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2010 EPL 90 68005

(http://iopscience.iop.org/0295-5075/90/6/68005)

A Letters Journal Exploring the Frontiers of Physics

June 2010

EPL, **90** (2010) 68005 www.epljournal.org
doi: 10.1209/0295-5075/90/68005

Diversity of game strategies promotes the evolution of cooperation in public goods games

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received 24 November 2009; accepted in final form 15 June 2010
published online 14 July 2010

PACS 87.23.Kg – Dynamics of evolution
PACS 02.50.Le – Decision theory and game theory
PACS 87.10.Rt – Monte Carlo simulations

Abstract – We propose a mechanism allowing strategy diversity instead of a common combination of cooperation and defection to study how cooperation evolves in public goods games. Each individual is assigned a variable valued in the unit interval as its cooperation degree. Thus, diverse cooperation degrees express the diversity of game strategies in the way of multiple contributions of players, and the investment in the common pool is positively correlated with cooperation degrees correspondingly. Moreover, we also define two particular roles named *altruist* and *egotist* defined locally since they depend on the behavior of their neighboring players. Numerical simulations show that the proposed diversity of strategies can substantially evoke the emergence and maintenance of cooperation. Notably, we also find that no player will act as a long-term exploiter (*egotist*) or exploitee (*altruist*) in the whole evolutionary process.

Social dilemmas capture the fundamental puzzle of the evolution of cooperation. Evolutionary game theory provides a competent theoretical framework to address the subtleties of cooperation among selfish individuals [1–4] and the prisoner's dilemma seems particularly suited in this respect [4,5]. However, competitions among a group of entities instead of two individuals are also widespread in many realistic situations. The public goods game (PGG), being proposed to illustrate the problem of cooperation and cheating through group interactions, is regarded as a prisoner's dilemma game with more than two participants and attracts also much attention to study the emergence of cooperative behavior [6–9]. Cooperation and defection are the two strategies that are usually at the heart of such social dilemma. In a typical PGG played in interaction groups of size N, each player must independently and simultaneously make its decision, to cooperate (contribute an amount c to the public goods) or to defect (contribute nothing). The collected sum is multiplied by a factor r $(1<r<N)$ and is redistributed to the N players equally, irrespective of their individual contributions. The maximum total income is achieved if all players contribute maximally. In this case each player receives rc, thus the final payoff is $(r-1)c$. Players are faced with the temptation of being free-riders, *i.e.*, to take advantage of the common pool without contribution. In other words, any individual investment is a loss for the player because only a portion $r/N<1$ will be repaid. Consequently, rational players invest nothing —hence to establish a social dilemma.

Various mechanisms aimed at find under what conditions the cooperation emerges in the frame of PGG have been explored. Among the more prominent are punishment [10–13], optional participation [8,14,15], image score effect [16–18] and different interaction topologies [19]. Hauert *et al.* have introduced the voluntary participation in PGG [7,20]. They found that this voluntary participation efficiently prevents defectors from spreading within the population and results in a substantial willingness to cooperate. The study of Szabó *et al.* shows that the introduction of loners leads to a cyclic dominance of the strategies and promotes substantial levels of cooperation in the PGG on a square lattice [21]. Social diversity by means of heterogeneous graphs was introduced by Santos and Pacheco, who have investigated that diversity associated with the number and the size of the PGG can promote strong cooperation [22,23].

However, herein it is worth mentioning that, to our knowledge, in most previous studies of PGG, a common simplifying assumption is that players can adopt one

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of the two feasible actions, cooperation and defection. Note that these may not be realistic assumptions in complicated real-life situations. Actually, agents often have inhomogeneities in personal features, such as the often-observed different economic status and contributive inclinations in real world. They usually have various choices to invest any part of their wealth according to their own personal situations, instead of just none or all. In addition, the act of giving may be more important than the amount given in a more generalized perspective. This may be of particular relevance whenever the survival of the community is at stake, in which case any help is necessary.

Enlightened by this idea, within our model we consider continuous set of strategies capable of representing a remarkably rich variety of contributive behavior, by introducing a variable named cooperation degree. For the sake of an easier depiction of cooperation degree, it is reasonable to assume that the proportion of actual investment to the total available possession is defined as the cooperation degree. Evidently such setup leads the value of cooperation degree to be distributed in the unit interval [0,1], resulting the highly heterogeneous strategy distribution and also the two aforementioned behavior are included in both limits, 0 for full defectors and 1 for full cooperators in the traditional definition.

