Adaptive mesh refinement computation of acoustic radiation from an engine intake

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Abstract

A block-structured adaptive mesh refinement (AMR) method is applied to the problem of computing acoustic radiation from an aeroengine intake. The aim is to improve the efficiency of the computation through reduction of computational cells. A parallel implementation of the adaptive mesh refinement algorithm is achieved using message passing interface. It combines a range of 2nd- and 4th-order spatial stencils, a 4th-order low-dissipation and low-dispersion Runge-Kutta scheme for time integration and several different interpolation methods. To solve the problem of acoustic radiation from an aeroengine intake, the code is extended to support body-fitted grid structures. The problem of acoustic radiation is solved with linearised Euler equations. The results demonstrate the efficiency of the current adaptive mesh refinement algorithm.

1 Introduction

STRINGENT noise regulation requirements for modern aircraft have promoted research into efficient and accurate numerical methods capable of predicting aircraft noise, e.g. engine intake noise radiation. The physical process of noise generation and radiation is governed by the Navier-Stokes equations. At present, a full numerical solution of noise generation, propagation and radiation process using the Navier-Stokes equations is not feasible. However, certain aspects of the noise propagation and radiation process can be modelled by linearised equations. For example, in the duct upstream of the rotor-stator region of an aeroengine, where nonlinear and viscous noise generation effects are minimal, the propagation of the rotor-stator noise can be studied using the inviscid linearised equations about the mean flow. A significant amount of research has been undertaken to develop theoretical and computational methods to predict engine tone noise propagation and radiation. However, the development of a cheap and quick computational method is still a challenging job. Of the three main numerical approaches for engine duct noise propagation and radiation problems, boundary element (BE) methods[1] are confined to problems of acoustics through uniform mean flows. Finite/infinite element (FE/IE) methods[2] are generally restricted to acoustic propagation through irrotational mean flows. Computational aeroacoustic (CAA) methods based upon the Euler or linearised Euler equations (LEE) are much general in terms of governing physics[3]. However, CAA methods are more expensive. Realistic engineering applications of CAA methods call for continuous research into efficient computational schemes/methods.

AMR is efficient and effective in treating problems with multiple spatial and temporal scales[4]. It represents computational domain as hierarchical refinement levels and increases points per wavelength only in areas of interest. The computational efficiency is improved by reducing the number of computational cells. This work extends an earlier effort[5] where a block-structured AMR code was constructed and tested against benchmark problems on rectangular meshes. In order to solve aeroacoustic problems of practical significance, e.g. acoustic radiation from a general aeroengine intake, the current code is extended to support body-fitted meshes and works on parallel machines using message passing interface (MPI) library.

The rest of this paper includes a brief description of the parallel AMR algorithm. The basic idea is explained and the terms of AMR are defined, followed by an introduction of the numerical schemes. A benchmark problem is then solved to illustrate the convergence rate of these schemes on an adaptively refined mesh. Finally, AMR is applied to solve the problem of acoustic radiation from a generic aeroengine intake. The case is governed by linearised Euler equations (LEE) and computed on a mesh that is adaptively refined according to the criterion of perturbation velocity gradients. Issues such as short wavelength spurious wave generation at coarse-fine block interfaces on the hierarchical meshes are discussed. Filter [6], artificial selective damping [7], and multigrid prolongation[8] are used to treat the problem. The

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accuracy of the prediction is compared with the earlier efforts\[3, 9, 10\]. The results demonstrate both the accuracy and the efficiency of the current AMR method. Costs and the parallel speedup performance are given at the end of the paper.

