Non-linearities in flight control systems

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ABSTRACT

Non-linear behaviour is usually undesirable for the operation of any system and it needs to be minimised in flight control systems in order to provide the pilot with a predictable and well-behaved aircraft. The non-linear response characteristics exhibited by all aircraft and their flight control systems can be characterised by using non-linear functions to simulate and analyse their behaviour. This paper explores the types of physical non-linearities that exist in aircraft flight control systems and provides some examples. It describes how the non-linear stability might be analysed and addresses the subject of actuation systems modelling. The final section provides recommendations for minimising non-linear behaviour and gives some general guidelines on how to deal with non-linear characteristics.

NOMENCLATURE

A	input signal amplitude
B	output signal amplitude
D	dead-zone magnitude
e	error signal
f	frequency (Hz)
$G(s)$	transfer function
k	gain
k_1	inner-loop gain
k_2	outer-loop gain
m	intermediate output signal
M	amplitude limit
R	rate limit
s	Laplace variable
t	time
τ	time constant
u	input signal
y	output signal
ω	frequency (rads/sec)
ω_n	natural frequency
ζ	damping
ϕ	phase lag

1.0 INTRODUCTION

The handling and control difficulties experienced by pilots that arise from problems with their aircraft's flight control system are not usually due to 'software faults' as we might have been led to believe. They are nearly always caused by aerodynamic or system non-linearities, and a lack of appreciation of their significance by the system's designers. The transition from linear modelling and design through to non-linear implementation and testing can be difficult and only tends to be achieved satisfactorily with experience. Control theory and advanced computing provide an excellent foundation for establishing a design, which then needs to be carefully implemented and tested, taking advantage of physical knowledge and the lessons learned from past projects.

Control systems are almost universally designed by using linear models and a set of related design techniques and associated criteria. It is important to recognise that these linearised models are obtained by making simplifying assumptions about the non-linear vehicle to be controlled, and that the linearised models have an associated set of limitations that need to be understood by the designers. Non-linearities have previously been classified as 'parasitic' and 'intended'[1], essentially depending on whether they are associated with the physical mechanics of the hardware, or the functional aspects within the flight control computing. With the development of fly-by-wire systems, the reduction in the mechanical complexity of the systems has reduced the parasitic non-linearities. However, the intended type has increased, along with the growth in digital computing capabilities.

There has been a rapid development of digital computing during the last decade and a strong emphasis on process improvements to reduce costs. It is considered to be timely to review the subject of

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non-linearities in relation to modern flight control systems with digital control laws, where the designers can introduce complex non-linear functions into the flight software. These might include on-board aircraft models, adaptive control algorithms and automatic limiting functions.

Although it is desirable to design systems that behave linearly and to minimise non-linearities, it is recognised that non-linearities are not always detrimental, since they can be used very effectively as part of the control laws, to give a desired effect. We now describe the main non-linearities and their natural physical occurrence in a flight control system. This leads to a description of their potential use as a control law functional element and the means of analysing the non-linearities by using linear techniques.

2.0 THE TYPES OF NON-LINEARITY

2.1 Amplitude limits

The basic characteristics of amplitude limits, which are sometimes referred to as authority or saturation limits, are shown in Fig. 1. These can be asymmetric and exist in a flight control system because of an actuation system's travel limits, available sensor ranges, and signal scaling or authority limits within the flight control computing.

Flight control laws can contain such functions for limiting the authority of part of the system, possibly for safety considerations. When carrying out a stability analysis for the unsaturated case, the function assumes the gain as defined by its local gradient, as indicated in Fig. 1. The effect of an amplitude saturation on the control loop stability can be assessed by progressively reducing the non-linearity's gain towards zero, in accordance with its describing function analysis(2,3).

The saturation function has an attenuating effect for large sinusoidal inputs, but it does not introduce any phase shift. It is noted that for the extreme case of 'hard saturation', where the input places the output well into the saturation region, the gain to be used in a stability analysis is zero, since a small perturbation of the input produces no change in the output.

2.2 Rate limits

This non-linearity received great attention during the last decade, because of the JAS39 Gripen(4) and YF-22 accidents(5). It occurs naturally in any actuation system, when the rate of travel becomes limited. For example, in a hydraulic actuator as its main control valve maximum flow capability is attained. In flight, the rate will vary according to the aerodynamic loading on the control surface and will be asymmetric, depending on whether the surface is moving with or against the load. A rate limit function is often used in the control laws in order to limit the rates of pilot or autopilot command signals. It is sometimes used to limit the rate of the demand signal to an actuation system, to protect the system from excessive commands and physical wear, and to provide consistent rate limiting behaviour.

Stability analysis that includes the effects of the rate saturation, can be carried out by using a describing function analysis(2,3). Rate limiting affects the gain and phase of the output relative to those of the input, since it introduces attenuation and an 'effective delay' to the input signal, as indicated in Fig. 2. This delay can have a significant effect on the system's stability characteristics. There are a range of possible non-linear compensation schemes(6,7,8) to recover this loss in phase by reversing the direction of the output, when the input changes direction. The analysis of this non-linearity is described further in Section 3.

2.3 Dead-zone

The dead-zone characteristic, which is sometimes called threshold or dead-space, is shown in Fig. 3 and arises naturally in mechanical and electrical systems where the first part of the input is needed to overcome some initial opposition at the output. For example, an overlapped valve in an actuation system, where the piston moves over a finite distance but no flow occurs until the valve becomes open. A simpler example is where a force is applied to a body on a surface, but there is no motion until the force is sufficient to overcome the effects of friction.

In flight control laws, if zero command is required for a certain position of a physical input device (e.g. the pilot's stick), then a dead-zone function can be included immediately downstream of the device to give this effect. This is particularly useful in any system, which has an integrating function downstream of an input device that has characteristics which make it difficult to achieve an accurate zero command (e.g. a stick with poor self centring). Without such a dead-space, the pilot would observe a continual drift in aircraft response due to a small and unintentional command, which would be continually integrated to produce a gradual control surface response.

When the operating condition results in the input signal being within the dead-zone, then this can be assessed in a stability analysis by setting the function's gain to zero, since a small perturbation in the input gives no change in the output. However, this may not be a particularly useful analysis. If the dead-zone is on the command path and it might be appropriate to remove the non-linearity (or bias the input sufficiently) to carry out a more meaningful linear analysis. It

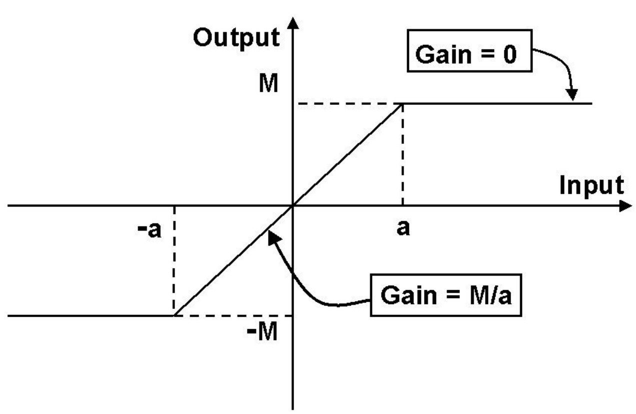


Figure 1. Amplitude saturation characteristics.

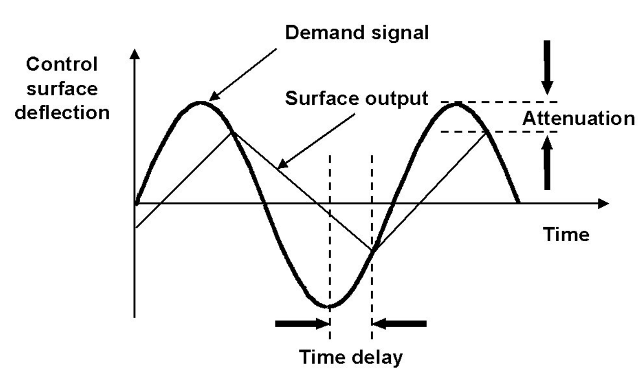


Figure 2. Example of actuator rate saturation characteristics.

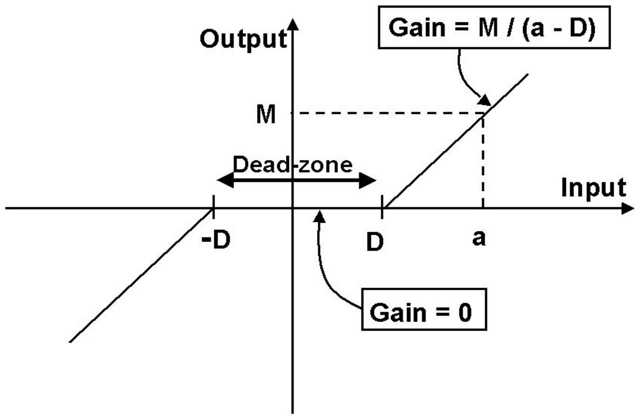


Figure 3. Dead-zone characteristics.