Here it is worth referring to the following two new roles: *Altruists* and *Egotists*. An *altruist* (A) is an individual whose cooperation degree is the highest one in its neighborhood, with the same idea, an *egotist* (E) is a player whose cooperation degree is lower than any one of its neighbors.

Now we would like to describe our presently introduced evolutionary model in ample detail. To eschew effects of complex host graph topologies, we employ a two-dimensional lattice network to establish the compulsory version of PGGs. Thus all individuals own the same k neighboring interactions, where k is the average degree of the underlying interaction topology. A given player x acts as an organizer of the common pool x with size $k+1$, where there occurs the PGG involving x itself and its neighboring players. Besides the PGG organized by itself, player x also engages in other k PGGs organized by its neighbors [22].

In order to demonstrate the rich variety of possible complex individual features, we consider the total wealth of each player in the following two cases. In one instance denoted by c-random, each player x has a randomly assigned variable c_x as its initial whole wealth, for simplicity, $c_x \in [0,1]$. In the other instance denoted by c-fixed, each one has the same fixed whole wealth c_x, as an additional simplification but without loss of generality, we normalize c_x to unity. In any case, the total wealth c_x of player x is equally distributed among all the PGGs that it engages in. After each contributing period, the collective sum in each PGG multiples by a constant factor η, where $\eta = r/(k+1)$ is a renormalized enhancement factor on the public goods. Then the public goods of each PGG

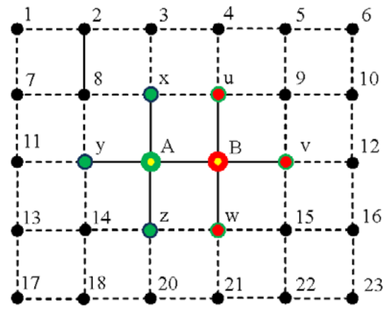

Fig. 1: (Color online) Small part of the square lattice indicating the relevant configuration for computing the maximum payoff difference M between sites A and B.

is equally shared to all participants, irrespective of their individual contributions. Thus the payoff P_{xy} of player x associated with the neighborhood centered at individual y can be expressed as

$$P_{xy} = \frac{r\sum_{i=0}^{k} c_i\delta_i}{k+1} - \frac{c_x\delta_x}{k+1}, \tag{1}$$

where c_i and δ_i are the total wealth and the cooperation degree of individual i, respectively. The accumulated payoff P_x of player x is the sum of gains from all interactions in which it participates:

$$P_x = \sum_{y\in\Omega_x} P_{xy}, \tag{2}$$

where Ω_x denotes the community of x's nearest neighbors plus itself.

After this, each player imitates the strategy (namely, the cooperation degrees) of those neighboring players who has scored higher payoff. Evolution of strategies is performed in accordance with a synchronous Monte Carlo simulation procedure comprising the following elementary steps. Agent x with strategy δ_x will shift over to the strategy of another agent y, chosen randomly from x's k nearest neighbors with a probability w_{xy}, iff y's strategy has yielded higher payoff P_y, otherwise the original strategy δ_x is maintained. The probability w_{xy} can be written as

$$W_{xy}(\delta_x \leftarrow \delta_y) = \frac{P_y - P_x}{M}, \tag{3}$$

where M ensures the proper normalization and is given by the maximum possible difference between the payoffs of x and y.

Additionally, for the sake of clarity, we plot a small part of the square lattice as shown in fig. 1. Based on the statement mentioned in the above context, we take site A and B, for example, and compute the collecting payoffs of agents occupying the site of A plus B, and then compute the maximum difference M between them. Player A will participate $k+1 (k=4$ in this study) PGGs organized by x, y, z, B and itself. Assuming that the total wealth and the cooperation degree of individual i is c_i and δ_i, the collecting payoff of A in the system is given by

$$P_a = \frac{r\sum_{c_i\in\Omega_a} c_i\delta_i}{k+1} - (k+1)c_a\delta_a. \tag{4}$$

Similarly, the collecting payoff of B is given by

$$P_b = \frac{r\sum_{c_j\in\Omega_b} c_j\delta_j}{k+1} - (k+1)c_b\delta_b. \tag{5}$$