2 The Parallel AMR Algorithm

On parallel machines, the existing AMR applications [11, 12, 13] generally employ a block-structured AMR algorithm. It involves a) representing the two-dimensional (2D)/three-dimensional (3D) hierarchical computational domain in blocks, b) connecting the generated blocks in a quadtree/octree data structure, c) estimating local truncation errors at all grid points and identifying blocks with excessive errors, d) regridding the identified blocks by superimposing or removing blocks to accommodate changes in flow physics, and e) redistributing computational load between processors to maintain dynamic load balancing. This procedure is operated recursively until either a given refinement/coarsening level is reached or a predefined local truncation error level has been met. After regridding, the initial conditions of the newly generated blocks are inherited from their base blocks. This operation is referred to as the AMR prolongation operation. Conversely, after each computing step, the solutions on the finer blocks should be used to update the solutions of the corresponding base blocks to maintain the desired accuracy. This is known as the AMR restriction operation. To solve partial differences of cells located near a block boundary, an extra area surrounding each block is required. This operation is referred to as the ghost construction in the following description.

In Fig. 1 a schematic of the block-structured AMR method employed in this work is given. The example is a benchmark problem of acoustics scattering [14] solved with body-fitted multi-block AMR. The display style is chosen to illustrate the hierarchical structure of the adaptively refined mesh more clearly. In reality meshes on the fine levels are superimposed on the coarse meshes. In this example, a nested mesh consisting of three refinement levels is created at the start of the computation. The refinement ratio between the consecutive coarse and fine levels is two. The AMR regridding operation defines the relationships between the blocks as parents/children or sibling according to the type of their connection, stores the hierarchy information in the data structure of quadtree, and either refines or coarsens the hierarchical meshes based on the gradients of perturbation velocity. As the simulation progresses, the meshes are dynamically updated to reflect the evolving solution. The prolongation operation provides the initial solutions on the newly generated blocks, and the restriction operation updates the solutions on the coarse blocks.

Parallel strategies are different from the AMR operations of regridding, prolongation, restriction and ghost construction. To achieve high efficiency, separate communication subroutines were designed in the aforementioned AMR programs for each AMR operation. Consequently, these programs tend to be complex and inaccessible. For this work, a simplified AMR library was developed as a starting point for our work in the area of CAA. The main goal is to realise the parallel AMR operations as simply as possible. The essential decision is to combine the parallel communication of the AMR operations together to reduce the code complexity. It is accepted that some efficiency is sacrificed for the simplicity of the code. To clarify the concerns over the potential cost penalties, the cost of each AMR operation is revealed by profiling the whole code.

3 Numerical Schemes

A CAA algorithm includes ingredients such as high-order spatial stencils, temporal schemes, inflow/outflow and surface conditions. The examples presented in this paper all use the explicit form of buffer zone techniques [15] as the outflow condition. To examine the global convergence rate on an adaptively refined mesh, the benchmark case of a Gaussian pulse propagation in a stationary medium [16] is used. The problem is governed by the following equations:

\[
\frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0, \\
\frac{\partial v'}{\partial t} + \frac{\partial p'}{\partial y} = 0, \\
\frac{\partial p'}{\partial t} + \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0,
\]

with the initial condition given by

\[
p'(x, y, 0) = e^{-(\frac{x^2+y^2}{2})}, \quad u'(x, y, 0) = 0, \quad v'(x, y, 0) = 0,
\]

where \(x\) and \(y\) are the Cartesian coordinates; \(t\) is time; \(u'\) and \(v'\) are velocity perturbations in the \(x\) and \(y\) directions respectively; \(p'\) is pressure perturbation. The computation domain covers an area of \(-8 \leq x \leq 8\) and \(-8 \leq y \leq 8\).
4 Acoustic radiation from an engine intake

In a previous work [5] the method of AMR was used in the computation of acoustic radiation along and away from an unflanged cylindrical duct. Here the method is extended to a generic aeroengine intake with a realistic background mean flow, as shown in Fig. 4, where a high engine power setting is used. The condition is referred to as sideline condition in the rest of the paper. This section begins with an introduction of the governing equations and numerical methods. Calculations are based upon the radiation of realistic acoustic modes generated by the engine fan and fan/stator flow interactions. The computational results of the far-field directivity are compared with the results computed on a fixed mesh solving linearised Euler equations and the predictions obtained from an established FEM solver [10]. The cost of AMR operations is illustrated by profiling the code on both the single- and multi-processors.

