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Real-time location of coherent sound sources by the observer-based array algorithm

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Abstract
Acoustic arrays have become an important tool in noise identification for aerospace measurement applications, and the conventional beamforming algorithm has been adopted as a processing technique of choice. In most practical cases the beamforming computations have to be conducted off-line due to extensive computational time requirements. An alternative algorithm with real-time capability has been proposed. The algorithm has a form similar to a classical observer whilst working in the frequency domain for the array processing. The performance of this observer-based algorithm is studied here in a simulation case and in an experimental case by comparing it to a conventional beamforming method. In this paper it is shown that the observer-based algorithm could release the coherence restriction between the background noise and the signal of interest. The proposed observer-based algorithm also has the capability of operating over sampling blocks recursively. The convergence rate of this recursive algorithm is also satisfactory for the simulation case. As a result, a great deal of experimental time could be saved as any testing defects could be revealed instantaneously and corrected on-site. In addition, this innovative approach provides an alternative perspective from which many techniques already in use could be extended to this new application area of array processing.

Keywords: beamforming, microphone array, aeroacoustics

(Some figures in this article are in colour only in the electronic version)

1. Introduction
The popularity of air travel has increased dramatically in the last few decades. This has led to an increase in air traffic and drawn attention to noise pollution. The development of aero-engines with high bypass ratio ducts has shifted the focus of noise reduction efforts from engines to airframe [1]. The main noise contribution comes from high lift devices, landing gear, wheel wells and sharp trailing edges. The airframe noise is particularly evident during a landing when the engines operate at a low power regime and the high lift devices and landing gear are deployed. Resulting noise pollution puts great pressure on communities near airports. The challenging environmental targets set by the Advisory Council for Aeronautics Research in Europe (ACARE) in their Strategic Research Agenda include a reduction of 50% of the perceived noise level by 2020 [1].

Airframe noise is produced by a fluid–structure interaction of aerodynamic surfaces and the surrounding turbulent flow [2]. The distribution of noise sources has to be localized before any development of an efficient noise control strategy can take place. Microphone arrays [3] are being increasingly used in aerospace industry [4] for this purpose. Various array processing algorithms [5], such as conventional beamforming
[5] and robust adaptive beamforming [6], have been proposed. The former is still the method of choice in aerospace for its robust performance in a noisy aeroacoustic testing environment. However, compared to advanced beamforming methods [6], the resolution of conventional beamforming [5] is quite limited. In addition to some post-processing algorithms [7–9], a phased array with hundreds of, or even thousands of, microphones is required by aerospace industry to improve the resolution and to avoid potential spatial aliasing [10].

However, it is extremely difficult to process a huge amount of data obtained from a large capacity array in real time, which is the preferred capability for field tests. Recently, a new approach with real-time computational capability has been proposed [11]. This is the so-called observer-based algorithm capable of generating acoustic images recursively over sampling data. It is difficult and time-consuming to conduct high quality experiments due to frequently occurring problems such as damaged cables and connectors, and occasionally occurring problems such as microphone failure and abnormal facility condition. A real-time algorithm could be quite helpful for practical field tests since any defects occurring in experiments can be instantaneously discovered and accordingly resolved on-site. A stationary defects occurring in experiments can be instantaneously could be quite helpful for practical field tests since any problems such as damaged cables and connectors, and occasionally occurring problems such as microphone failure and abnormal facility condition. A real-time algorithm could be quite helpful for practical field tests since any defects occurring in experiments can be instantaneously discovered and accordingly resolved on-site. A stationary phase difference between the facility background noise and the total noise (background noise plus model noise) was assumed previously for the real-time observer algorithm [11]. In this work a more generic case with an instantaneously varying phase difference that leads to a nonlinear output for measurements has been taken into account. The observer-based algorithm has been further extended to address the issue.