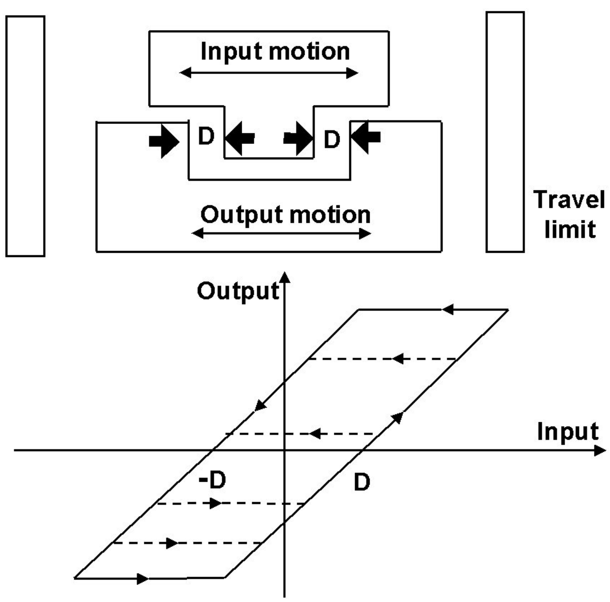


Figure 4. Backlash characteristics.

is possible to perform a describing function analysis[2,3] for this non-linearity, which only affects the gain characteristic, by introducing an attenuating effect for small inputs.

2.4 Hysteresis/backlash

Hysteresis is a non-linear characteristic that is normally associated with the characteristics of electromagnetic materials. A particular form of hysteresis in mechanical systems is backlash, a moving dead-zone, the characteristics of which are shown in Fig. 4. This occurs naturally in all mechanical systems due to the finite gaps between linkages or gears, commonly known as 'free play'. A good example is that of a bicycle chain and chain wheel. The non-linear characteristics of hysteresis are more complicated than those previously described, since the hysteresis usually has inherent dead-zone and amplitude limiting behaviour.

A hysteresis function can be introduced into an aircraft's flight control laws to prevent limit-cycle oscillations between two system states, where automatic switching between the states is required. Where the states meet, it might be possible to operate at the switching condition and invoke a limit cycle. A hysteresis function can be added and 'opened out' until the limit cycle cannot occur; i.e. the system definitely operates in one state or the other and there is no possibility of 'automatic indecision', sometimes referred to as 'hunting'.

When the operating condition leads to the input signal being within the hysteresis' dead-zone, then a stability analysis involves setting the function's gain to zero (but as noted previously, although theoretically correct, this may not be a particularly useful analysis). It is again possible to perform a describing function analysis[2,3]. This non-linearity affects the gain and phase, and introduces attenuation for both small and large inputs. The small inputs are affected by the inherent dead-zone and the large inputs by the authority limits. Hysteresis also introduces an effective delay, as the gaps between the mechanical components are traversed, i.e. there is no output displacement until contact is made, therefore affecting the phase.

2.5 Jump resonance

Strictly speaking, this is not a well-defined non-linear function such as those described above, but is a non-linear phenomenon (Fig. 5), which can result in 'cliff-edge' system behaviour, with a sudden jump between two system states. In the literature, it is usually associated with non-linearity in springs, due to changes in spring stiffness with frequency. A jump resonance type of characteristic can arise in actuation systems due to low acceleration capability, possibly as a result of low servo valve limits[9], but such a feature can usually be designed out if it exists[10]. The authors are unaware of any deliberate application of jump resonance, since it can produce a very abrupt loss in phase and is best avoided. The exact characteristics may be very complex and even chaotic. It is possible to perform an 'ad hoc' describing function analysis for this non-linearity[11]. Jump resonance affects both gain and phase due to the change in damping characteristics, but it is the rapid change in phase that is the major effect for a closed-loop system.

2.6 Non-linear gearing/shaping

This type of non-linearity (Fig. 6) is possibly the most common and occurs in most mechanical systems due to changes in mechanical advantage between an input and output, as a result of the physical geometry variations associated with operating point or position. For flight control, a good example is the variation of control surface effectiveness with an aircraft's angle-of-attack. This type of non-linear characteristic is also found in many mechanical circuits.

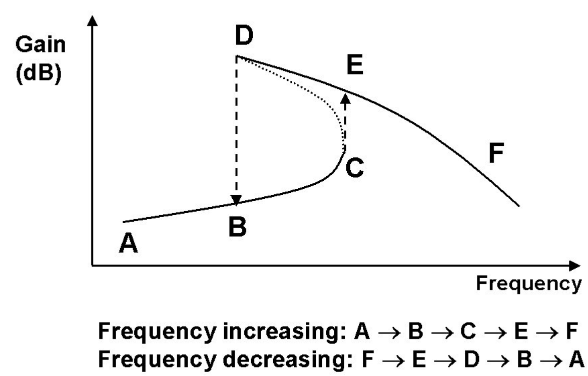


Figure 5. Jump resonance characteristics.

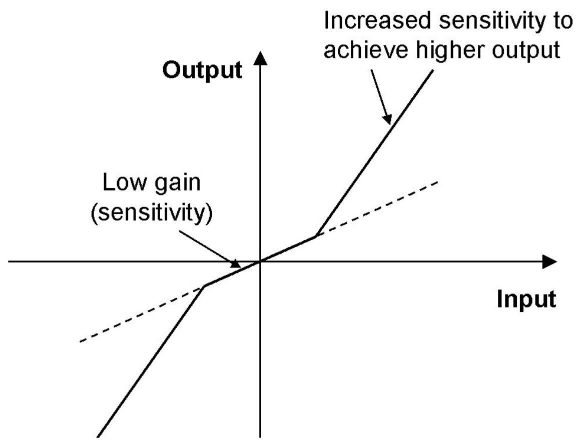


Figure 6. Non-linear shaping function.

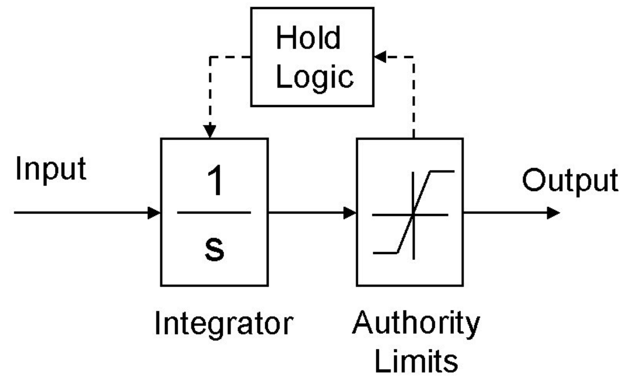


Figure 7. Limited integrator function.

Within an aircraft's flight control laws, it is usual to provide such a function immediately downstream of the inceptor to act on the pilot's command. This will be designed to make best use of the available range of travel of the inceptor. The aim will be to give acceptably low sensitivity about the datum (trim) inceptor positions to allow precision control, with increasing sensitivity (gradient) for larger inceptor inputs to allow full command capability.

For a stability analysis, this non-linearity does not introduce any phase shift and is readily analysed by calculating the local gradient at the function's steady operating point.

2.7 Other non-linearities

Some of the most common types of non-linearity have been described and discussed. Several other types can be found in literature, associated with electrical or mechanical physical devices, such as a rectifier, relay or detent[1].

In the design of flight control systems, it is often the case that some quite ingenious, and possibly complicated, functions are created by the flight control law designers, either to solve a particular non-linear stability problem or to enhance performance in the presence of non-linearities. Such non-linear functions usually involve a combination of the fundamental non-linearities and linear transfer functions. The simplest example is the limited integrator shown in Fig. 7. It 'freezes' the integral action when the upper or lower output limit is reached and only becomes active again, when the input is such that it will bring the output off the limit. This prevents integrator 'wind-up', whereby the integrator output continues to increase beyond the output's limiting value and (incorrectly) remains on the limit, even though the input has changed its sign. This is a popular function in modern control systems.