4.1 Governing Equations

The governing equations are linearised Euler equations. If the acoustic disturbances are restricted to the multiples of the blade passing frequency and propagate on an axisymmetric mean flow field without swirl, it will be possible to write the disturbances in terms of a Fourier series, e.g. for the pressure disturbance $p'$ at a single frequency $k$ the series is

$$p' = \sum_{m=0}^{\infty} p'_{m}(x, r)e^{i(kt-m\theta)},$$

where $x$ is the axial coordinate, $r$ the radial coordinate and $\theta$ the circumferential angle. Consequently, there are two important relations for the circumferential velocity disturbance $w'$ and the pressure disturbance $p'$ correspondingly. They are

$$\frac{\partial w'}{\partial \theta} = -\frac{m}{k} \frac{\partial w'}{\partial t}, \quad \frac{\partial^2 p'}{\partial \theta^2} = mk^2.$$

The resulting set of equations are generally called 2.5D equations [21]. The complete governing equations in cylindrical coordinates for a single blade passing frequency $k$ are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho' u_0 + \rho_0 u')}{\partial x} + \frac{\partial (\rho' v_0 + \rho_0 v')}{\partial r} - \frac{\rho_0}{kr} u' + \frac{\rho' v_0 + \rho_0 v'}{r} = 0,$$

$$\frac{\partial u'}{\partial t} + u_0 \frac{\partial u'}{\partial x} + v_0 \frac{\partial u'}{\partial r} + (u' + \frac{\rho'}{\rho_0} u_0) \frac{\partial u_0}{\partial x} + (v' + \frac{\rho'}{\rho_0} v_0) \frac{\partial u_0}{\partial r} = 0,$$

$$\frac{\partial v'}{\partial t} + u_0 \frac{\partial v'}{\partial x} + v_0 \frac{\partial v'}{\partial r} + (u' + \frac{\rho'}{\rho_0} u_0) \frac{\partial v_0}{\partial x} + (v' + \frac{\rho'}{\rho_0} v_0) \frac{\partial v_0}{\partial r} = 0,$$

$$\frac{\partial p'}{\partial t} + u_0 \frac{\partial p'}{\partial x} + v_0 \frac{\partial p'}{\partial r} + \frac{mk}{\rho_0} p' + \frac{w_0'}{r} = 0,$$

where $u_0$ and $v_0$ are the axial and radial components of the mean flow, respectively. The terms $\rho_0$ and $u_0$ are the density and mean axial flow velocity, respectively. The primes denote perturbations from the mean values.
where superscript ('') and subscript (0) denote perturbation and mean properties respectively. \( p' = C_0^2 \rho' \), \( C_0 \) is sound speed. Other definitions are the same as those appearing in Eqns. (1-3).

The incident wave is defined as follows:

\[
\begin{align*}
\rho'(x, r, \theta, t) & = a[J_m(k_ar) + c_1 Y_m(k_ar)\cos(kt - k_ar x - m\theta)], \\
u'(x, r, \theta, t) & = \frac{k_a}{k - k_a M_j} \rho', \\
v'(x, r, \theta, t) & = -\frac{a}{k - k_a M_j} \frac{d[J_m(k_ar) + c_1 Y_m(k_ar)]}{dr} \sin(kt - k_ar x - m\theta), \\
w'(x, r, \theta, t) & = -\frac{a M_j k [J_m(k_ar) + c_1 Y_m(k_ar)]}{r(k - k_a M_j)} \sin(kt - k_ar x - m\theta), \\
p'(x, r, \theta, t) & = a[J_m(k_ar) + c_1 Y_m(k_ar)\cos(kt - k_ar x - m\theta)],
\end{align*}
\]

where \( M_j \) is nondimensional velocity inside the duct; \( a \) is fixed at \( 10^{-4} \) to ensure small relative changes in density (as required for LEE); \( J_m \) and \( Y_m \) are the \( m^{th} \) order Bessel functions of the first and second kind respectively; \( k_a \) is the axial wave number and \( k_r \) is the radial wave number. \( k_r \) is the \( n^{th} \) solution of the following equation determined by the hard-wall boundary conditions of the duct