The main objective of this work is to propose and develop the observer-based beamforming algorithm and to validate its performance in terms of coherent noise rejection and recursive operations. The paper is organized as follows. The conventional beamforming algorithm and its limitations are briefly discussed in section 2. Section 3 summarizes the observer theory in classical control. An improved observer-based algorithm is developed particularly for array signal processing in section 4. The performance of the new method is evaluated in both a numerical simulation and an experimental case. Details are given and discussed in section 5. A summary of this work is presented in section 6.

2. Classical beamforming formulations

Given a microphone array with \( m \) microphones, the output \( y(t) \) denotes the time domain measurements of microphones, \( y \in \mathbb{R}^{m \times 1} \). For a single noise source \( x \in \mathbb{R}^1 \) in a free sound propagation space, we have

\[
y(t) = \frac{1}{4\pi r} x(t - \tau), \quad \tau = \frac{r}{C},
\]

where \( C \) is the speed of sound, \( r \in \mathbb{R}^{m \times 1} \) is the distance between the noise source \( x \) and microphone, and \( \tau \) is the related sound propagation time. For most aeroacoustic applications, a beamforming operation is generally conducted in the frequency domain [8]. The frequency domain version of equation (1) is

\[
Y(j\omega) = \frac{1}{4\pi r} X(j\omega) e^{-j\omega r} = G(r, j\omega)X(j\omega),
\]

where \( G(r, j\omega) \) is the sound propagation vector in free space.

To reduce the detrimental effect of the background noise in a closed-section wind tunnel, special adjustments for tunnel design are required. Interested readers can refer to the literature [12] for more information. In addition, the processing algorithm has to be modified to address the interference issue from background noise. In particular, if background noise is generated by a tunnel drive or from a coherent noise source nearby, equation (2) is modified to

\[
Y_B(j\omega) = G(r, j\omega)X_B, \quad Y_{BS}(j\omega) = G(r, j\omega)X_{BS},
\]

where the symbol \( \omega \) is omitted for simplicity. The subscript \( B \) denotes the results of experiments without the presence of a test model. Hence only the data from the background noise are acquired during the tests. The measurement \( Y_B \) comes solely from the background noise. The subscript \( BS \) denotes the experiments measuring both the background noise and the sound of interest from the test model. Due to the presumed linearity of the sound process, \( X_{BS} = X_B + X_S \). Hence \( Y_{BS} \) reflects the combined effect of the sound from the test model and the background noise from the testing facility.

As the sound signal of interest can be regarded as an almost stationary and ergodic process, statistical operations can be used to approximate the test model sound \( X_S \).

\[
A_B = Y_B Y_B^* = G(X_B X_B^*) + G^*, \quad A_{BS} = Y_{BS} Y_{BS}^* = G(X_{BS} X_{BS}^*) + G^* = G((X_B X_B^*) + (X_S X_S^*) + (X_B X_S^*) + (X_S X_B^*)) + G^*,
\]

\[
\langle A_S \rangle = G(X_S X_S^*) + G^* \approx \langle A_{BS} \rangle - \langle A_B \rangle,
\]

where the superscript * denotes the complex conjugate and \( \langle \rangle \) denotes an ensemble average. Equation (5) is valid if \( \langle X_B X_S^* \rangle = 0 \) and \( \langle X_S^* X_S \rangle = 0 \). In other words, the background noise and noise (signal of interest) from a testing model should be incoherent, which is normally the case for the experiments measuring both the background noise and the sound of interest from the test model. Due to the presumed linearity of the sound process, \( X_{BS} = X_B + X_S \). Hence \( Y_{BS} \) reflects the combined effect of the sound from the test model and the background noise from the testing facility.