The limited integrator is an example of how linear and non-linear functions can be used together in a harmonious way. Another example is shown in Fig. 8, which shows an amplitude-dependent filter. If the gradient of the non-linearity is set to unity at its centre, then the transfer function of the filter is $(1 + \tau_1 s)/(1 + \tau_2 s)$, giving phase advance or phase retard, depending on the relative size of the time constants. For larger inputs, that invoke the non-linearity, the effective denominator time constant is modified by the change in the gradient of the non-linearity. It can be arranged in such a way, that for large inputs, the numerator and denominator time constants become equal and the filter becomes an 'all pass' filter (unit gain). In Fig. 8 it is possible to obtain the same linear dynamics by taking the input to the differential term from just upstream of the integral term and

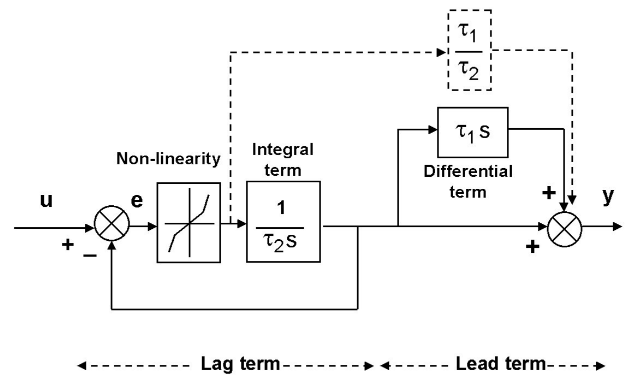


Figure 8. Non-linear command filter.

replacing the differentiator τ_1 s with the scaling factor τ_1/τ_2 as indicated by the dashed lines in Fig. 8. This is easier to implement, as it avoids the use of a pure differentiator.

This type of filter was used on the pitch command path of the Experimental Aircraft Programme (EAP) aircraft[12] to give good pitch attitude tracking for small pilot commands and a rapid normal acceleration (g) response for large pilot commands. This type of filter is also used on the Eurofighter Typhoon aircraft.

With modern digital flight control laws, it is likely that geometric functions will be used to convert between aircraft axis systems. Taking this one step further, complete on-board aircraft models have been included as part of aircraft flight control laws[13]. Further non-linearities are introduced within the air data system and its calibration functions, and by the gain scheduling of the flight control laws. Overall, this presents a large and complex analysis challenge and appropriate automated tools are needed.

It is important to note that a vehicle's aerodynamic characteristics can include many of the non-linear effects described above. For example, if a control surface is ineffective over part of its range, such as a spoiler operating within an aircraft's boundary layer, then this is effectively an aerodynamic dead-zone. Hysteresis is sometimes seen in aerodynamic behaviour, particularly in the transitions between attached and separated flow, which can result in a limit-cycle oscillation commonly known as 'wing rock'[14].

An example of aerodynamic non-linearity that leads to hysteresis is a pitch-up characteristic on an otherwise conventionally stable aircraft. Such a hysteresis is illustrated in Fig. 9, where the upper graph shows the pitching moment characteristics of an aircraft for several different values of elevator angle (η) plotted against angle-of-attack.

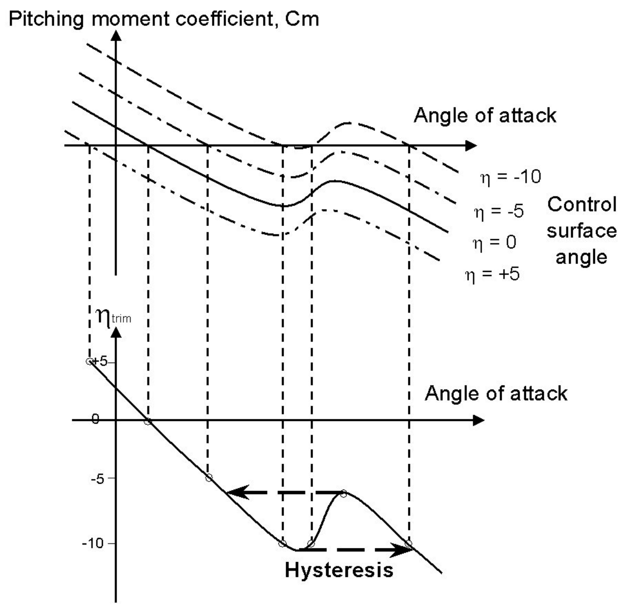


Figure 9. Example of aerodynamic hysteresis.

This exhibits a pitch-up (localised instability) at some angles of attack. From this data can be extracted the trimmed elevator angle (i.e. the elevator angle that gives zero pitching moment, denoted η_{trim}) as a function of angle-of-attack, as illustrated in the lower graph of Fig. 9. Over most of the operating range, where the aircraft is stable, this results in a unique relationship between trimmed elevator angle and trimmed angle-of-attack. Thus, the pilot can accurately control the angle-of-attack by varying the elevator angle via the mechanical linkage from his stick. However, in the region of the pitch-up, there are multiple angles-of-attack that are trimmed by the same elevator angle, and the aircraft will tend to rapidly transition through the unstable region from one stable trim solution to another. As the pilot increases the (negative) elevator angle to increase the angle-of-attack, the aircraft will 'pitch up' to a much higher angle of attack, as illustrated by the rightwards dashed arrow in Fig. 9. Similarly, when the pilot reduces the elevator angle through this region, the aircraft will rapidly reduce in angle-of-attack, following the leftwards dashed arrow. The relationship between elevator angle and resulting angle-of-attack can clearly be seen to exhibit hysteresis.

Finally, it should be noted that for pilot-in-the-loop control, the pilot could be regarded as a non-linear adaptive control element. This topic is complex and is not covered further in this paper but can be found in references that address pilot modelling and aircraft handling qualities(15).

3.0 DESCRIBING FUNCTION ANALYSIS

3.1 Describing function principles

Non-linear systems, by definition, are not usually amenable to analysis by linear methods, with their associated criteria for stability and robustness, such as gain and phase margins.

Describing functions (strictly speaking, 'sinusoidal input describing functions') are a very useful method for assessing the effect of a single dominant non-linearity on the stability and robustness of a linear system. A describing function is an approximate frequency response function, that is analagous to a linear transfer function, which describes the relationship between the output and the input of the non-linearity.

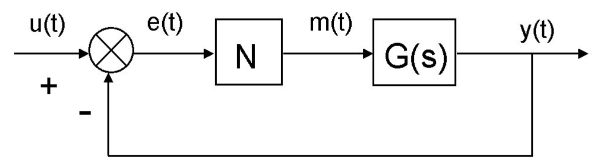


Figure 10. System with non-linear element.

Consider a closed-loop system, with a single non-linear element, as shown in Fig. 10.

A sinusoidal input signal $e(t) = E\ \mathrm{Sin}(\omega t)$ to the non-linear element N will produce a distorted output signal $m(t)$. For most naturally occurring non-linearities, this output signal will be a periodic cycle of the same fundamental frequency as the sinusoidal input. The output can therefore be expressed as a Fourier series, i.e. as the sum of an infinite number of sinusoids of integer multiples of the fundamental excitation frequency.

$$m(t) = \frac{M_0}{2} + M_1 \mathrm{Sin}(\omega t + \phi_1) + M_2 \mathrm{Sin}(2\omega t + \phi_2) + \dots \qquad \dots (1)$$

The describing function assumes that the response of the non-linear element can be approximated by the ratio of the fundamental component of the output $m(t)$ to the input sinusoid $e(t)$. Thus the describing function is given by:

$$|N(\omega)| = \frac{M_1}{E} \qquad \text{and} \qquad \angle N(\omega) = \phi_1 \qquad \dots (2)$$

This might appear to be a gross approximation, but it is justified for two reasons. Firstly, for most practical non-linearities, the magnitudes of the harmonic components of the output (M_2, M_3, M_4 etc.) reduce as frequency increases. Secondly, the linear parts of the system $G(s)$ will, in most cases, act as a low pass filter, attenuating the higher frequency components of the non-linear output. For these reasons, the fundamental component of the non-linear output will dominate the non-linear contribution to the system output $y(t)$. Therefore, in the closed-loop system, the fundamental frequency component will also dominate the non-linear contribution to the error signal $e(t)$, and hence dominate the stability of the system.

3.2 Validity

The describing function method is valid where:

1. The non-linear element is time-invariant.
2. There is only a single non-linear element in the system, or a group of non-linearities that can be easily combined and regarded as a single non-linearity. In reality, most systems feature multiple non-linear elements but for the purposes of analysis most can be considered to be insignificant and a single dominant non-linearity can usually be analysed.
3. The linear part of the system does not feature any high-frequency resonances that could cause the amplitudes of higher frequency harmonics to become significant. The describing function is most effective when the linear element attenuates the higher frequency harmonics, which are neglected as part of the analysis.