\[
\frac{d[J_m(y_{outer} k_r)]}{dr} \frac{d[Y_m(y_{inner} k_r)]}{dr} - \frac{d[J_m(y_{inner} k_r)]}{dr} \frac{d[Y_m(y_{outer} k_r)]}{dr} = 0,
\]

where \( y_{outer} \) and \( y_{inner} \) are the height of the inlet duct inner wall and the inner hub radii in the inflow boundary. \( k_a \) is calculated from

\[
k_a = \frac{k}{1 - M_j^2} \left( -M_j \pm \sqrt{1 - \frac{k^2(1 - M_j^2)}{k^2}} \right),
\]

the selection of plus or minus (\( \pm \)) signs in the parenthesis is determined by the propagation direction of the spinning wave. Plus (+) is for the positive propagation direction in the axial coordinate, and vice versa. The constant \( c_1 \) satisfies the following relations

\[
c_1 = \frac{\frac{d}{dr}[J_m(y_{outer} k_r)]}{\frac{d}{dr}[Y_m(y_{outer} k_r)]}
\]

and

\[
c_1 = \frac{\frac{d}{dr}[J_m(y_{inner} k_r)]}{\frac{d}{dr}[Y_m(y_{inner} k_r)]}.
\]

On the centerline boundary where \( r = 0 \) a singularity exists. It is treated by using l’Hopital’s rule to approximate \( 1/r \) by \( \partial/\partial r \) at the singularity. To solve these equations, \( 4^{th} \)-order spatial and temporal schemes [17, 18, 19] are employed.

### 4.2 Numerical Issues

#### 4.2.1 Curvilinear Coordinate System

In an earlier work [22] it was shown that a Cartesian mesh with low-order immersed boundary method [23] performed much poorly than a body-fitted mesh to solve acoustic propagation problems with curved geometries. There are also other attempts of using AMR for body-fitted multi-block meshes [11, 24], where curved geometries were allowed to be transformed into and simulated using a uniform computational domain. This can be achieved by using the coordinate transformation given by Eqns. (13-15), which represent a transformation from the physical to the computational coordinates. For simplicity the time variance of both coordinate systems is not considered.

\[
\xi = \xi(x, r), \quad \eta = \eta(x, r).
\]

The first order spatial derivatives of the governing equations are evaluated using the chain rule:

\[
\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta}, \\
\frac{\partial}{\partial r} = \frac{\partial \xi}{\partial r} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial r} \frac{\partial}{\partial \eta}.
\]
with the transformation metrics defined as

\[
\frac{\partial \xi}{\partial x} = J \left( \frac{\partial r}{\partial \eta} \right), \quad \frac{\partial \xi}{\partial \eta} = J \left( -\frac{\partial x}{\partial \eta} \right),
\]
\[
\frac{\partial \eta}{\partial x} = J \left( -\frac{\partial r}{\partial \xi} \right), \quad \frac{\partial \eta}{\partial \eta} = J \left( \frac{\partial x}{\partial \xi} \right).
\]

(15)

\( J \) is the transformation Jacobian relating the geometric properties of the physical space to the uniform computational space and is given by

\[
J = \left[ \frac{\partial x}{\partial \xi} \frac{\partial r}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial r}{\partial \xi} \right]^{-1}.
\]

(16)