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\]

\[
\langle A_S \rangle = G(X_S X_S^*) + G^* \approx \langle A_{BS} \rangle - \langle A_B \rangle,
\]

where the superscript * denotes the complex conjugate and \( \langle \rangle \) denotes an ensemble average. Equation (5) is valid if \( \langle X_B X_S^* \rangle = 0 \) and \( \langle X_S^* X_S \rangle = 0 \). In other words, the background noise and noise (signal of interest) from a testing model should be incoherent, which is normally the case for aeroacoustic tests. As a result, the averaged sound source power \( \langle (X_S X_S^*) \rangle \) can be obtained from equation (6), that is,

\[
\langle X_S X_S^* \rangle = G^* \langle (A_{BS}) - \langle A_B \rangle \rangle G^{**},
\]

where the superscript * denotes Moore–Penrose pseudoinverse and \( G^* = (G^* G)^{-1} G^* \).

The operations of equations (4)–(7) are conducted for every scanned point in succession to generate a sound pressure image, which helps to identify positions and strengths of dominant noise sources. More details on a typical beamforming algorithm and its implementations can be found in [4, 10]. It is worthwhile to emphasize that the assumption implicitly followed in equations (4)–(7) supposes no coherence between \( X_B \) and \( X_S \), otherwise \( \langle X_B X_S^* \rangle \neq 0 \), \( \langle X^*_B X_S \rangle \neq 0 \), and the approximation in equation (6) is invalid.

In summary, the typical delay and sum beamforming adopted in aeroacoustics have two shortcomings: the statistical operation of \( \langle \rangle \) is time-consuming and cannot be recursively conducted for real-time noise identification; and the assumption of no coherence between the background noise
3. Classical observer formulations

A new algorithm proposed by our group [11, 13] can address the aforementioned shortcomings. The basic idea is based on observer theory in classical control. For convenience of the readers, the theory is briefly introduced in this section. More details can be found in a linear control textbook [14].

First, a continuous time-invariant linear system can be represented by

\[ \dot{x}(t) = Ax(t) + Bu(t), \]
\[ y(t) = Gx(t), \]

where \( x \) is the system state, the symbol \( \dot{x} \) denotes \( dx/dt \), \( y \) is the output, and \( u \) is the control input. For simplicity, scalar symbols are used here but a similar vector form applies to a multi-input and multi-output system as well. In classical linear control, \( A \) is the state matrix, \( B \) is the input matrix and \( G \) is the output matrix. The dynamics of \( x \) is described by equation (8). Equation (9) is the measurement equation that has a form similar to equation (2) (where \( G = e^{-j\omega t}/(4\pi r) \)). It is worthwhile to emphasize that equations (8), (9) are in the time domain.

A classical state observer (the so-called Luenberger observer) can be constructed to approximate \( x \) from \( y \). It has the form

\[ \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y - \hat{y}), \]
\[ \hat{y}(t) = G\hat{x}(t), \]

where \( \hat{x} \) is the approximation of \( x \), and \( L \) is the observer gain. The estimation error is \( \hat{e} = x - \hat{x} \), and its dynamics satisfies

\[ \dot{\hat{e}} = (A - LG)e, \]

which is achieved by subtracting equation (8) from equation (10). It can be shown that \( \hat{e} \) converges to zero when \( t \to \infty \), as long as all eigenvalues of the matrix \( A - LG \) have negative real parts. The problem boils down to finding a proper \( L \), which has already been resolved in Matlab.

4. Observer-based beamforming

The aforementioned observer is formulated in the time domain, whereas classical beamforming is generally conducted in the frequency domain. For consistency, the array measurements and sound source in the time domain are represented by the Fourier series. All variables in the time domain can be expressed by

\[ y(t) = \sum_{n=-\infty}^{\infty} Y_n e^{j\omega t}, \quad x(t) = \sum_{n=-\infty}^{\infty} X_n e^{j\omega t}, \]