3.3 Evaluation

Consider the output of a non-linear element, which is a periodic function $m(t)$ with period $T = 2\pi/\omega$. This can be expressed as a Fourier series as follows:

$$m(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \mathrm{Cos}(n\omega t) + \sum_{n=1}^{\infty} B_n \mathrm{Sin}(n\omega t) \qquad \ldots (3)$$

where

$$A_n = \frac{2}{T}\int_{t_0}^{t_0+T} M(t)\mathrm{Cos}(n\omega t)\,\mathrm{d}t \qquad \ldots (4)$$

and

$$B_n = \frac{2}{T}\int_{t_0}^{t_0+T} M(t)\mathrm{Sin}(n\omega t)\,\mathrm{d}t \qquad \ldots (5)$$

Therefore,

$$M_n = \sqrt{A_n^{\,2} + B_n^{\,2}} \qquad \ldots (6)$$

and

$$\phi_n = \mathrm{Tan}^{-1}\frac{A_n}{B_n} \qquad \ldots (7)$$

Evaluating describing functions in this way is a tedious process, but in practice it can often be circumvented by looking up pre-evaluated describing functions in tables[3,16].

It should be noted that the describing function of a static non-linearity (one with no energy-storage elements) is independent of the input frequency ω. The describing function will, however, be a function of the input amplitude.

3.4 Stability

The beauty of the describing function is that it allows the stability and robustness of a closed-loop system to be assessed in the frequency domain in a similar way to linear systems. Consider the system of Fig. 10, where the non-linear element is represented by a describing function $N(j\omega)$. The closed-loop frequency response of the system is given by:

$$\frac{G(j\omega)N(j\omega)}{1+G(j\omega)N(j\omega)} \qquad \ldots (8)$$

For a sustained oscillation, the characteristic equation must satisfy:

$$1+G(j\omega)N(j\omega) = 0 \qquad \ldots (9)$$

Therefore,

$$G(j\omega)N(j\omega) = -1 \qquad \ldots (10)$$

or, expressed another way:

$$G(j\omega) = \frac{-1}{N(j\omega)} \qquad \ldots (11)$$

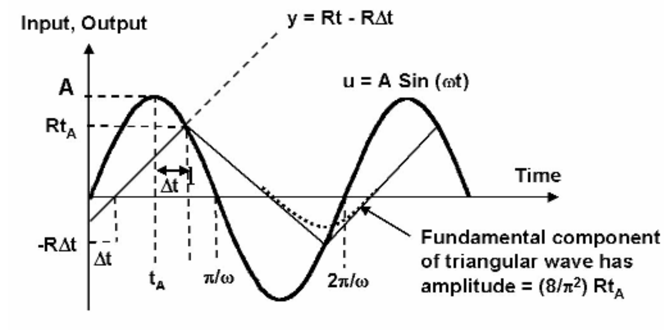


Figure 11. Rate saturation analysis.

Describing functions can therefore be used to show stability and robustness of the closed-loop non-linear system in a graphical form such as on Nichols or Nyquist plots, by using these results.

The Nichols or Nyquist plot of the linear part of the system, $G(j\omega)$, can be plotted as usual, and the reciprocal of the describing function $-1/N(j\omega)$ is plotted on the same axes. The possibility of a limit-cycle oscillation is then indicated where the two loci intersect. The $-1/N(j\omega)$ locus therefore corresponds to the classical '–1 point' in linear analysis. Some understanding of the nature of the possible oscillation can be obtained by considering the conditions necessary for this intersection, such as the frequency and amplitude of the input.

Alternatively, one can plot the locus $G(j\omega)N(j\omega)$ as a series of lines, corresponding to different input amplitudes, on a conventional Nyquist or Nichols diagram. In this case, of course, the conventional '–1 point' represents the point at which an oscillation could occur. This method is less elegant than the first, but gives a better graphical indication of system robustness, in terms of gain and phase margins.

3.5 Rate limit describing function

The rate limit characteristic is one of the most important non-linearities occurring within aircraft flight control systems. Despite this, its describing function is rarely covered in control textbooks. It is possible to develop a describing function representation for a fully developed rate limit, as follows. Consider the effect of a rate limit non-linearity on a sinusoidal input signal as shown in Fig. 11. Here, the rate limit R has caused full rate saturation of the input signal A Sin(ωt), such that the output signal $m(t)$ is a triangular wave.

The gain and phase relationships between the input $e(t)$ and the output $m(t)$ can be derived as follows. Consider the point at which the output equals the input, where:

$$e(t) = m(t) \quad \text{and} \quad t = t_A + \Delta t \qquad \ldots (12)$$

and therefore,

$$\begin{aligned} A\mathrm{Sin}(\omega(t_A+\Delta t)) &= R(t_A+\Delta t) - R\Delta t \\ &= Rt_A \end{aligned} \qquad \ldots (13)$$

Also,

$$t_A = \frac{\pi}{2\omega} = \frac{1}{4f}$$

$$\Delta t = \frac{\phi}{\omega} = \frac{\phi}{2\pi f} \qquad \ldots (14)$$

Substituting for t_A and Δt in equation (13) gives:

$$A\mathrm{Sin}\left(\left(\frac{1}{4f}+\frac{\phi}{2\pi f}\right)2\pi f\right)=\frac{R}{4f} \qquad \ldots (15)$$

or,

$$\mathrm{Sin}\left(\frac{\pi}{2}+\phi\right)=\frac{R}{4Af} \qquad \ldots (16)$$

which gives a phase lag of:

$$\angle N(\omega)=\phi=\mathrm{Sin}^{-1}\left(\frac{R}{4Af}\right)-\frac{\pi}{2} \qquad \ldots (17)$$

The gain is given by:

$$|N(\omega)|=\frac{2R}{\pi^2 Af}$$

$$=\frac{8}{\pi^2}\mathrm{Sin}\left(\phi+\frac{\pi}{2}\right) \qquad \ldots (18)$$

Note that as the rate limit decreases, the gain approaches zero, and the phase lag approaches 90 degrees.

These expressions were validated[17] by comparing their gain and phase characteristics with the results from a transfer function analysis (TFA) of a simulated rate limit. This covered a range of frequencies and amplitudes, as shown in Fig. 12. The matches obtained for gain and phase are excellent, where full rate saturation has occurred. Errors can be seen at the lower frequency end of the phase curves, where phase lag is less than 30 degrees. This is because the output from the rate limit function is not triangular, as assumed in the analysis, but contains an arc of a sine wave. Overall, the effect of the rate limit is well represented by this describing function.

The expressions for gain and phase contain the ratio (R/Af), the rate limit divided by input amplitude and frequency. This gives the useful result that the describing function can be simply plotted as a fixed relationship between gain and phase, as a function of (Af/R), as shown in Fig. 13.

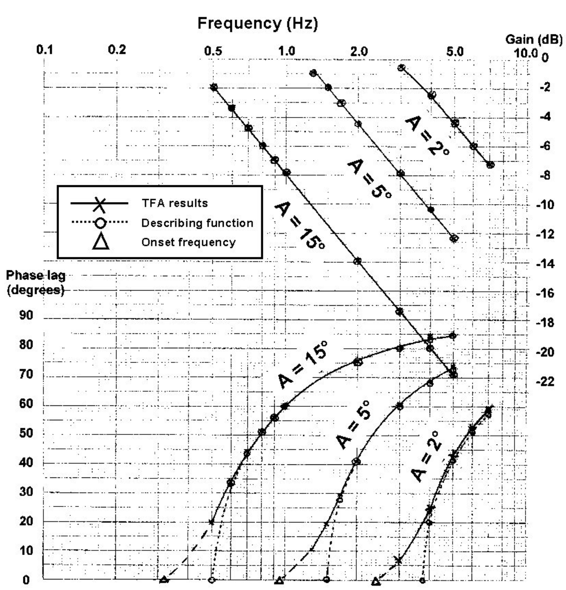


Figure 12. Validation of the rate limit describing function.

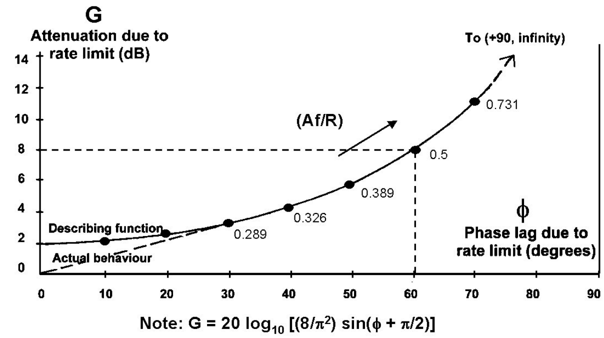


Figure 13. Rate limit describing function characteristics.

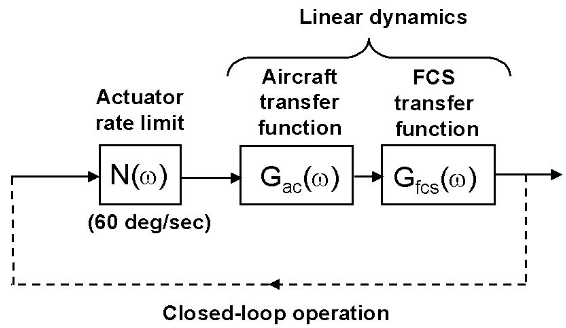


Figure 14. Block diagram.