4.2.2 Absorbing Numerical Noise

By using \( \epsilon \)-pseudospectra analysis [25] to analyze the stability of the employed spatial schemes, it is discovered that high-order, e.g. 4\textsuperscript{th}-order, spatial schemes are more susceptible to numerical errors than its low-order counterpart on an adaptively refined mesh. A 10\textsuperscript{th}-order explicit filter [26] or an artificial selective damping [18] was used in CAA applications to absorb high-frequency numerical nuisance. In addition, a multigrid prolongation operation[8] can also be used to remove these spurious waves. Their effects are compared in Fig. 5, where the mesh is adaptively refined to capture the acoustic propagation. Firstly, the damping technique is employed with the same coefficients used in a cavity flow simulation [7]. Figs. 5(a)-5(b) show the method fails to absorb numerical noise generated around the lip of the intake. It reveals the fact that the coefficients of artificial selective damping have to be adjusted case by case. Figs. 5(c)-5(d) indicate that the filter removes some of spurious waves, but it is not so effective as the multigrid prolongation operation, by which solutions on the coarse blocks continuously update their counterparts on the fine blocks with a linear interpolation to absorb the short wavelength spurious waves. The results are shown in Figs. 5(e)-5(f). With this method, it is found in the later discussion that there is a slightly increase in the dissipation level surrounding the area of the intake lip. It could be remedied by using a high-order interpolation, e.g. 6\textsuperscript{th}-order polynomial interpolation, in the multigrid prolongation method. In the rest of the paper both the filter and the multigrid prolongation methods are employed to remove the spurious waves.

4.3 Results and Discussion

With the aforementioned techniques, the acoustic propagation in and radiation from the aeroengine intake is solved with AMR. In the computation, typically two to three refinement levels are used. Once the waves reach the outflow boundary of the computation domain, the finest blocks span the whole computational domain and the regridding operation is stopped to improve efficiency. After that junction, the computation only proceeds on the top level blocks, while the multigrid prolongation operation from the coarse blocks to the fine blocks is maintained to remove the spurious waves in the computational domain.

Two background mean flow configurations have been used. One has a stationary medium, the other represents a sideline mean flow. Figs. 6-7 compare the results of instantaneous perturbation pressure computed on an adaptively refined mesh with the same predictions computed on a fixed mesh[10]. There is little difference between the two results in the case of the stationary medium, while for the sideline mean flow case the radiation pattern is slightly influenced by the AMR method. In order to show the difference much more clearly, the far-field directivity results computed with various strategies are compared in Fig. 8. The far-field directivity is estimated through an integral solution of Ffowcs-Williams and Hawkins equations [27]. The integration surfaces are displayed in Figs. 6-7.

By using the multigrid prolongation operation to absorb the spurious waves at the coarse and fine block interfaces, it appears that both the peak level and the peak radiation angle agree well with that of Richards [10]. For the stationary medium case, the peak radiation is well predicted at 47.0 deg (Fig. 8(a)). This compares well with the LEE prediction (47.27 deg) of Richards [10]. For the sideline case, the peak radiation angle is at 59.9 deg (Fig. 8(b)) which compares favourably with 59.4 deg predicted by the LEE and 60.8 deg by the FEM [10]. The dynamic range of the prediction is typically higher than 50 dB which is good enough for most of the engineering applications. Using filter alone, the level of the peak radiation is 1.35dB higher than that given by the LEE method. It may be caused by the inadequacy of the filter method in treating the short wavelength spurious waves around the lip of the aeroengine intake. With the multigrid prolongation operation, the peak radiation level is 0.55dB lower than that given by the LEE method. It perhaps reflects the fact that the operation introduces an excessive level of dissipation surrounding the lip of the aeroengine intake. Both operations produce predictions which do not follow the decaying envelope at low observation angles to the axisymmetrical...
axis ($\phi \leq 25$ deg). The dynamic range of the prediction is somewhat smaller than that given by a LEE computation\[10\] on a fine mesh without AMR. This is as expected as the order of the spatial scheme on an adaptively refined mesh is demonstrated earlier to be less than 4. This particular feature might also be influenced by the spurious waves generated at the fine-coarse interfaces in the AMR operations. As shown in Fig. 5, the multigrid prolongation operation absorbs the spurious waves better than the filter method. This leads to a dynamic range of the prediction around 12dB better than the filter operation (Fig. 8(a)). With the multigrid prolongation operation, the overall prediction results cover a dynamic range of approximately 60dB. The accuracy suffers slightly as the observation angle approaches 120 deg, the discrepancy in pressure level being at most 2.2dB.