Figure 1 shows a practical aeroacoustic measurement setup in a closed-section wind tunnel. It shows a test scenario when the array output contains combined noise from the model and the facility. A separate test without the presence of the test model can be conducted to evaluate the background facility noise, that is, \( Y_B = GX_B \) for each background noise source. Similarly, we can have \( Y_{BS} = GX_{B2} + GX_S \), given the background noise source and model noise source satisfy the superposition principle. It is natural to assume that the signals for two cases, \( X_B \) and \( X_{B2} \), have the same amplitude. The only difference between \( X_B \) and \( X_{B2} \) is a phase shift \( \phi \) due to time delay between two testing scenarios, and we can have \( Y_{BS} = GX_B e^{j\phi} + GX_S \). As a result, the governing equations describing the two scenarios together are

\[ \begin{pmatrix} X_{B|k+1} \\ X_{S|k+1} \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} \begin{pmatrix} X_{B|k} \\ X_{S|k} \end{pmatrix}, \]
\[ \begin{pmatrix} Y_{B|k} \\ Y_{BS|k} \end{pmatrix} = \begin{pmatrix} G & 0 \\ G e^{j\phi} & G \end{pmatrix} \begin{pmatrix} X_{B|k} \\ X_{S|k} \end{pmatrix}, \]
where $\phi$ was assumed as already known in our previous work [11], and the whole system is linear. A Luenberger observer in discrete time can be designed to approximate $X_B$ and $X_S$. Following equation (10), the observer is

$$
\begin{align*}
\left( \begin{array}{c}
\hat{X}_B[k+1] \\
\hat{X}_S[k+1]
\end{array} \right) &= \left( \begin{array}{cc}
A & 0 \\
0 & A
\end{array} \right) \left( \begin{array}{c}
\hat{X}_B[k] \\
\hat{X}_S[k]
\end{array} \right) \\
&+ L \left[ \left( \begin{array}{c}
Y_B[k] \\
Y_B[k]
\end{array} \right) - \left( \begin{array}{c}
\hat{Y}_B[k] \\
\hat{Y}_B[k]
\end{array} \right) \right].
\end{align*}
$$

(19)

$$
\left( \begin{array}{c}
\hat{Y}_B[k] \\
\hat{Y}_B[k]
\end{array} \right) = \left( \begin{array}{cc}
G & 0 \\
G e^{j\phi} & G
\end{array} \right) \left( \begin{array}{c}
\hat{X}_B[k] \\
\hat{X}_S[k]
\end{array} \right).
$$

(20)

In equations (19), (20), $Y_B$ and $Y_B^S$ are DFT results of the time domain samples from the array. Figure 2 shows some sampling data from a practical experiment conducted in a closed-section wind tunnel. The data shown here are from a single microphone of the array and consist of four blocks of samples. Each block contains 4096 time domain samples for the efficient computation of DFT. Figure 2 suggests that the aeroacoustic measurements are almost stationary and ergodic, and therefore implies that $A$ could be an identity [15]. In essence the $X[k]$ is presumably invariant over sampling blocks, whereas the estimation $\hat{X}$ is recursively adjusted with the oncoming new information from the measurements (that is $(Y[k] - \hat{Y}[k])$, the so-called innovation in control), which reflects any potential variations in experimental environment and developments of the signal of interest. It is also worthwhile to mention that the duration of one sampling block is assumed small compared to the variation timescale.

The above-mentioned observer-based beamforming algorithm has been applied to a simple simulation case given the prior knowledge of $\phi$ in the previous work [11]. It showed that for each sampling block the computation cost of the observer-based algorithm was comparable to that of a classical beamforming. In addition, given a constant $\phi$, the observer-based algorithm can recursively approximate the sound of interest and simultaneously reject interferences from the spatial coherent background noise.