It is noted that the describing function is inaccurate for less than 30 degrees of phase lag, due to the previously mentioned reasons. The actual behaviour for non-triangular wave outputs is approximately linear, as indicated by the onset frequencies in Fig. 12.

3.6 Flight control applications

3.6.1 Rate limit

To illustrate how the describing function might be used to assess the effect of a rate limit non-linearity on the stability of a flight control system for an aircraft, consider the system shown in Fig. 14. Here a non-linearity $N(\omega)$, representing the actuator rate limit of 60 degrees per second, has been placed in series with the FCS and aircraft transfer functions.

The open-loop frequency response of the linear part of this system, $G_{fcs}(\omega)G_{ac}(\omega)$, has been plotted as a Nichols plot in Fig. 15. Plotted on the same axes is the gain and phase relationship from the rate limit describing functions derived above, plotted as the inverse function $-1/N(j\omega)$. The contour for this inverse function is constant and the distance along it varies as a function of (Af/R).

The two curves intersect at a frequency of about 0·4 Hertz. This indicates the possibility of a limit cycle in the closed-loop system, for a value of (R/Af) that would give an attenuation of 8dB and a phase lag of 60 degrees. By consulting Fig. 13 it is seen that this can occur when (Af/R) is approximately 0·5. The intersection with the linear frequency response shows that the potential limit cycle can only occur at a frequency of 0·4Hz, and therefore, it will only occur at an amplitude of A = 1·25R, i.e. 75 degrees for a rate limit of 60 degrees per second. Such a large amplitude input would be extremely unlikely in practice or even impossible, as this is well in excess of

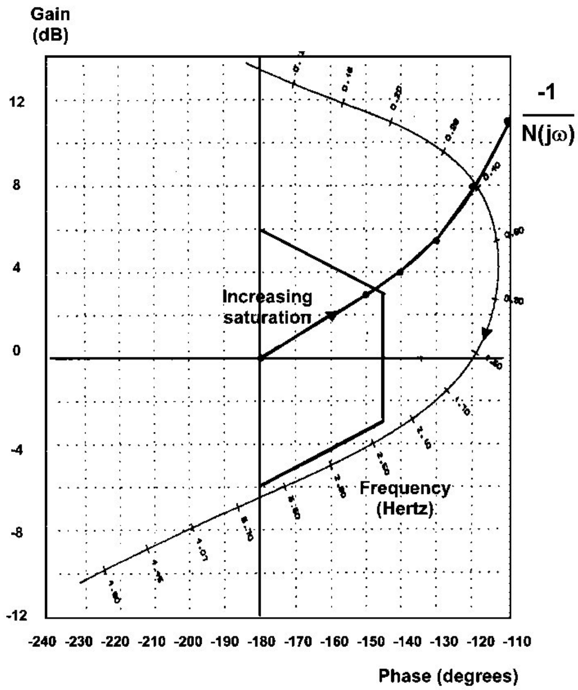


Figure 15. Rate limit example.

typical physical limits on control surface travel. Therefore, it is concluded that a limit cycle due to rate limiting would not exist in this case.

It should be noted that for a basically unstable airframe, the two curves of Fig. 15 will always intersect.

3.6.2 Hysteresis

Let us now consider the effects of hysteresis caused by mechanical backlash of 0·1 degrees, in a control surface actuation mechanism of a guided missile. The describing function for a hysteresis of width $2d$, excited by an input sinusoid of amplitude E_m is given by the following expressions:

$N = 0,\ E_m \leq d;$ otherwise:

$$|N| = \frac{\sqrt{A_1^{\,2} + B_1^{\,2}}}{E_m} \quad \ldots (19)$$

and

$$\angle N = \phi = \mathrm{Tan}^{-1}\left(\frac{A_1}{B_1}\right) \quad \ldots (20)$$

where

$$A_1 = \frac{4d}{\pi}\left(\frac{d}{E_m} - 1\right) \quad \ldots (21)$$

and

$$B_1 = \frac{E_m}{2}\left[1 - \frac{2}{\pi}\mathrm{Sin}^{-1}\left(\frac{2d}{E_m} - 1\right) - \frac{4}{\pi}\frac{\sqrt{dE_m - d^2}}{E_m^{\,2}}\left(2d - E_m\right)\right] \quad \ldots (22)$$

Note that, unlike the rate limit case described above, the describing function for a hysteresis element is not a function of the input frequency, but of the input amplitude and the size of the hysteresis. Taking d equal to 0·05 degrees, the describing function was evaluated for a number of different input amplitudes, and the results are plotted in Fig. 16.

The effect of this non-linearity on the stability of the closed-loop system is assessed in Fig. 17, where the frequency response of the linear parts of the system is shown on a Nichols plot. Note that the low moments of inertia of the missile give rise to a high-frequency short-period mode.

The linear frequency response has then been shifted by the phase lag and gain attenuation of the hysteresis, as estimated by its describing function. It can be seen that there is the possibility of a limit cycle occurring at a frequency of approximately 4 Hertz and with amplitude of about ±0·08 degrees. It would then need to be decided whether this is acceptable, based on the likely effects of such an oscillation. If the oscillation is not acceptable then the actuation system's backlash would need to be reduced in order to provide acceptable behaviour. Indeed, this type of analysis would enable a specification of the maximum amount of backlash to be defined.

Describing function analyses of this kind have proved to be very accurate in predicting limit cycles, which have then been confirmed by using non-linear simulations. Occasionally, the reverse process is used, whereby the analysis is used to explain the cause of an oscillation that is evident from time histories. Sometimes a stability problem can be due to a combination of non-linearities and an alternative ad hoc approach has to be used in the analysis. The next section gives an example.

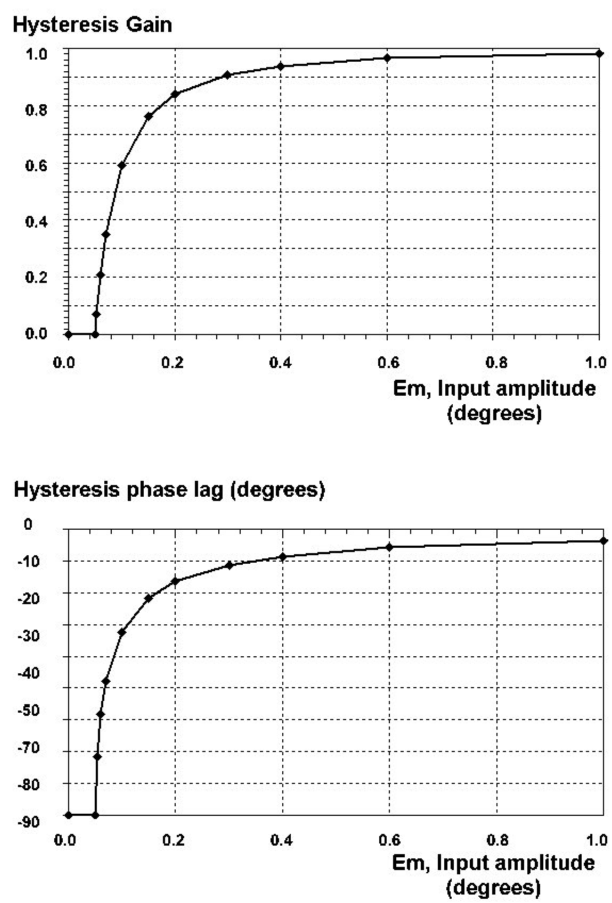


Figure 16. Hysteresis gain and phase.

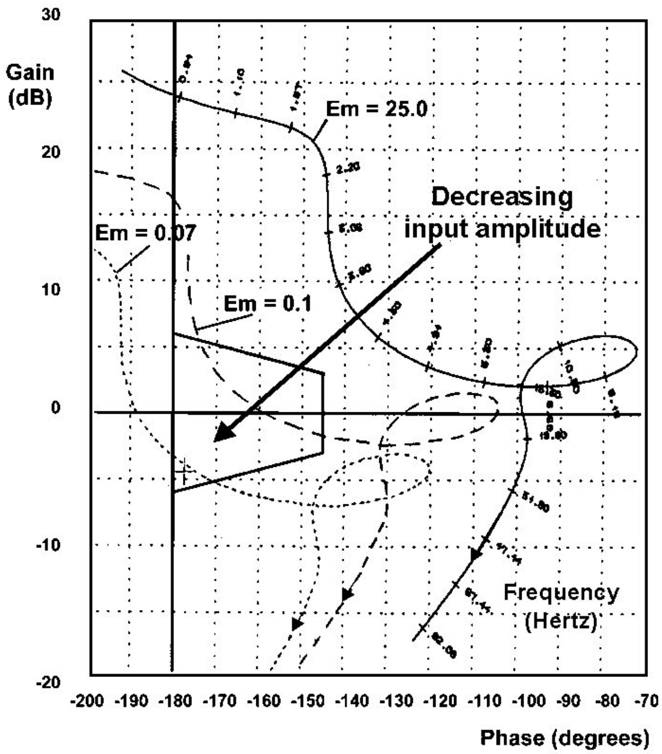


Figure 17. Hysteresis Nichols plot.

