An Observer for Phased Microphone Array Signal Processing with Nonlinear Output

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Abstract

Acoustic arrays have become an important testing tool in noise identification for industry applications, where the typical beamforming algorithm has been adopted as a classical processing technique. In most practical cases the beamforming computations have to be conducted off-line due to the excessive computational requirements. An alternative algorithm with a real-time capability is proposed. The algorithm is similar to a classic observer while array processing is performed in the frequency domain. The performance of this observer-based algorithm is studied here through comparing with the typical beamforming method, particularly for a case of coherent noise sources. In this paper it is shown that the observer-based algorithm could resolve the coherence restriction between the background noise and the signal of interest. The proposed algorithm is also beneficial for its capability of operating over sampling blocks recursively. The convergence rate of this recursive algorithm is fast enough to satisfy the requirements for practical cases. The experimental efforts could be saved extensively as any testing defects could be revealed instantaneously and corrected on site. In addition, this innovative approach provides an alternative perspective, from which many techniques already developed in control and filtering could be extended to this new application area of array processing.

Keywords: state observer; aeroacoustic beamforming; convergence rate; coherent sources; array processing

1. Introduction

The aircraft nuisance noise has a great negative effect on communities near airports. The noise is produced by the complex interaction of aerodynamic surfaces and the surrounding turbulence flow [1]. The distribution of noise sources must be identified firstly for the proper development of a more silent design [2]. Phased microphone array [3] is being increasingly used in aerospace industry [4] to determine noise source distributions. Various array processing algorithms [5], such as typical beamforming[5] and robust adaptive beamforming[6], have been proposed. The typical beamforming method is the most popular for its robust performance in a noisy aeroacoustic testing environment.

The diameter of a microphone array is becoming larger than ever before to meet the strict needs in aeroacoustic tests. The modern arrays have larger numbers of microphone elements to improve resolution and to avoid spatial aliasing[7]. However it is extremely difficult to process the huge amount of data produced by the array elements in real-time if the typical beamforming algorithm is adopted. An alternative algorithm which borrows the idea of state observer in control theory is proposed in this work. This so-called observer-based algorithm can be especially useful for field tests for its real-time capability, by which aeroacoustic sources can be identified recursively over sampling blocks and any defects in on-site experiments can be discovered instantaneously.

The performance of the observer-based algorithm compared to the typical beamforming algorithm is studied in this work, particularly for a case of coherent noise sources. The background knowledge, derivation and related mathematical equations are also included. The typical beamforming algorithm and its limitations are described in Section 2. Section 3 introduces the basic observer-based algorithm and raises another problem described by the phase difference ϕ. This problem is successfully solved in Section 4, and a summary of this work is presented in Section 5.

2. Typical Beamforming Formulations

For an array with N microphones, the output y consists of measurements of the N microphones, which says $y \epsilon \mathbb{R}^{N\times 1}$. The time series $y(t)$ is subsequently cut into K blocks, DFT (discrete fourier transform) can be performed over each block to produce the counterpart in frequency domain, that is, $Y(\omega)|_k$ for the Kth block at an angular frequency of ω. The size of each block

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is typically chosen to be 4096 which enables the efficient computation of DFT. Generally for most aero-acoustic applications beamforming is conducted in frequency domain which includes the assumption that the signal of interest is almost stationary and its statistical properties remain stable during the sampling.

Assuming that a noise distribution has M sources at distinctive locations, $X^{M\times 1}=[X_1,\dots,X_M]' \in \mathbb{C}^{M\times 1}$, the measurement of microphones should satisfy

$$Y^{N\times 1}|_k = G^{N\times M}X^{M\times 1}|_k\ , \qquad (1)$$

where $G^{N\times M}$ is steering vector matrix, which is also known as propagation matrix. To produce an acoustic image, the microphone array scans over the plane of interest which is divided by a set of grid points. For each grid point, the measurement of microphones can be simplified to $Y|_k = GX|_k$. The solution of the typical beamforming algorithm is given by[7]

$$X|_k = w'\ Y,\ w = G/||G||^2\ , \qquad (2)$$

where w is the weighting vector that can be obtained using a pseudoinverse over Eq. (1). The steering vector $\mathrm{G}=\frac{1}{4\pi r}\mathrm{e}^{-j\omega\tau}$, where $\mathrm{r}\epsilon\mathbb{R}^{N\times 1}$ is the distance between the noise source X and each microphone, and $\tau = r/c$, *c* is the speed of sound. So it is clear that this time the dimension of *G* is $N\times 1$.

When there is background noise besides signal of interest, Eq. (1) can be written as

$$Y_B = GX_B\ , \qquad (3)$$

$$Y_{BS} = GX_{BS}\ , \qquad (4)$$

the subscript of B denotes the experiments without the installment of any model. The measurement Y_B comes solely from background noise. The subscript of BS denotes the experiments with the presence of a model, and Y_{BS} records the combined effect of the sound from the test model and noise from the testing facility.

