Kirchhoff's circuit laws

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Kirchhoff's circuit laws are two equalities that deal with the conservation of charge and energy in electrical circuits, and were first described in 1845 by Gustav Kirchhoff. Widely used in electrical engineering, they are also called Kirchhoff's *rules* or simply Kirchhoff's *laws* (see also Kirchhoff's laws for other meanings of that term).

Both circuit rules can be directly derived from Maxwell's equations, but Kirchhoff preceded Maxwell and instead generalized work by Georg Ohm.



6 External links

Kirchhoff's current law (KCL)

This law is also called **Kirchhoff's point rule**, **Kirchhoff's junction rule** (or nodal rule), and **Kirchhoff's first rule**.

The principle of conservation of electric charge implies that:

At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

Adopting the convention that every current flowing towards the node is positive and that every current flowing away is negative (or the other way around), this principle can be stated as:

$$\sum_{k=1}^{n} I_k = 0$$

n is the total number of branches with currents flowing towards or away from the node.



This formula is also valid for complex currents:

$$\sum_{k=1}^{n} \tilde{I}_k = 0$$

The law is based on the conservation of charge whereby the charge (measured in coulombs) is the product of the current (in amperes) and the time (which is measured in seconds).

Changing charge density

Physically speaking, the restriction regarding the "capacitor plate" means that Kirchhoff's current law is only valid if the charge density remains constant in the point that it is applied to. This is normally not a problem because of the strength of electrostatic forces: the charge buildup would cause repulsive forces to disperse the charges.

However, a charge build-up *can* occur in a capacitor, where the charge is typically spread over wide parallel plates, with a physical break in the circuit that prevents the positive and negative charge accumulations over the two plates from coming together and cancelling. In this case, the sum of the currents flowing into *one* plate of the capacitor is *not* zero, but rather is equal to the rate of charge accumulation. However, if the displacement current d**D**/dt is included, Kirchhoff's current law once again holds. (This is really only required if one wants to apply the current law to a point on a capacitor *plate*. In circuit analyses, however, the capacitor as a whole is typically treated as a unit, in which case the ordinary current law holds since exactly the current that enters the capacitor on the one side leaves it on the other side.)

More technically, Kirchhoff's current law can be found by taking the divergence of Ampère's law with Maxwell's correction and combining with Gauss's law, yielding:

$$\nabla \cdot \mathbf{J} = -\nabla \cdot \frac{\partial \mathbf{D}}{\partial t} = -\frac{\partial \rho}{\partial t}$$

This is simply the charge conservation equation (in integral form, it says that the current flowing out of a closed surface is equal to the rate of loss of charge within the enclosed volume (Divergence theorem)). Kirchhoff's current law is equivalent to the statement that the divergence of the current is zero, true for time-invariant ϱ , or always true if the displacement current is included with **J**.

Uses

A matrix version of Kirchhoff's current law is the basis of most circuit simulation software, such as SPICE.

Kirchhoff's voltage law (KVL)

This law is also called Kirchhoff's second law. Kirchhoff's loon (or

mesh) rule, and Kirchhoff's second rule.

The directed sum of the electrical potential differences(voltage) around any closed circuit must be zero.

Similarly to KCL, it can be stated as:

$$\sum_{k=1}^{n} V_k = 0$$

Here, n is the total number of voltages measured. The voltages may also be complex:

$$\sum_{k=1}^{n} \tilde{V}_k = 0$$

This law is based on the conservation of "energy given/taken by potential field" (not including energy taken by dissipation). Given a voltage potential, a charge which has completed a closed loop doesn't gain or lose energy as it has gone back to initial potential level.

This law holds true even when resistance (which causes **dissipation** of energy) is present in a circuit. The validity of this law in this case can be understood if one realizes that a charge in fact doesn't go back to its starting point, due to dissipation of energy. A charge will just terminate at the negative terminal, instead of positive terminal. This means all the energy given by the potential difference has been fully consumed by resistance which in turn loses the energy as heat dissipation.

To summarize, Kirchhoff's voltage law has nothing to do with gain or loss of energy by electronic components (resistors, capacitors, etc). It is a law referring to the potential field generated by voltage sources. In this potential field, regardless of what electronic components are present, the gain or loss in "energy given by the potential field" must be zero when a charge completes a closed loop.

Electric field and electric potential

Kirchhoff's voltage law as stated above is equivalent to the statement that *a single-valued electric potential can be assigned to each point in the circuit* (in the same way that any conservative vector field can be represented as the gradient of a scalar potential). Then the sum of the changes in this potential that occur as one makes an imaginary traverse around any closed loop in the circuit should be equal to zero.

This could be viewed as a consequence of the principle of conservation of energy. Otherwise, it would be possible to build a perpetual motion machine that passed a current in a circle around the circuit.

Considering that electric potential is defined as a line integral over an electric field, Kirchhoff's voltage law can be expressed equivalently as

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0,$$

which states that the line integral of the electric field around closed loop C is zero.



In order to return to the more special form, this integral can be "cut in pieces" in order to get the voltage at specific components.

This is a simplification of Faraday's law of induction for the special case where there is no fluctuating magnetic field linking the closed loop. Therefore, it practically suffices for explaining circuits containing only resistors and capacitors.

In the presence of a changing magnetic field the electric field is not conservative and it cannot therefore define a pure scalar potential—the line integral of the electric field around the circuit is not zero. This is because energy is being transferred from the magnetic field to the current (or vice versa). In order to "fix" Kirchhoff's voltage law for circuits containing inductors, an effective potential drop, or electromotive force (emf), is associated with each inductance of the circuit, exactly equal to the amount by which the line integral of the electric field is not zero by Faraday's law of induction.

See also

- Kirchhoff's laws
- Faraday's law of induction
- Gauss's law
- Maxwell's equations
- Conservation of energy
- Mesh analysis
- SPICE (Simulation Program with Integrated Circuits Emphasis)

References

- Paul, Clayton R. (2001). Fundamentals of Electric Circuit Analysis. John Wiley & Sons. ISBN 0-471-37195-5.
- Serway, Raymond A.; Jewett, John W. (2004). *Physics for Scientists and Engineers (6th ed.)*. Brooks/Cole. ISBN 0-534-40842-7.
- Tipler, Paul (2004). Physics for Scientists and Engineers: Electricity, Magnetism, Light, and Elementary Modern Physics (5th ed.). W. H. Freeman. ISBN 0-7167-0810-8.

External links

- http://www.sasked.gov.sk.ca/docs/physics/u3c13phy.html
- MIT video lecture (http://academicearth.org/lectures/basic-circuit-analysis-method-kvl-and-kclmmethod) on the KVL and KCL methods

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