However the time delay $\phi$ is actually largely unknown and volatile in most practical measurements. It has been suggested in the previous work [11] that any unknown time delay $\phi$ could be approximated by checking the correlation between $Y_B$ and $Y_B^S$. It is easy to see that a statistical operation such as correlation to some extent is inappropriate for real-time computation. An alternative idea with a recursive approximation is developed in this work and the observer algorithm is extended accordingly for a more practical case when the relationship between $Y$ and $X$ is nonlinear due to the unknown $\phi$. It can be written as

$$
\begin{align*}
\left[ \begin{array}{c}
\hat{X}_B \\
\hat{X}_S \\
\phi
\end{array} \right] &= A \left[ \begin{array}{c}
X_B \\
X_S \\
\phi
\end{array} \right], \\
\left[ \begin{array}{c}
Y_B \\
Y_B^S
\end{array} \right] &= G(X_B, X_S, \phi),
\end{align*}
$$

(21)

where $A$ is an identity matrix, and $G(X_B, X_S, \phi)$ is the right-hand side of equation (18). A new observer is proposed here for the problems with nonlinear outputs $G(X_B, X_S, \phi)$. It has the form

$$
\dot{X} = A\dot{X} + LG^{**}(\hat{X}_B, \hat{X}_S, \hat{\phi})(Y - \hat{Y}) = A\dot{X} + LG^{**}(\hat{X}_B, \hat{X}_S, \hat{\phi})
\times [G(X_B, X_S, \phi) - G(\hat{X}_B, \hat{X}_S, \hat{\phi})],
$$

(22)

where the symbol $^*$ is the complex conjugate, the definition of $\dot{\phi} \triangleq \partial/\partial X$, and $G'(X_B, X_S, \phi)$ is

$$
\begin{pmatrix}
\frac{\partial Y_B}{\partial X_B} & \frac{\partial Y_B}{\partial X_S} & \frac{\partial Y_B}{\partial \phi} \\
\frac{\partial Y_B^S}{\partial X_B} & \frac{\partial Y_B^S}{\partial X_S} & \frac{\partial Y_B^S}{\partial \phi}
\end{pmatrix} = \begin{pmatrix}
G & 0 & jG e^{j\phi} X_B \\
G e^{j\phi} & G & 0
\end{pmatrix}.
$$

(23)

The above-mentioned observer-based beamforming with nonlinear output is an extension of a former one in the time domain [16]. The estimation error is $E \triangleq (X_B, X_S, \phi)^T - (\hat{X}_B, \hat{X}_S, \hat{\phi})^T$, where the superscript $T$ is the transpose of a vector. The estimation error dynamics (with respect to the sampling data block number) is nonlinear but can be linearized around zero through a Taylor series expansion,

$$
E = [A - LG^{**}(\hat{X}_B, \hat{X}_S, \hat{\phi})G'(\hat{X}_B, \hat{X}_S, \hat{\phi})]E.
$$

(24)

$G^{**}(\hat{X}_B, \hat{X}_S, \hat{\phi})G'(\hat{X}_B, \hat{X}_S, \hat{\phi})$ is a positive definite Hermitian matrix. To understand why $L$ is chosen let us first study a simple scalar case as an example, where $E = \alpha E$ and $
alpha = [1 - LG^{**}(\hat{X}_B, \hat{X}_S, \hat{\phi})G'(\hat{X}_B, \hat{X}_S, \hat{\phi})]$. It is easy to see that $E$ approaches zero as $\alpha < 0$. Given a negative
value of $\alpha$, we can calculate the observer weight $L$ by $L = (1 - \alpha)/[G''(X_B, \phi)G'(X_B, \phi)]$. A similar calculation can be simply performed in Matlab for more complex vector cases.

In summary, the observer-based approach can be conducted in the following sequence: (1) define the invariant matrix $A$ and matrix $G$; (2) make an initial guess of $(X_B, \phi)$; (3) calculate $(Y_B, Y_{BS})$ iteratively for each sampling data block; (4) conduct the discrete forms of equations (22), (23) recursively over sampling data blocks to update the approximation $\hat{X}_i$; (5) repeat steps (2)–(4) for all gridpoints in the scanned plane.