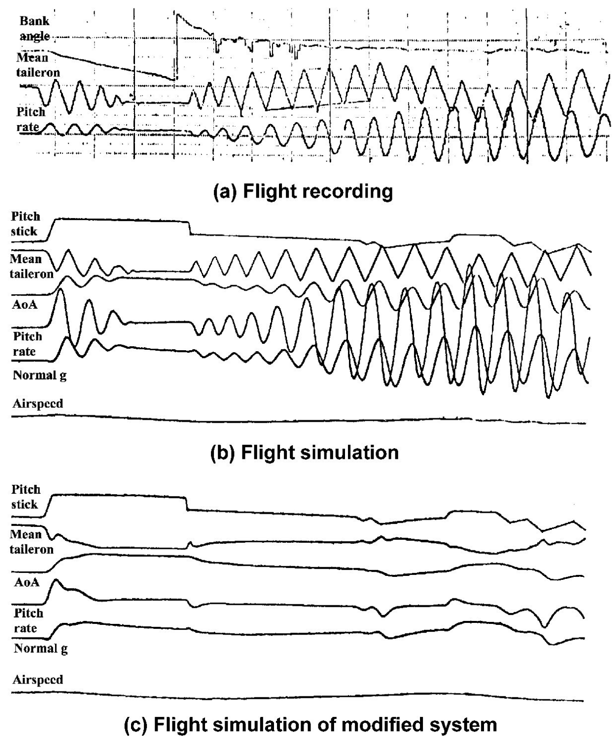


Figure 18. Tornado aircraft time histories.

4.0 TORNADO SPILS EXPERIENCE

4.1 Flight oscillation

During 1981, a large amplitude in-flight oscillation occurred during the development of the Tornado aircraft's Spin Prevention and Incidence Limiting System (SPILS). In early flights, rate-limited oscillations had been encountered, which exhibited adequate damping characteristics. These were only seen when the pilot pulled the stick rapidly to its aft stop, to test the incidence limiting capability of the system, and only at specific flight conditions. Comparisons with the simulation model, which included actuator rate limiting, showed the in-flight oscillations to be somewhat worse, with slightly lower damping. However, the test pilots considered the aircraft response to be acceptable and flight testing was therefore allowed to continue to further investigate the system.

A severe large amplitude rate-limited oscillation was encountered during the 42nd flight with this system and this was despite the system having (apparently) acceptable stability margins. Following a detailed analysis of the flight incident, the aircraft's instability was found to be associated with a combination of specific conditions and non-linear behaviour. To provoke the oscillation, it was necessary:

- To drive the taileron actuators hard into rate and acceleration limiting.
- To have the aircraft in the speed range where the aircraft/FCS loop gain was highest.
- To hold the airspeed constant and hence maintain the highest loop gain, by being in a dive.
- To have the pitch stick positioned about 50% aft of centre to maximise the combined feedback through the Command and Stability Augmentation System (CSAS) and SPILS.

Such a worst case combination had not been encountered in previous flights. Some difficulties in simulating the oscillation were encountered. However, following a detailed taileron actuation system modelling exercise, which included the effects of acceleration limiting due to current limiting in the servo amplifier driving the first stage actuator, a good simulated match of the incident was obtained, as indicated by Figs 18(a) and 18(b). This actuation system model played a significant part in evaluating the design modifications.

The solution to this stability problem involved an actuation system outer loop modification and control law non-linear compensation. This compensation, which was tested in 1981-82, is identical in purpose to the rate limiting algorithms promoted in the mid-1990s. These modifications, which are described in the next section, led to a dramatic increase in the augmented aircraft's stability, effectively recovering the linear behaviour, as shown by Fig. 18(c).

4.2 The design solution

Figure 19 shows a simplified version of the Tornado aircraft's pitch control laws. The top half shows the stability augmentation function which is based on proportional pitch rate feedback. The lower half

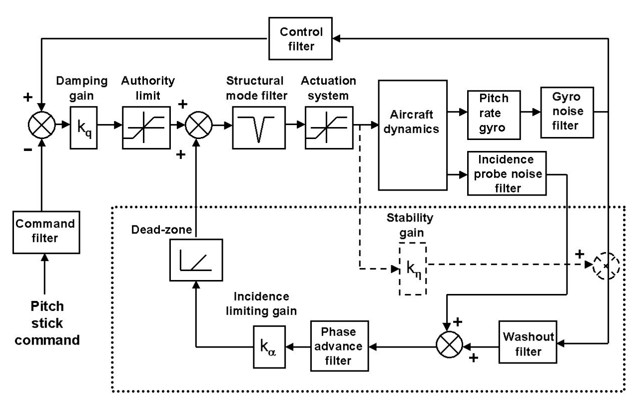


Figure 19. Simplified pitch CSAS and SPILS.

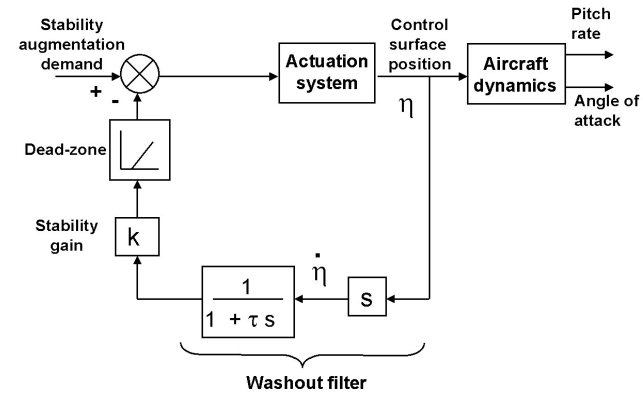


Figure 20. Modification functionality.

shows the incidence limiting function, which is based on proportional feedback of incidence to the tailerons.

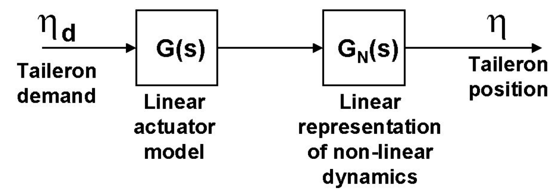
The solution to the aircraft's stability problem with the SPILS engaged was to feed back the aircraft's average tailpane angle through a gain which was scheduled with pitot-static pressure, to the existing washout filter, as indicated by the dashed elements in Fig. 19. This modification had a remarkable stabilising effect, as it countered any tendency for the actuation systems to become rate limited. The reason for this dramatic improvement can be explained by considering Fig. 20, which shows a simplified diagram of the functionality that had been introduced by this additional feedback term.

In this figure, the washout filter (a linear element) has been interchanged with the stability gain and is shown as a differentiation term and a first-order lag. The differentiator produces a taileron rate signal, which is then lagged and re-scaled. If the resulting signal is sufficiently large, it will produce an output from the dead-zone to 'back-off' the command that is tending to produce the rate limiting of the tailerons. This modification, besides being very effective, has two other important benefits:

- The additional feedback is 'washed out' in the steady state and does not affect the existing trim characteristics of the aircraft.
- Since the additional feedback does not have any effect until the dead-zone threshold is exceeded, it does not affect the aircraft's small amplitude handling characteristics.

In order to analyse the non-linear stability of the modification, it was necessary to consider the various non-linearities present. A describing function analysis was initially used to represent the actuation systems' rate limiting behaviour, but it was found to be insufficient to capture the combined effects of rate and acceleration limiting. Instead, an *ad hoc* actuator model was developed for use in the linear analysis. This 'quasi-linear actuator model' is shown in Fig. 21 and was used to carry out Nyquist and Nichols stability analysis of the re-designed system. The transfer function $G(s)$ was derived by matching rig test results for small amplitide frequency responses, and the term $G_N(s)$ was chosen to approximate the set of large amplitude frequency response results.