For the sideline case, the prolongation method is used to remove the spurious waves generated at the coarse and fine block interfaces. The AMR result is compared with the LEE prediction of Richards on a fixed mesh and a FEM prediction\[10\]. The results are presented in Fig. 8(b). The main peak angle and the peak level of the AMR result match the other two solutions well. The differences of the peak radiation level between these results are less than 0.5dB, while the peak radiation angles differ form each other by less than 0.7 deg. Towards the low observation range ($\phi \leq 22$ deg), the discrepancy in the pressure level increases again. This feature is the same as the previous case. The dynamic range of the prediction is still about 60 dB. Nevertheless, the prediction deteriorates toward the high observation angles, especially around the shadow interference dip angles at 88.3 deg, where there is 7dB difference between the AMR and the FEM results, and 5 dB between the AMR and the LEE prediction on a fixed mesh. It could be caused by the spurious wave generated above the lip of the intake, as indicated by the wiggles shown in the perturbation pressure contours (Fig. 5(f)).

In the AMR computation of the stationary medium case, the total number of cells increases from 13,872 to 41,616 as the wave propagates out of the intake. The computing time is 3,463 seconds on a desktop computer (Pentium IV 1.3GHz, 768MB). On a uniformly fine mesh the computing time is 5,400 seconds with the AMR code. In an earlier LEE computation with SotonLEE code which solves LEE\[21\], 7,560 seconds are required to achieve the same results on a uniform mesh of 81,600 cells. It suggests that on a uniform mesh, the efficiency of the AMR code is lower than that of SotonLEE code. The dynamic range of the prediction is somewhat smaller than that given by a LEE computation\[10\]. The results are presented in Fig. 8(b). The main peak angle and the peak level of the AMR result match the other two solutions well. The differences of the peak radiation level between these results are less than 0.5dB, while the peak radiation angles differ form each other by less than 0.7 deg. Towards the low observation range ($\phi \leq 22$ deg), the discrepancy in the pressure level increases again. This feature is the same as the previous case. The dynamic range of the prediction is still about 60 dB. Nevertheless, the prediction deteriorates toward the high observation angles, especially around the shadow interference dip angles at 88.3 deg, where there is 7dB difference between the AMR and the FEM results, and 5 dB between the AMR and the LEE prediction on a fixed mesh. It could be caused by the spurious wave generated above the lip of the intake, as indicated by the wiggles shown in the perturbation pressure contours (Fig. 5(f)).

In the AMR computation of the sideline case, the total number of cells increases from 13,872 to 41,616 as the wave propagates out of the intake. The computing time is 3,463 seconds on a desktop computer (Pentium IV 1.3GHz, 768MB). On a uniformly fine mesh the computing time is 5,400 seconds with the AMR code. In an earlier LEE computation with SotonLEE code which solves LEE\[21\], 7,560 seconds are required to achieve the same results on a uniform mesh of 81,600 cells. It suggests that on a uniform mesh, the efficiency of the AMR code is lower than that of SotonLEE code due to the introduction of AMR operations. Fig. 9 illustrates the process of regridding and dynamic load balancing within 4 processors which are differentiated by different colours. The computational load is redistributed evenly. For the sideline case, the AMR code is profiled and the cost of each operation is displayed in Fig. 10, where the number of processors ranges from 1 to 8. Fig. 10(a) indicates that nearly 30% of the computing time is consumed by the ghost construction operation. Other AMR operations together consume less than 6% of the total computing time. In Fig. 10(b) the parallel speedup performance of the AMR code is compared with that of SotonLEE code on a Beowulf cluster connected by Gigabit Ethernet. It is discovered that the communication cost of the AMR code is generally 1 to 3 times higher than that of SotonLEE. The cost is mainly the result of the expensive communication of the AMR ghost construction operation which is mostly consisted of memory movements and limited by the presenting network speed. Therefore the speedup performance of AMR deteriorates along with the increase of the processors number.