It has been found in numerical simulations (see the following section for details) that equations (22), (23) converge slowly. It is understandable because the estimation of the time delay $\phi$ in equation (22) is conducted for all gridpoints (generally tens of thousands for the cases studied here). The algorithm is modified below in equations (25)–(27) to improve the convergent speed:

$$\begin{align*}
\begin{bmatrix}
\hat{X}_{B|k+1} \\
\hat{X}_{S|k+1}
\end{bmatrix}
&= 
\begin{bmatrix}
A & 0 \\
0 & A
\end{bmatrix}
\begin{bmatrix}
\hat{X}_{B|k} \\
\hat{X}_{S|k}
\end{bmatrix} \\
&+ L
\begin{bmatrix}
Y_{B|k} \\
Y_{BS|k}
\end{bmatrix} - 
\begin{bmatrix}
\hat{Y}_{B|k} \\
\hat{Y}_{BS|k}
\end{bmatrix},
\end{align*}$$

(25)

$$\begin{align*}
\hat{Y}_{B|k}
&= (G e^{i\phi_a}) (\hat{X}_{B|k}),
\end{align*}$$

(26)

$$\begin{align*}
\hat{\phi}|_{k+1}
&= \hat{\phi}|_k + LH^T(Y|_k - \hat{Y}|_k),
\end{align*}$$

(27)

where $H \triangleq \partial Y/\partial \phi$ and $Y = (Y_B, Y_{BS})^T$. Equations (25), (26) have a similar form to the previous observer (equations (19), (20)), while the time delay $\phi$ is only estimated using equation (27) for dominant background noises. For example, if a dominant noise source is at $(0, 0, 1)$, $H$ is set accordingly to that position and the resulting $\hat{\phi}$ is used for the whole imaging plane gridpoints. In addition, $L$ can be easily selected to ensure the numerical stability of the recursive algorithm, and little performance difference has been found for various stable options of $L$ for which the main lobe shapes and point spread function values are quite similar.

5. Validations

The application of the algorithm and its benefits are first demonstrated here in a numerical simulation. Two coherent monopole sources of identical strength are considered. The background noise is $x_B(t) = p_a e^{j\omega t}$ at $(0.1, 1, 0)$ m, the signal of interest is $x_S(t) = p_a e^{j(\omega t + \phi)}$ at $(-0.1, 0.1, 1)$ m, $\omega = 2\pi f$, $f = 3$ kHz for the case and $p_a$ is the sound amplitude. A virtual microphone array is placed 1 m away. The array consists of 64 microphones. The intra-sensor spacing is almost 0.1 m if the approach of regular spacing of microphones is adopted, which can resolve the acoustical signal at a frequency up to 1700 Hz due to the classical half-wavelength criterion. It is however of interest to measure sound at much higher frequency for a scaled model in common aerospace experimental practices. The regular spacing approach violates the half-wavelength criterion and thus leads to spurious side lobes in acoustic images. The issue can be addressed by increasing microphone numbers, which however is not economical for high frequency measurements. An alternative approach is to place microphones irregularly, which can attenuate spatial aliasing and spurious lobes with limited microphones. A classical irregular layout of microphones is of a multi-arm spiral profile [10] as shown in figure 3, which has been adopted in this work for the particular purpose of reducing spatial aliasing at high frequency.

The sound fields of $x_B(t)$ (see figure 4(a)) and $x_{BS}(t)$ (see figure 4(b)) respectively generate data of $Y_B$ and $Y_{BS}$ for the simulation. It is assumed that $Y_S$ cannot be directly obtained which is generally the case for practical measurements with the background noise.