The resulting design was thoroughly assessed by using non-linear simulation, both off-line and with the pilot in the loop. A rigorous flight test programme validated the re-designed system and its simulation model. Figure 22 shows an example of deliberate pitch stick pumping by the pilot, to assess the modified aircraft's large amplitude pitch stability. The upper plots show the post-flight simulation results which match the flight case shown just below. By comparing the mean taileron responses between the model and the flight, it can be seen that an excellent match is obtained with the stability of the model being very representative of the aircraft. More importantly, the forced oscillation of the aircraft stops abruptly when the stick is centred, showing the aircraft to have a well-damped response for large amplitude commands. The confidence in the modelling allowed the re-designed system to be re-cleared for flight and enabled the flight test programme to continue.



$$G(s) = \frac{1}{(1 + s/25)(1 + 2(0.5)s/50 + s^2/50^2)}$$

$$G_N(s) = 1 \quad \text{if } \omega < \omega_n$$

$$= \frac{2(0.4)s/\omega_n}{(1 + 2(0.4)s/\omega_n + s^2/\omega_n^2)} \quad \text{if } \omega \geq \omega_n$$

Where ω_n is the saturation onset frequency and is given by
$\omega_n = (2\pi) \times (\text{Actuator rate limit}) / (4 \times \text{Demand amplitude})$

Figure 21. Quasi-linear actuator model.

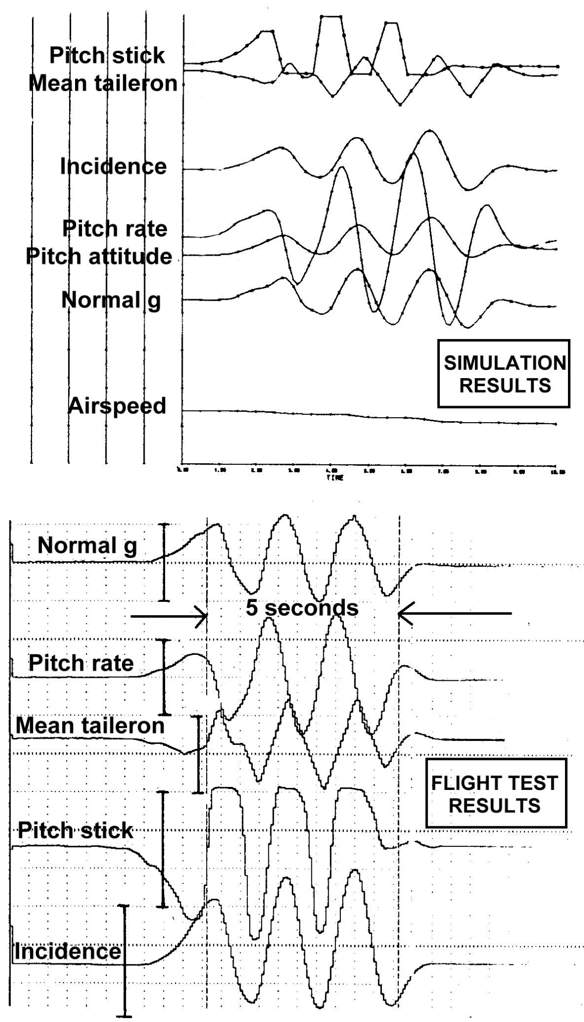


Figure 22. Model validation by flight matching.

The described re-design was very successful and has been flown in the Tornado aircraft since 1982. The stability characteristics of the aircraft with the SPILS engaged have been similar to those shown in Fig. 18(c).

4.3 Some lessons learned

The lessons learned from the Tornado SPILS experience are mainly associated with flight control system non-linearities, as follows:

- The SPILS was designed as an 'add-on' system to the existing CSAS, with the dictate that no significant changes should be made to the CSAS in the interests of minimising the costs and impact of the change. This compromised the design of the SPILS and the subsequent modifications to correct the problem found. A more integrated approach would have allowed a better design to be achieved.
- Even the accurate modelling of rate limiting, including actuation loading effects, may not provide an adequate representation for design and simulation, since an additional effective time delay is introduced as a rate limited actuator changes its direction. Acceleration limits should be accurately modelled and actuation system specifications should include adequate acceleration capability, to avoid the possibility of undesirable large amplitude characteristics.
- The system stability analyses and simulations need to identify the worst cases, including the combined effects of several non-linearities and maximum loop gains. It is necessary to understand the system non-linearities and to be aware that for a highly non-linear system, any sign of low damping for large amplitude responses is a potential warning sign for a 'cliff-edge' instability.
- The main area of concern was that the pilots would 'beat the system'. In this respect, the rapid fully aft stick pull had been assumed to be the worst case in the pitch axis, in that it induced significant pitch momentum and rate limiting behaviour as the incidence limit was being approached. In terms of overall system stability this was not the worst case, since the CSAS error authority limiting was occurring for the extreme stick commands, and this effectively reduced the feedback through the system. This effect, although known, was not fully appreciated when in combination with the non-linear actuation system behaviour described above.

The main lessons learned were the need to minimise and to fully understand non-linear actuation system behaviour, and to ensure that the effects were adequately taken into account during design and flight clearance. Lessons learned from this project and from other aircraft flight control systems are described in a report by RTO[18], which also describes some best practices.

An important aspect of the investigation was the need to fully understand and develop an actuation system model with sufficiently accurate characteristics. The next section describes the actuator modelling undertaken and includes some general guidelines regarding the modelling and analysis of actuation systems.

5.0 ACTUATION SYSTEM MODELLING

5.1 Model characteristics

Figure 23 shows the actuation model used for Tornado's port and starboard tailerons. For pitch control, there is a common symmetrical command to both surfaces, with a differential offset provided by the roll command. The taileron position command is limited by an electrical authority limit and is used with the taileron position feedback to produce a position error signal. The error signal is electrically amplified to produce a taileron rate command signal, which is then limited to produce the rate command to the inner loop. The feedback of main valve position is used in the inner loop to generate a rate error signal, which, via the gain of the servo valve, produces hydraulic fluid flow equivalent to a taileron acceleration command. The maximum acceleration capability is limited by the maximum flow through the spool valve, which is proportional to the spool valve's opening limit. The flow over a period of time is effectively a hydraulic integration process, resulting in a main valve position, which is limited by the valve's displacement. The flow through the main valve and hence the actuator rate is proportional to the valve opening. Finally, the taileron's position is achieved by the integration of the flow through the main valve over time. Both taileron position and rate are electrically measured to provide the outer and inner loop feedback signals.

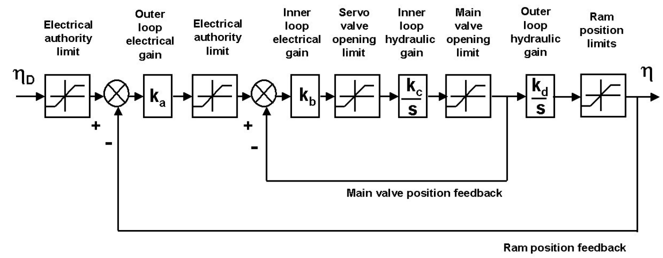
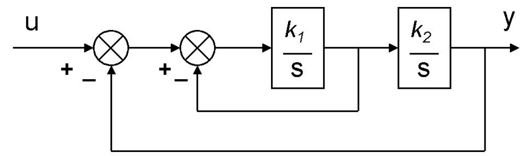


Figure 23. Actuation system model.



$$\omega_n = \sqrt{k_1 k_2} \qquad \zeta = 0.5\sqrt{k_1/k_2}$$

Outer-loop saturation: $k_2 \to 0 \Rightarrow \omega_n \to 0$ and $\zeta \to \infty$

Inner-loop saturation: $k_1 \to 0 \Rightarrow \omega_n \to 0$ and $\zeta \to 0$

Figure 24. Second-order linearisation.

In terms of its linear dynamics, the non-linear actuation system in Fig. 23 is a second-order system and can be represented as shown in Fig. 24. Increasing the inner-loop or outer-loop integral gains results in an increase in the natural frequency of the system. Increasing the outer loop gain also produces a reduction in damping and the inner loop gain will need to be increased, if the level of damping is to be maintained. Most significantly, a reduction in the inner-loop gain (a reduction of velocity feedback) also results in a reduction in damping.

From these basic considerations of the linear dynamics, we can get an appreciation of the effects of authority saturation in the inner or outer loops, for limits positioned at the inputs to the integrators. This is readily done because the describing function of such a limit is simply that of a constant gain until saturation occurs, with a gradual gain reduction with increasing input amplitude. This would be equivalent to decreasing k_1 or k_2 in Fig. 24. Saturation of the input to k_2 (i.e. rate limit) results in a reduction in natural frequency and actuator bandwidth, with an increase in damping. Saturation at the input to k_1 (i.e. acceleration limit) is likely to be more significant, as this reduces the actuator's bandwidth and its damping.

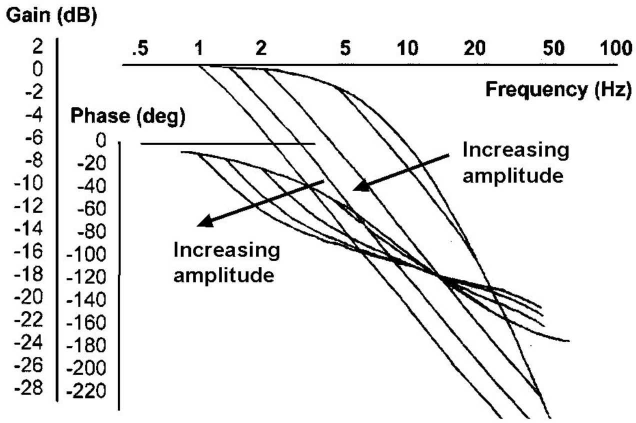


Figure 25. Effect of rate limits on frequency response for different amplitudes.

