltages. As a consequence, each branch constitutive relation must give current as a function of voltage; an admittance representation. For instance, for a resistor, $I_{branch} = V_{branch} * G$, where G (=1/R) is the admittance of the resistor.

Nodal analysis is possible when all the circuit elements' branch constitutive relations have an admittance representation. Nodal analysis produces a compact set of equations for the network, which can be solved by hand if small, or can be quickly solved using linear algebra by computer. Because of the compact system of equations, many circuit simulation programs (e.g. SPICE) use nodal analysis as a basis. When elements do not have admittance representations, a more general extension of nodal analysis, modified nodal analysis, can be used.

While simple examples of nodal analysis focus on linear elements, more complex nonlinear networks can also be solved with nodal analysis by using Newton's method to turn the nonlinear problem into a sequence of linear problems.

Contents

Method

1. Note all connected wire segments in the circuit. These are the *nodes* of nodal analysis.
2. Select one node as the ground reference. The choice does not affect the result and is just a matter of

convention. Choosing the node with most connections can simplify the analysis.

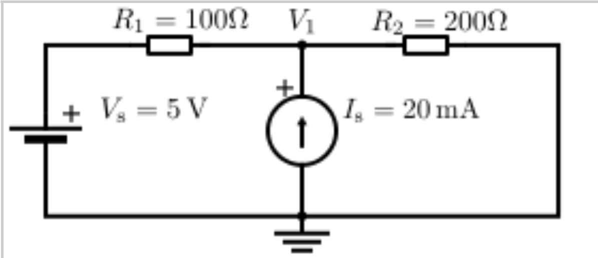
3. Assign a variable for each node whose voltage is unknown. If the voltage is already known, it is not necessary to assign a variable.
4. For each unknown voltage, form an equation based on Kirchoff's current law. Basically, add together all currents leaving from the node and mark the sum equal to zero.
5. If there are voltage sources between two unknown voltages, join the two nodes as a supernode. The currents of the two nodes are combined in a single equation, and a new equation for the voltages is formed.
6. Solve the system of simultaneous equations for each unknown voltage.

Examples

Basic case

The only unknown voltage in this circuit is V_1. There are three connections to this node and consequently three currents to consider. The direction of the currents in calculations is chosen to be away from the node.

1. Current through resistor R_1: $(V_1 - V_S) / R_1$
2. Current through resistor R_2: V_1 / R_2
3. Current through current source I_S: $-I_S$



Basic example circuit with one unknown voltage, V_1.

With Kirchoff's current law, we get:

$$\frac{V_1 - V_S}{R_1} + \frac{V_1}{R_2} - I_S = 0$$

This equation can be solved in respect to V_1:

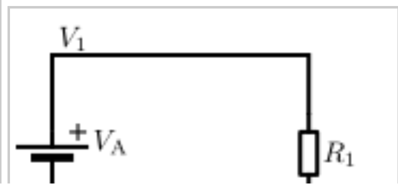
$$V_1 = \left(\frac{V_S}{R1} + I_S\right) : \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Finally, the unknown voltage can be solved by substituting numerical values for the symbols. Any unknown currents are easy to calculate after all the voltages in the circuit are known.

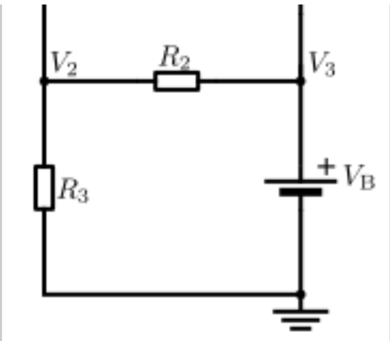
$$V_1 = \left(\frac{5\text{ V}}{100\,\Omega} + 20\text{ mA}\right) : \left(\frac{1}{100\,\Omega} + \frac{1}{200\,\Omega}\right) \approx 4.667\text{ V}$$

Supernodes

In this circuit, we initially have two unknown voltages, V_1 and V_2. The voltage at the positive terminal of V_B is already known because the other terminal is at ground potential.



The current going through voltage source V_A cannot be directly calculated. Therefore we can not write the current equations for either V_1 or V_2. However, we know that the same current leaving node V_2 must enter node V_1. Even though the nodes can not be individually solved, we know that the combined current of these two nodes is zero. This combining of the two nodes is called the supenode technique, and it requires one additional equation: $V_1 = V_2 + V_A$.



In this circuit, V_A is between two unknown voltages, and is therefore a supernode.

The complete set of equations for this circuit is:

$$\begin{cases} \frac{V_1 - V_\mathrm{B}}{R_1} + \frac{V_2 - V_\mathrm{B}}{R_2} + \frac{V_2}{R_3} = 0 \\ V_1 = V_2 + V_\mathrm{A} \end{cases}$$

By substituting V_1 to the first equation and solving in respect to V2, we get:

$$V_2 = \frac{(R_1 + R_2) R_3 V_\mathrm{B} - R_2 R_3 V_\mathrm{A}}{(R_1 + R_2) R_3 + R_1 R_2}$$

See also

- Mesh analysis
- Ybus matrix

External links

- Branch current method (http://www.allaboutcircuits.com/vol_1/chpt_10/2.html)
- Four-node problem solver (http://mazlumi.webng.com/node.asp)

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