Thus,

$$\begin{aligned} |P_a - P_b| &= \left| \frac{r(\sum_{c_i\in\Omega_a} c_i\delta_i - \sum_{c_j\in\Omega_b} c_j\delta_j)}{k+1} \right. \\ &\quad \left. + (k+1)(c_a\delta_a - c_b\delta_b) \right| \\ &\leqslant \left| \frac{8r}{k+1} + \left(k+1-\frac{3r}{k+1}\right)(c_b - c_a) \right|. \end{aligned} \tag{6}$$

Noticing $0 \leqslant \delta \leqslant 1$ and $0 \leqslant c \leqslant 1$, we get

$$|P_a - P_b| \leqslant \frac{5r}{k+1} + k + 1. \tag{7}$$

Since $\eta = r/(k+1)$ and $k = 4$ in our adopted network, thus we have

$$|P_a - P_b| \leqslant 5\eta + 5. \tag{8}$$

Therefore, $M = 5\eta + 5$.

Initially, the computer simulations were started from a random initial strategy distribution, where the cooperation degrees (δ) are assigned to players at random. After every such iteration cycle, the individual wealth for the subsequent round robins remains fixed to initial c_x for $\forall x$, thus restarting the cycle. Simulations of the evolutionary process via the Monte Carlo algorithm were performed for a large enough number of iteration cycles 10^6 on networks hosting $N = 10^4$ players. The structure of connectivity is restricted to regular graphs with $k = 4$. The final results shown below were averaged over 100 independent realizations of the initial conditions to warrant appropriate accuracy.

We start the study by visually inspecting the characteristic spatial distributions of the average cooperation degree δ_{ave} in the whole population as a function of the multiplication factor η in two different cases of c-fixed (red line) and c-random (blue line). The simulations presented in fig. 2 indicate clearly that the average cooperation level monotonously increases with the increment of η, albeit that the rate of the increase differs slightly depending on the two representative cases of total wealth. Note that for a fixed value of initial wealth (red line), all the realizations lead the system to a configuration in which all group members make the maximum contribution to the group project. It is long known that a sharp transition from defection to cooperation takes place at $\eta = 1$ in well-mixed populations. Moreover, the outstanding importance of social diversity for maintaining cooperative behavior has been described in ref. [22], where it has been argued that cooperators become predominant at $\eta \approx 0.7$ on regular graphs and $\eta \approx 0.6$ on scale-free graphs. While this number decreases to $\eta = 0.2$ (red line) and $\eta = 0.3$ (blue line) in our results (see fig. 2) which yield exclusive dominance of cooperators who tend to contribute all the wealth to the public goods even in highly unfavorable conditions. Thus, irrespective of the two cases of c-fixed and c-random, the presented results in fig. 2 evidence the sizable impact of strategy diversity on the emergence and evolution of cooperation.

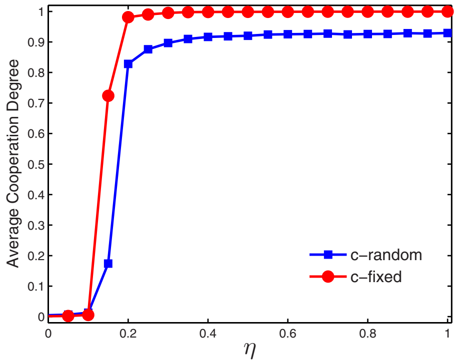

Fig. 2: (Color online) The simulation results of average cooperation degree δ_{ave} as a function of η for two different cases of initial wealth c: c-random and c-fixed.

The impact of the diversity of the initial wealth distribution on the evolution of cooperation is also corroborated nicely by results presented in fig. 2. As shown in fig. 2, the red line takes on a rising trend when $\eta > 0.1$, and the maximum value 1 of δ_{ave} may come into being when η is approaching to 0.2. The blue line (c-random) monotonously increase with a slightly lower ascending velocity than the red line, while its maximum value is about 0.9. Even more obvious differences brought by the two initial wealth distributions are clearly demonstrated in fig. 2. Forming a striking contrast to the various colors and slow changing velocity in figs. 3(a)–(d), $\eta \geqslant 0.15$ are able to sustain red clusters with green or blue borderlines scattered across the spatial grid in figs. 3(e)–(h), indicating that the majority of players have already been incline to contribute all their wealth. In addition, $\eta \geqslant 0.3$ can facilitate cooperation to the point of domination and leads to the extinction of defectors finally owning to the large single contiguous cluster of cooperators in figs. 3(e)–(h), while players adopting different cooperation degrees still coexist on a square lattice for a wide range of η in figs. 3(a)–(d). Cooperation can be better revitalized and maintained by the equality of initial wealth (c-fixed) than the inequality of initial wealth (c-random) in the collection activities of public goods, as was found by results in fig. 3.

