Norton's theorem

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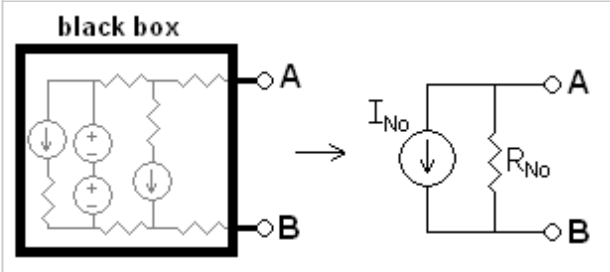
Norton's theorem for linear electrical networks, known in Europe as the **Mayer–Norton theorem**, states that any collection of voltage sources, current sources, and resistors with two terminals is electrically equivalent to an ideal current source, I, in parallel with a single resistor, R. For single-frequency AC systems the theorem can also be applied to general impedances, not just resistors. The **Norton equivalent** is used to represent any network of linear sources and impedances, at a given frequency. The circuit consists of an ideal current source in parallel with an ideal impedance (or resistor for non-reactive circuits).

Norton's theorem is an extension of Thévenin's theorem and was introduced in 1926 separately by two people: Hause-Siemens researcher Hans Ferdinand Mayer (1895–1980) and Bell Labs engineer Edward Lawry Norton (1898–1983). Only Mayer actually published on this topic, but Norton made known his finding through an internal technical report at Bell Labs.[1]

Contents



Any black box containing only voltage sources, current sources, and resistors can be converted to a Norton equivalent circuit.

Calculation of a Norton equivalent circuit

The Norton equivalent circuit is a current source with current I_{No} in parallel with a resistance R_{No}. To find the equivalent,

1. Find the Norton current I_{No}. Calculate the output current, I_{AB}, with a short circuit as the load (meaning 0 resistance between A and B). This is I_{No}.
2. Find the Norton resistance R_{No}. When there are **no dependent sources** (i.e., all current and voltage sources are **independent**), there are two methods of determining the Norton impedance R_{No}.

   - Calculate the output voltage, V_{AB}, when in open circuit condition (i.e., no load resistor — meaning infinite load resistance). R_{No} equals this V_{AB} divided by I_{No}.

   or

   - Replace independent voltage sources with short circuits and independent current sources with open circuits. The total resistance across the output port is the Norton impedance R_{No}.

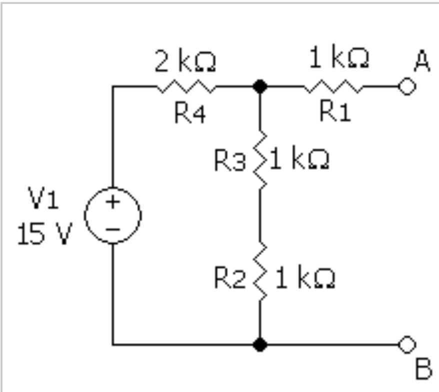
   or

   - Use a given Thevenin resistance: as the two are equal.

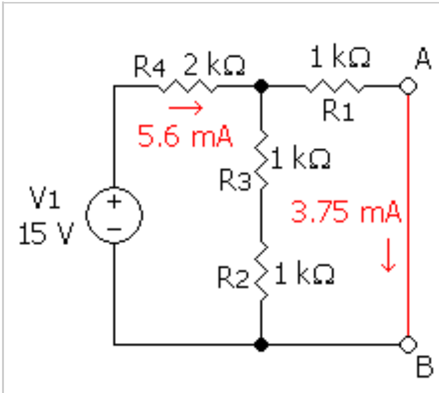
However, when there are **dependent sources**, the more general method must be used. This method is not shown below in the diagrams.

- Connect a constant current source at the output terminals of the circuit with a value of 1 Ampere and calculate the voltage at its terminals. The quotient of this voltage divided by the 1 A current is the Norton impedance R_{No}. This method must be used if the circuit contains dependent sources, but it can be used in all cases even when there are no dependent sources.

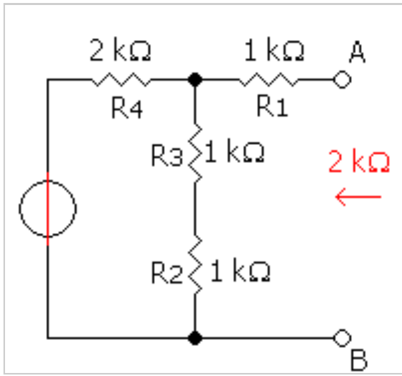
Example of a Norton equivalent circuit



Step 0: The original circuit



Step 1: Calculating the equivalent output current



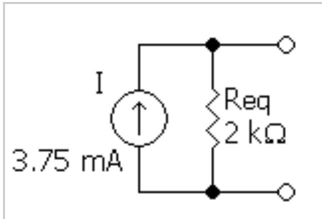
Step 2: Calculating the equivalent resistance

In the example, the total current I_{total} is given by:

$$I_{\text{total}} = \frac{15\text{V}}{2\,\text{k}\Omega + 1\,\text{k}\Omega \| (1\,\text{k}\Omega + 1\,\text{k}\Omega)} = 5.625\text{mA}$$

The current through the load is then, using the current divider rule:

$$I = \frac{1\,\text{k}\Omega + 1\,\text{k}\Omega}{(1\,\text{k}\Omega + 1\,\text{k}\Omega + 1\,\text{k}\Omega)} \cdot I_{\text{total}}$$
$$= 2/3 \cdot 5.625\text{mA} = 3.75\text{mA}$$



Step 3: The equivalent circuit

And the equivalent resistance looking back into the circuit is:

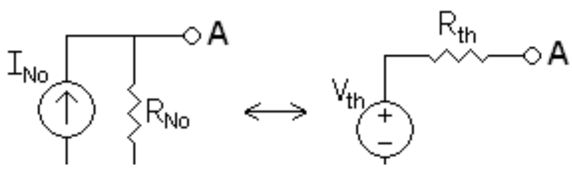
$$R = 1\,\text{k}\Omega + 2\,\text{k}\Omega \| (1\,\text{k}\Omega + 1\,\text{k}\Omega) = 2\,\text{k}\Omega$$

So the equivalent circuit is a 3.75 mA current source in parallel with a 2 kΩ resistor.

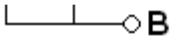
Conversion to a Thévenin equivalent

A Norton equivalent circuit is related to the Thévenin equivalent by the following equations:

$$R_{Th} = R_{No}$$
$$V_{Th} = I_{No} R_{No}$$



$$V_{Th}/R_{Th} = I_{No}$$



See also

- Thévenin's theorem

References

1. ^ Norton Biography at ECE/Rice University website (http://www-ece.rice.edu/~dhj/norton/)

External links

- Origins of the equivalent circuit concept (http://tcts.fpms.ac.be/cours/1005-01/equiv.pdf)
- Norton's theorem at allaboutcircuits.com (http://www.allaboutcircuits.com/vol_1/chpt_10/9.html)

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