One solution is to approximate $X_S$ with the information of both $Y_B$ and $Y_{BS}$. Figure 5(a) shows the acoustic image from $Y_{BS}$. Figure 5(b) shows the acoustic image of $X_S$ with the aforementioned conventional beamforming (equation (7)), where the image is averaged over 100 sampling blocks. Although the main part of the signal of interest $X_S$ is restored from $Y_B$ and $Y_{BS}$, the detrimental effect caused by the coherent noise $X_B$ is still partially retained and can be clearly seen in figure 5(b). The observer algorithm (equation (22)) is applied to the same data. Figure 5(c) shows the recursive result at the 10th block. The interference from the coherent background noise is still visible, but most side lobes are well suppressed. Figure 5(d) shows the recursive result of the observer-based algorithm at the 2000th block. It can be seen that the interference from the coherent noise is almost completely rejected, and the sound of interest is satisfactorily restored.

In the simulation the actual time delay $\phi = 1.0134$ rad (initially specified in the simulation), while the initial guess of the observer is $\hat{\phi} = 1.5$ rad. The estimation of the time delay is recursively conducted here for every gridpoint in the
Figure 4. Sound sources in the numerical simulation: (a) a monopole source as a background noise and (b) two coherent monopoles.

Figure 5. Simulation results of sound pressure contours in decibel: (a) two coherent sources, (b) background noise is removed by conventional beamforming, (c) and (d) observer results at the 10th block and the 2000th block, respectively.
acoustic image. Figure 6 shows that the approximation error $\max(\phi|_k - \hat{\phi}|_k)$ converges to zero in about 3000 iterations of blocks (the related sampling time is almost 300 s). The relatively slow convergence urges the adoption of a simple observer algorithm (equations (25)–(27)), where $\phi$ is only estimated at the dominant background noise location, which is $(0.1, -0.1)$ for this case. New simulation data are generated to validate the method and figure 7 shows the performance of convergence, where the solid line denotes the actual $\phi$ and the dashed line denotes its estimation (\hat{\phi}). It can be seen that for this new simulation the phase shift between the two spatially coherent noise sources varies every 100 blocks, which imitates any potential changes during measurements. The rest of the setup of the new simulation case is identical to the previous one. The estimation of $\phi$ can quickly trace variations in less than 5 to 10 blocks (0.5 to 1 s correspondingly). A persistent stationary error of about 5° remains, which is presumably caused by only estimating $\phi$ at a single gridpoint.

The computational costs of conventional beamforming and the observer-based beamforming have been profiled on a laptop with Intel Core 2 Duo processor (P8400 @ 2.26 GHz) and 3 GB memory. The code has been developed in Matlab without particular optimization and the resulted acoustic images contain 14 400 gridpoints. The computational time for one single block case and 100 blocks case are evaluated. For a single block case, the required time is 3.6 s for conventional beamforming and 19.3 s for observer-based beamforming. The extra computational cost for observer-based beamforming is largely caused by the computation of $L$, which can be calculated off-line and only needs to be done once. For 100 block case, the required time is 62.9 s for conventional beamforming and 128 s for observer-based beamforming. The latter time for observer-based beamforming is almost double that of conventional beamforming, because both the background noise and the signal of interest are simultaneously calculated in equations (26)–(27). The reward of the complex operation is a gradual rejection of the spatially coherent background noise. In addition, although the overall cost is more expensive, the observer can be recursively calculated in each block and the average computational time is around 1 s per block. In contrast, it is impossible for conventional beamforming to be recursively conducted and thus to evenly distribute its computational costs within each sampling block (see equation (8)). In on-site experimental practices it is possible to achieve at least 12× computational time speedup by implementing algorithms with C language instead of using Matlab. Hence it is easy to ensure real-time capability for observer-based beamforming, considering that the sampling duration is almost 0.1 s per block. A pipeline can be designed with one thread to acquire data and the other thread to process observer-based beamforming.

It is also worthwhile to mention that aeroacoustic images for practical experiments are separately generated at various
Figure 9. Experimental results of sound pressure contours in decibel: (a) two coherent sources, (b) background noise is removed by conventional beamforming, (c) and (d) observer results at the 1st block and the 100th block, respectively.

chosen frequencies. The results presented in figures 5–7 are only for $f = 3$ kHz. The same findings for the observer-based beamforming algorithm are discovered at other frequencies.