It is evident that individuals will follow with interest in their partners' payoff and strategies due to the selfish and rational identity and the payoff-based strategy update

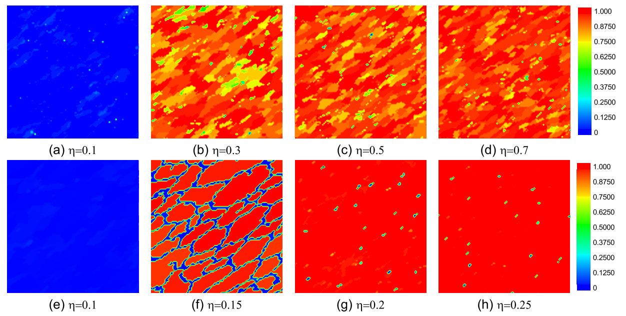

Fig. 3: (Color online) The distribution illustration of cooperation degrees in dependence on different values of the multiplication factor η and initial wealth c. The color code indicates the value of the cooperation degree. The snapshots are a contractible picture of a large image scale of the system on the full 100×100 lattices at a certain time when the system reaches a steady state. Each data point on the lattice denotes a player, and the color on the point correspondingly expresses as the cooperation degree of the player. (a)–(d) c-random. (e)–(h) c-fixed.

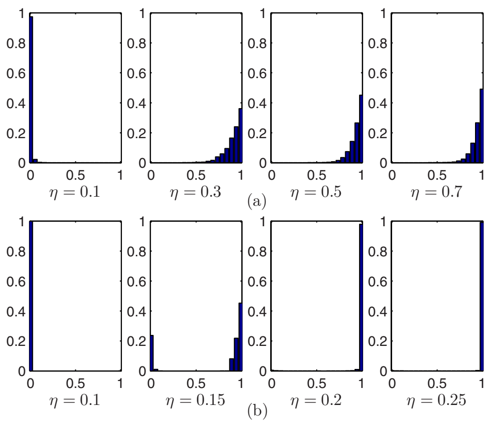

Fig. 4: (Color online) The proportion distribution of cooperation degrees for different values of the multiplication factor η and initial wealth c. The X-axis represents the value of the cooperation degree. The Y-axis is the proportion of individuals with such cooperation degree to the whole population. (a) c-random. (b) c-fixed.

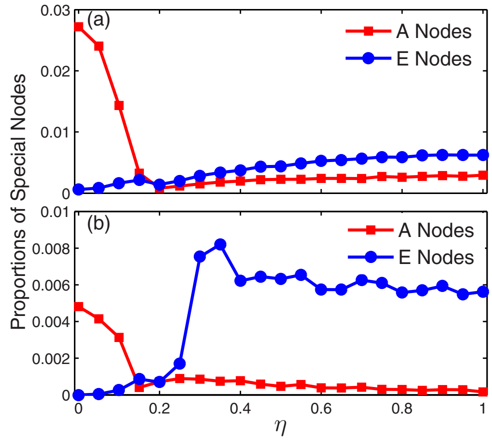

Fig. 5: (Color online) The proportions of A's (blue line) and E's (red line) to the whole population at the final stationary state *vs.* the multiplication factor η and initial wealth c. (a) c-random. (b) c-fixed.

rule. In particular, the latter will drive players to imitate successful strategy most probably, and thus lead to no much difference in the cooperation degrees among most nodes after the system converges to the equilibrium. As can be noticed, this conjecture can be strengthened by the relatively monotonous colors presented in fig. 3(d) and fig. 3(h), clearly evidencing that individual strategies do not take on diversity when the system reaches the final steady state. Importantly, the distribution of cooperation degrees, presented in figs. 4, can help us intuitively observe the strategy adoption in the population when the system reaches its final stationary state. The wide range of cooperation degrees in figs. 4(a) is largely in agreement with the emergence of various colors in figs. 3(a)–(d). Moreover, the results in figs. 4(b) are identical with those in figs. 3(e)–(h) showing that a large portion of the players' cooperation degrees are close to 1 for $\eta \geqslant 0.25$.