5 Summary

While the essential idea of AMR is not difficult to grasp, its parallel implementation is far from trivial. In this work the main AMR operations are explained and their parallel communications are bundled together to simplify the code developing efforts. The convergence rates of a range of 2nd- and 4th-order spatial schemes on an adaptively refined mesh are studied by solving the benchmark problem of Gaussian pulse propagation. The AMR method is applied to the prediction of spinning-mode radiation from a generic engine intake, with or without an axisymmetric mean flow. To model curved geometries, the AMR code is extended to support body-fitted grids. Filter, artificial selective damping and the multigrid prolongation are employed to absorb spurious waves generated in the AMR computation. Their effects are compared, and the multigrid prolongation method is shown to be the preferred method for the problem. The accuracy and the efficiency of the AMR method is demonstrated by the predicted far-field directivity, which compares well with those given by a LEE computation on a uniform mesh and FEM. In terms of computation efficiency, the adaptively refined mesh represents a saving of up to 40% compared with a uniform mesh. Relied on MPI, the computation loads are shown to be distributed evenly within the processors. The conclusions about the cost of each AMR operation and its efficiency are also studied. The ghost construction operation in the AMR computation appears to be the bottleneck of the AMR parallel performance. In order to attain a higher efficiency on current parallel machines, it is suggested to separate the parallel communication of the ghost construction operation from the other AMR operations to obtain an optimal performance.
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References


Figure 1: Block-based AMR for an acoustic scattering problem. Solid lines denote blocks boundaries. Each block contains $21 \times 21$ cells.

Figure 2: Perturbation pressure contours of 2D Gaussian pulse propagation. 16 equi-spaced contours between $\pm 0.05$. Negative contours are dashed. Straight lines are blocks’ borders. Solutions on the thick central line in (b) are used to compute the convergence rate.
Figure 3: $L_2$-norm error of the pressure, $N$ is the number of cells in a block in one coordinate axis. △: 2$^{nd}$-order explicit scheme and 2$^{nd}$-order interpolation, X: 4$^{th}$-order explicit scheme and 2$^{nd}$-order interpolation, ○: 4$^{th}$-order explicit scheme and 6$^{th}$-order interpolation, +: 4$^{th}$-order DRP scheme and 6$^{th}$-order interpolation, -.-: 2$^{nd}$-order slope and --: 4$^{th}$-order slope.

Figure 4: Mach number contours of the mean flow around an aeroengine intake. Freestream Mach number is 0.25, ambient pressure is 94250 Pa, intake Mach number is set to 0.55, and intake pressure is 79687 Pa.
Figure 5: Effect of spurious wave treatment methods. Perturbation pressure contours. Gray lines denote the boundaries of blocks. $m = 12, n = 1, k = 20$. 

(a) Damping, $t = 1.25$.  
(b) Damping, $t = 3$.  
(c) Filter, $t = 1.25$.  
(d) Filter, $t = 3$.  
(e) Prolongation, $t = 1.25$.  
(f) Prolongation, $t = 3$.  

$X$
Figure 6: Perturbation pressure contours. \( m = 26, n = 1, k = 41.9 \). Stationary medium.

Figure 7: Perturbation pressure contours. \( m = 13, n = 1, k = 16.7 \). Sideline mean flow.

Figure 8: Far-field directivities for the aeroengine intake. (a): \( m = 26, n = 1, k = 41.9 \), stationary medium and (b) \( m = 13, n = 1, k = 16.7 \), sideline mean flow.
(a) 20 blocks, $t = 0.566$.  
(b) 32 blocks, $t = 1.698$.  
(c) 52 blocks, $t = 2.83$.  
(d) 60 blocks, $t = 11.32$.  

Figure 9: Perturbation pressure contours, $m = 12$, $n = 2$, $k = 20$. The computing load is distributed over 4 processors, which are denoted by different colours.

(a) 

Time percentage (%) 

100 

0 

1 

2 

3 

4 

5 

6 

7 

8 

9 

Processors

(b) 

Time percentage (%) 

100 

0 

1 

2 

3 

4 

5 

6 

7 

8 

9 

Processors

Figure 10: The parallel performance of the AMR code. (a) The time percentage of parallel AMR operations with processors numbers. ⬤: CAA Computing, △: ghost construction, ◆: restriction, ◄: interpolation, ▶: regridding and □: prolongation. (b) Parallel speedup. - - - ideal speedup, △: the result of SotonLEE code and ◆: the result of the AMR code.