To obtain the testing model sound X_S, the following operations can be conducted:

$$A_B = Y_B Y_B{}^*\ ,\ A_{BS} = Y_{BS}Y_{BS}{}^*\ , \qquad (5)$$

$$< A_S >= (1/4\pi r)^2 < X_S X_S{}^* > \approx < A_{BS} > - < A_B >, \qquad (6)$$

where the superscript of * denotes the conjugate transpose, and <> denotes the arithmetic averaging. The averaged sound source $< X_S X_S{}^* >$ can be straightforwardly calculated from Eq. (6). X_S is then given by $||X_S|| = \sqrt{(G)^* A_S G/||G||^4}$. Here again employed the assumption of stationary signal which enables us to use arithmetic average of signals in the-calculation of X_S. Another assumption adopted is that there is little coherence between X_B and X_S. Therefore, $< X_B X_S{}^* >= 0, < X_B{}^*\ X_S >= 0$, and the approximation in Eq. (6) is valid. However, the time-consuming operation of (<>) prevents identifying noise sources in real-time, while the assumption of little coherence could be invalid in some cases of coherent or extended sources.

3. Observer-Based Algorithm

Let us begin by giving a brief introduction to the state observer. A time-invariant linear system can be represented in continuous-time in the following form:

$$\dot{x}(t) = Ax(t) + Bu(t)\ , \qquad (7)$$

$$y(t) = Cx(t)\ , \qquad (8)$$

where *x* is the state vector, the symbol of "." denotes *d/dt*, *y* is the output. *A* is the state matrix, *B* is the input matrix, and *C* is the output matrix. Eq. (7) describes the dynamics of *x*. Eq. (8) is the measurement equation, which has a similar form to Eq. (2).

The observability of the above system is defined through the matrix[8]:

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{M-1} \end{bmatrix}, \qquad (9)$$

if and only if the rank of *O* equals the rank of *A*, the system is observable.

A classical state observer (so-called Luenberger observer) can then be used to approximate *x* from *y*, which has the form of

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t))\ , \qquad (10)$$

$$\hat{y}(t) = C\hat{x}(t), \qquad (11)$$

where $\hat{x}(t)$ is the approximation of *x*, and *L* is observer gain. The estimation error can be given by

$$\dot{e} = (A - LC)e\ , \qquad (12)$$

where $e \triangleq x - \dot{x}$ denotes the estimation error, which converges to zero when $t \to \infty$, as long as the all the eigenvalues of the matrix *(A-LC)* have negative real parts.

An innovative method which has a form similar to the classical state observer in control theory is proposed in this work. A state equation has to be included along with the measurement equation to form a complete linear system. The system is

$$X|_{k+1} = AX|_k\ , \qquad (13)$$

$$Y|_k = GX|_k\ . \qquad (14)$$

A is an identity for any stationary signal process. The observability of this system can be studied by forming the observability matrix:

$$O = \begin{bmatrix} G \\ GA \\ \vdots \\ GA^{M-1} \end{bmatrix}, \qquad (15)$$

where the dimension of *G* is $N\times M$. From Eq. (15) it is easy to see that $rank(O) \le min(N,M)$, while $rank(A) = M$. Hence the number of signals of interest (*M*) should be less than the number of microphones (*N*) in an array to maintain observability. At the same time, the positions of microphones must ensure that the steering matrix be of full rank.

Hence, an observer can be designed to approximate the state *X*. Specifically, the linear system equations for each gridpoint is:

$$X|_{k+1} = AX|_k\ , \qquad (16)$$

$$Y|_k = GX|_k\ . \qquad (17)$$

Note here that the dimension of *G* in Eq. (14) is $N\times M$, while the dimension of *G* in Eq. (17) is $N\times 1$. For this case the observability can easily be maintained since the ranks of O and A are both 1. The corresponding observer has the form of

$$\hat{X}|_{k+1} = A\hat{X}|_k + L(Y|_k - \hat{Y}|_k), \tag{18}$$

$$\hat{Y}|_k = G\hat{X}|_k. \tag{19}$$

The symbol of ^ denotes the estimation of the state by the observer, and L is observer gain. We can obtain the whole approximation of X which is a matrix of $M \times 1$, provided that there are M such observers, each for one component of X.

The estimation error is given by

$$\hat{e}_{X|_{k+1}} = (A - LG)\hat{e}_{X|_k}, \tag{20}$$

where $\hat{e}_{X|_k} = X|_k - \hat{X}|_k$ is the estimation error by the observer. For this discrete-time system, $\hat{e}_{X|_{k+1}}$ converges to zero when $k \to \infty$ as long as all the eigenvalues P of the matrix (A-LG) is within a unit circle[8]. The observer gain L at $P = 0$ equals the weighting vector w of the typical beamforming algorithm and varies at other values of P.