Furthermore, it remains of interest to investigate the behavior of *altruists* and *egotists* defined in the previous description. A more meaningful result in fig. 5 is that A's and E's account for tiny ratios in the population for a wide spectrum of η in the whole evolving process. E's will gain higher payoff than their neighbors by exploiting them, but the intrinsic rational personality reminds the neighbors to do not allow long-term exploitation by E's. Therefore the payoff-based strategy adoption rule will facilitate the prevalence of E's strategies in their neighborhood. Nevertheless, the widespread mutual exploitation will decline of the number of E's. Coming back to the real world, when two coequal players or companies are playing with each other, defection occurring at irregular intervals will also not last for a long time. If one player fails by being exploited by the counterpart, it will switch its strategy to another to shake off the adverse circumstances in the next round. So A's and E's are temporary roles played by individuals in a short period of time. In other words, there is no node which acts as an A or E for a long time, thus indicating the scenario where the number of A's and E's keep a steady and small value in the population.

Another noteworthy phenomenon is the proportion of A's to the whole population is lower than E's across the long span of η. Evidently, the proportion of A's gets an upper hand than E's when $\eta < 0.2$, yet it falls rapidly, and then maintains at a lower value than that of E's henceforth. As we know A's will get lower payoffs than their neighbors, while E's can obtain greater payoffs by exploiting their neighbors. Therefore, A's will abandon their current strategies to adopt the successful strategies of their neighbors in the next game ground. The number of A's is bound to decrease and maintains in a nearly extinct state. Meanwhile, E's with high payoffs will be easily copied by their neighbors. Thus, E's will get short-term increase in their number. However, mutual exploitation is destructive to the improvement of the overall payoffs of individuals participating within the PGG. So E's cannot be widely prevalent for a long time and will account for only a small proportion to the population ultimately.

Finally, we take into account some interesting extensions after presenting main results of the original model. Ultimatum Game (UG) is another kind of game extensively used to model altruistic behavior [24]. Two players have to agree on how to split a sum of money. The proposer makes an offer. If the responder accepts, the deal goes ahead. If the responder rejects, neither player gets nothing. In both cases, the game is over. Obviously, rational responders should accept even the smallest positive offer, since the alternative is getting nothing. Here we draw analogies with the investigations in the frame of UG in ref. [25], where the asymptotic behavior of pure populations with different topologies using two kinds of strategic update rules: natural selection and social penalty are investigated. They have discussed the emergence of fairness in the different settings and network topologies. Our model, with a continuous strategy of PGG, can be seen as an expansion of UG, that individuals contribute a certain percentage of their wealth to the entire group, not to only one person. Moreover, the abundance of highly generous individuals observed dictate the emergence of fairness, thus coupling our findings with considerations of continuous strategy in the frame of PGG. Therefore, this presently extension manifests a nice view on future studies to investigate the thorough interplay between the two social dilemmas.

Summing up, we have investigated the effects of diverse strategies on the emergence and evolution of cooperative behavior in the frame of public goods games. Players participate in games with cooperation degrees uniformly distributed in the range [0,1]. Strategies are of diverse choices and not just two options of cooperation and defection. Furthermore, we define two special nodes named *altruist* and *egotist*. Our study thus supplements previous works examining the impact of diverse strategies and, moreover, demonstrates that the observed promotion of cooperation can be attributed to the proposed diverse strategies in the public goods game. Interestingly, the numerical investigations suggest that players acting as altruist or egotist account for very small ratios in the population. In addition, it should be emphasized that the inequality of initial wealth distribution inhibits and delays the emergence and evolution of cooperation to a certain extent, being in contrast with the case that each individual is assigned the same initial wealth. Our work may provide an alternative way to promote cooperation by the introduction of strategy diversity and may be helpful in reflecting the realistic phenomenon in social systems.

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We are grateful to the anonymous referees for their constructive comments and suggestions which helped us improve our manuscript. We also acknowledge F. Fu and X. Chen for their helpful assistance in preparing this paper. This work was supported by the National Natural Science Foundation of China (NSFC) under Grants Nos. 60674050, 60736022, 10972002, and 60974064.

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