The observer algorithm has also been validated in an experiment. A microphone array was constructed in our group at Peking University. Figure 8 shows the experimental setup. The diameter of the array is 1 m and the layout of microphones is identical to the layout shown in figure 3. The array is manufactured using printed circuit board (PCB) technology, which can ensure good position accuracy (less than 0.0002 m) with economical manufacturing cost. Preamplifiers can be easily integrated with microphones soldered on the PCB, thus making it a convenient way to construct an array for laboratory research. Other mechanical manufacturing techniques should be specifically considered for particular wind tunnel experiments and that is beyond the topic of this paper. Two speakers driven by an Agilent signal generator produce the spatially coherent sound. An NI PXI-1033 chassis inserted with 4 PXI-4496 cards has been used to simultaneously sample 64 channels of microphones at 44 k samples s$^{-1}$. Each sample is digitally quantized to 24 bits. The data stream is cut into various blocks, and each block contains 4096 samples for the efficiency of DFT. All experimental setups are almost identical to the previous numerical simulations. For example, the distance between the two speakers is $0.2\sqrt{2}$ m, and the bottom right one is regarded as the background noise and can be separately measured.

The typical beamforming algorithm (equation (7)) has to be applied to the complete dataset. In contrast, the observer algorithm (equations (25)–(27)) is able to be recursively applied to each sampling block. Figure 9 shows the experimental results. The findings are quite similar to the numerical results in figure 5. The bottom-right coherent noise (see figure 9(a)) cannot be completely rejected with conventional beamforming (see figure 9(b)). In addition, the operation in conventional beamforming can only be conducted for the whole 100-block dataset. In contrast, the proposed observer-based beamforming can be recursively operated for each block and almost completely rejects the interference from
the coherent noise source at the 100th block. It is also clearly shown by comparing figures 9(c) and (d) that the processing results asymptotically approach the actual appearance of a monopole noise source. In summary the observer algorithm outperforms the typical beamforming in terms of its recursive capability and the suppression of coherent interferences. The present results are preliminary and more experiments are still being conducted.

6. Summary

A new approach, the so-called observer-based algorithm, has been proposed as an alternative to classical beamforming. It is important to note that the beam patterns of both classical beamforming and the observer-based algorithm are comparable. Related discussion can be found in the previous work [11]. Advanced signal processing techniques can be employed to further improve the resolution and accuracy of classical beamforming as well as the observer-based algorithm. Those deconvolution type of algorithms have expensive computational costs and can only be conducted offline. Interested readers can refer to the literature [7–9].

The detailed algorithm and theoretical background of the observer-based beamforming method have been given in this paper. From the theoretical perspective it can be seen that the observer-based beamforming avoids the assumption of incoherence between the background noise and the signal of interest in classical beamforming. In addition, the observer-based beamforming can be recursively operated for each block whereas the classical beamforming can only be conducted once all data acquirements are available. As a result, it is claimed that the observer-based beamforming has the distinctive feature of coherent noise rejection and holds potential for real-time computation.

The observer-based beamforming algorithm has been validated in both numerical simulations and an experiment. The quality of resulting acoustic images and computational costs have been compared to those of classical beamforming. The results confirm that the coherent background noise can be gradually rejected with the observer-based beamforming. Any variations in coherence can be quickly captured in less than 1 s. The real-time capability of the observer-based beamforming has been demonstrated by performing the calculations over sampled blocks one by one. The results suggested that, in addition to the recursive algorithm, a pipeline configuration with a software implementation in C language can achieve real-time performance in practical experiments. In addition, this innovative observer-based beamforming approach adopts an alternative perspective of classical control to resolve issues in sensor array measurements. Following the same perspective many techniques already developed in control could be extended to this new application area of array signal processing.

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