When there is background noise, the linear system can be modified to state the background noise explicitly:

$$\begin{bmatrix} X_B|_{k+1} \\ X_S|_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} X_B|_k \\ X_S|_k \end{bmatrix}, \tag{21}$$

$$\begin{bmatrix} Y_B|_k \\ Y_{BS}|_k \end{bmatrix} = \begin{bmatrix} G & 0 \\ Ge^{j\emptyset} & G \end{bmatrix} \begin{bmatrix} X_B|_k \\ X_S|_k \end{bmatrix}. \tag{22}$$

The corresponding state observer are:

$$\begin{bmatrix} \hat{X}_B|_{k+1} \\ \hat{X}_S|_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} \hat{X}_B|_k \\ \hat{X}_S|_k \end{bmatrix} + L\left(\begin{bmatrix} Y_B|_k \\ Y_{BS}|_k \end{bmatrix} - \begin{bmatrix} G & 0 \\ Ge^{j\emptyset} & G \end{bmatrix} \begin{bmatrix} \hat{X}_B|_k \\ \hat{X}_S|_k \end{bmatrix} \right) \tag{23}$$

It is easy to conduct separate experiments in acoustic applications to acquire Y_B and Y_{BS} respectively. The symbol of ϕ denotes the phase difference of the background noise between the two experiments. ϕ can be approximated in practical applications by checking the correlation between Y_B and Y_{BS}. However, for some cases it is impossible to achieve the desired accuracy. The resolution of this problem is discussed in the following section.

4. New Observer-Based Algorithm

We can introduce another system for the estimation of ϕ. The system is given as follows:

$$\emptyset_{k+1} = \emptyset_k, \tag{24}$$

$$\begin{bmatrix} Y_B|_k \\ Y_{BS}|_k \end{bmatrix} = \begin{bmatrix} G & 0 \\ Ge^{j\emptyset} & G \end{bmatrix} \begin{bmatrix} X_B|_k \\ X_S|_k \end{bmatrix}. \tag{25}$$

The measurement function is the same as Eq. (22), however, because this time the state is ϕ rather than X, the system is no longer linear. We can get the approximation of the state ϕ using the equations from the extended kalman filter (EKF) as follows:

$$\hat{\emptyset}|_{k+1} = \hat{\emptyset}|_k + P|_k^- H^*(HP|_k^- H^* + R)^{-1}(Y|_k - \hat{Y}|_k), \tag{26}$$

where $H = \partial Y / \partial \emptyset$, Y is the output term in Eq. (25). $P|_k^-$ represents the covariance of priori estimation error of the EKF, R is the noise term.

Though Eq. (26) seems somewhat complicated, by choosing proper values for the coefficients in Eq. (26) we can come to

$$\hat{\emptyset}|_{k+1} = \hat{\emptyset}|_k + m(Y|_k - \hat{Y}|_k). \tag{27}$$

m is a constant larger than 0 . Similar idea can be found in literature[9]. The estimation error is given by

$$e_{\emptyset_{k+1}} = (1 - mH^*H)e_{\emptyset_k}. \tag{28}$$

Thus, we can easily calculate out proper value for m to make sure that the error converges.

In summary, the observer-based approach can be implemented through Eq. (23) and Eq. (27) recursively. Applications of the algorithm and its benefits are demonstrated in Figure 1. Two coherent monopole sources of the identical strength are shown in Figure 1(a), which collectively produce the Y_{BS} at a microphone array of 63 sensors. In addition, it is assumed that Y_B of the coherence noise can be measured separately. The source of the interest X_S [the top left one in Figure 1(a)] is approximated by typical beamforming and the observer-based algorithm, respectively. Figure 1(b) shows the estimation of X_S by typical beamforming that is averaged over 100 blocks. It can be seen that the beamforming outcome is detrimentally affected by the coherent noise. Figure 1(c) shows the recursive result of observer-based algorithm at the 10th block. Figure 1(d) shows the recursive result of observer-based algorithm at the 500th block. It can be seen that the interference from the coherent noise is almost completely suppressed, and the sound of interest is satisfactorily restored. Figure 2 shows that the approximation of ϕ quickly converges in the iterations. Finally, it is worthwhile to mention that aeroacoustic images are normally generated separately at various single frequencies. The results presented in Figures 1-2 are at f = 3 kHz. The same working procedure of the aforementioned observer-based algorithm can be followed straightforwardly to achieve results at other frequencies.

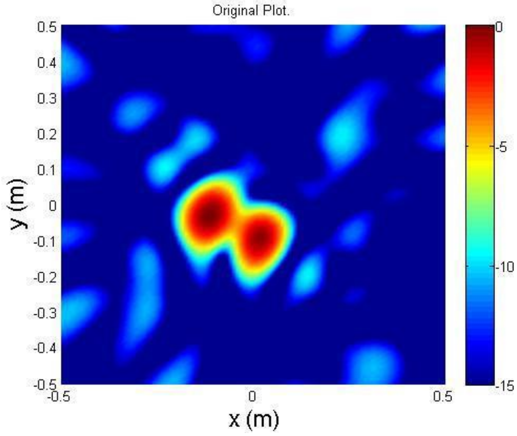

Fig. 1(a) Two coherent sources

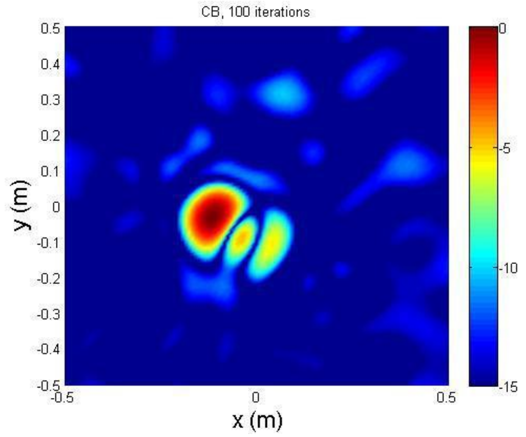

Fig. 1(b) Conventional beamforming, after 100 blocks.

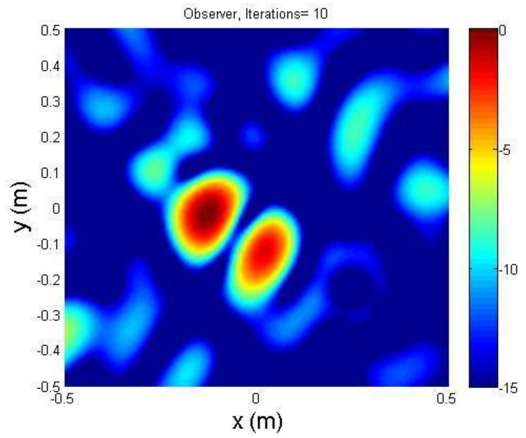

Fig. 1(c) Observer, after 10 blocks.

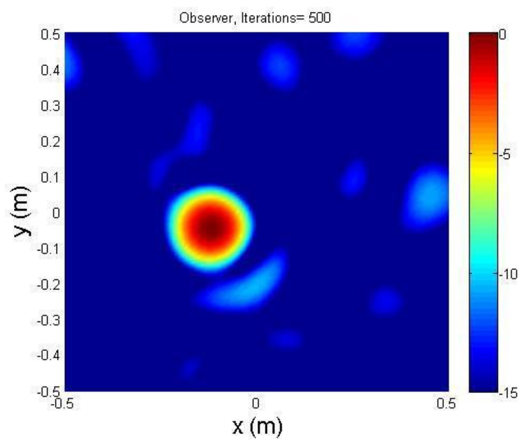

Fig. 1(d) Observer, after 500 blocks.

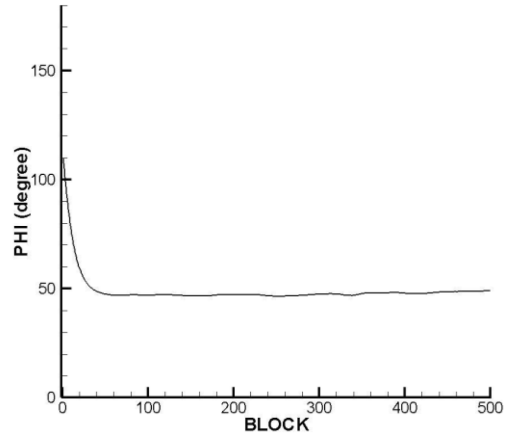

Fig.2 Estimation of Φ during 500 blocks..

5. Summary

A new approach has been proposed as an alternative of typical beamforming. The corresponding signal processing can be conducted in real-time because the observer-based algorithm is recursive, and the defects in measurements can be detected instantaneously and rectified accordingly. More importantly, the algorithm can identify coherent noise sources. Compared to classical beamforming, the incoherence assumption is not present in the new approach, which is especially valuable and interesting for array signal processing. The other assumptions generally adopted in the beamforming methods, such as a free space of sound propagation, are still accepted in this observer-based method. It is important to note that the beam patterns of both classical beamforming and the observer-based algorithm are comparable. Related discussion, along with the investigation of convergence error and speed, can be found in the reference[2]. Advanced signal processing techniques can be employed to further improve the resolution and accuracy of classical beamforming, as well as the observer-based algorithm. Those algorithms have expensive computational costs and can only be conducted off-line. Interested readers can refer to the references[10-12].

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