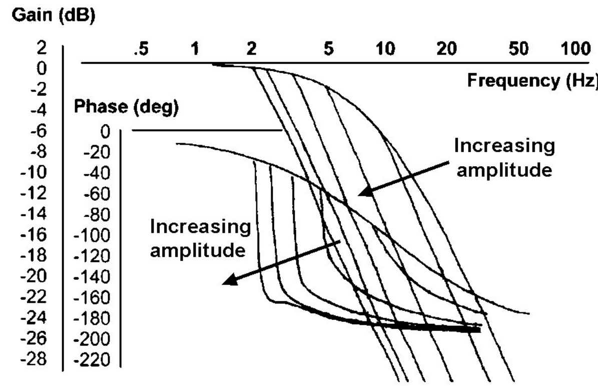


Figure 26. Effect of low acceleration limits for different amplitudes.

In the next two sections, the effects of rate and acceleration saturation are highlighted, in terms of the actuation model's frequency response characteristics.

5.2 Rate limiting effects

Figure 25 is an extract from a study performed at the University of Bristol[9], where a parameterised non-linear model, similar to that of Fig. 23, was used with a transfer function analyser program. This time-domain facility used a sinusoidal input of a user-defined magnitude and frequency to excite the actuator simulation model. By correlation of the output with the input, the gain and phase of the fundamental component of the output can be determined, relative to the input. By stepping through a frequency range, a frequency response can be determined for any given input amplitude. By using different input amplitudes and over-plotting the frequency responses, the effects of input amplitude and frequency variations can be shown on one plot. In the original study, many combinations of rate and acceleration limits were evaluated in this way. Figure 25 is a typical case of rate limiting behaviour, where high acceleration limits do not significantly influence the responses.

In Fig. 25, the rate limit is being progressively invoked as the increasing input amplitude increases the level of rate saturation and reduces the bandwidth of the actuator. It is noted that for the most extreme cases, as the bandwidth is reduced, over a narrow frequency range (5-10 Hertz) the actuator has characteristics similar to an open-loop integrator, since the phase lag is close to 90 degrees and the gain is reducing at 20dB/decade. This effect is easily confirmed by a limiting analysis of Fig. 24, where the transfer function from input to output is given by:

$$\frac{k_1 k_2}{k_1 k_2 + k_1 s + s^2} \quad \text{. . . (23)}$$

As k_2 tends towards zero, the transfer function's gain also tends towards zero. However, in this limiting case, the denominator polynomial becomes $s(k_1 + s)$, an integrator and a first-order lag with a time constant determined by the actuator's inner-loop gain.

5.3 Acceleration limiting effects

Figure 26 is also taken from Ref. 9 and shows the frequency response variations for an extreme case of low acceleration limits in an actuation system, for varying input amplitudes.

Although at first sight, the results of rate and acceleration limiting might seem to be broadly similar, in that there is a reduction in bandwidth with increasing input amplitude, there are some very important differences. In particular, in Fig. 26 it can be seen that the gain is reducing at 40dB/decade, which is typical of a second-order lag or a double integrator. For the cases of extreme acceleration saturation it is evident that the actuator is behaving more like a double integrator, especially from consideration of its phase response. It is noted that for the worst case, a genuine 'cliff edge' situation can occur, with a massive loss in phase (tending towards 180 degrees) over a very small frequency range — leading to the jump resonance type of characteristic described in Section 2. Again, this effect is confirmed by theoretical considerations by taking the transfer function derived from Fig. 24 and considering the limit as k_1 tends towards zero. As previously, the transfer function gain and bandwidth tend towards zero but this time, the denominator dynamics become s^2, a double integrator.

5.4 Saturation analysis

It is possible to predict the frequencies at which rate or acceleration saturation occur, via a simple saturation analysis. Let us consider the second-order system from Fig. 24 and set the natural frequency to 30 radians/second and the damping to 0·7. If we then set the actuation rate limit to 60 degrees/second and the acceleration limit to 1,200 degrees/second^2, we have the model shown in Fig. 27, where the limits have been scaled to incorporate the effects of the scaling on the integrators.

By considering this example, we can determine the combinations of amplitudes and frequencies for which saturation occurs and which limit is being invoked.

Let us consider the output signal y as being sinusoidal with amplitude A and frequency ω:

$$y = A \operatorname{Sin}(\omega t) \quad \text{. . . (24)}$$

The output rate is therefore:

$$\dot{y} = A\omega \operatorname{Cos}(\omega t) \quad \text{. . . (25)}$$

and its acceleration is:

$$\ddot{y} = -A\omega^2 \operatorname{Sin}(\omega t) \quad \text{. . . (26)}$$

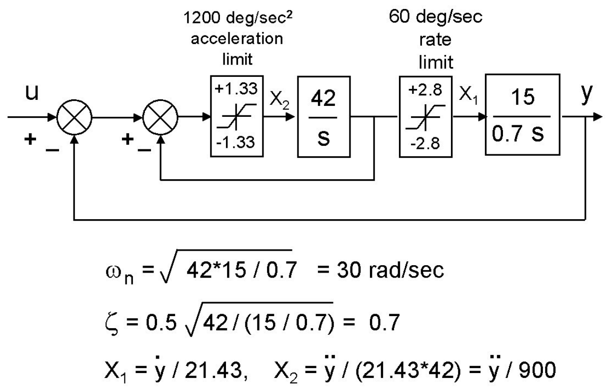


Figure 27. Simplified actuator model with rate and acceleration limits.

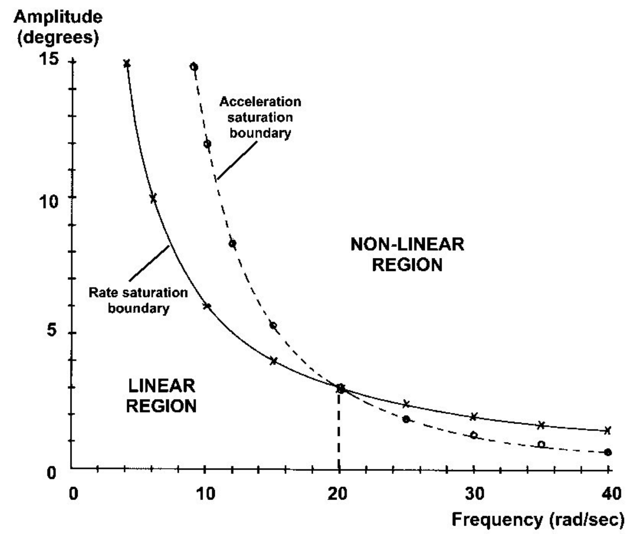


Figure 28. Saturation boundaries for rate and acceleration limiting.

At the point where limiting occurs, X_1 and X_2 in Fig. 27 are equal to the authority limits immediately upstream. By substituting for the magnitudes of the rate and acceleration it is shown that:

$$A_{RL} = 60/\omega \qquad \text{. . . (27)}$$

and

$$A_{AL} = 1{,}200/\omega^2 \qquad \text{. . . (28)}$$

Where A_{RL} and A_{AL} are the amplitudes at which rate and acceleration limiting occur, for each frequency ω. The values in these equations are as expected from the original assumptions. It is easier to interpret these equations, if we overplot them as shown in Fig. 28. Below the two curves, we have a region where the actuator model is behaving linearly and for low frequencies and large amplitudes, rate-limiting behaviour will occur. For high frequencies, and small amplitudes, acceleration limiting will occur before the actuator has attained its rate limit. For this example, rate and acceleration limiting will occur simultaneously if the output amplitude reaches ±3 degrees at a frequency of 20 radians/second.

In practice, all actuators have rate and acceleration limits that will be reached. It is essential to design these to give satisfactory performance and a saturation analysis will be a useful step, since it should give an indication of the likely frequency response behaviour of the system. This can then be supported by more detailed actuator modelling, transfer function analysis and describing function analysis, as required. Further information on actuation systems design can be found in Refs 19 and 20.

6.0 CLOSING REMARKS

The subject of non-linearities in aircraft flight control systems has been reviewed, starting from a description of the basic types of non-linearity, their natural physical occurrence and how they behave in combination. There is no universal theory for dealing with non-linearities since each case tends to be different and the possible methods of assessment will need to be carefully considered. Usually, this is underpinned by a comprehensive amount of non-linear simulation, based on validated models, with assumptions made about possible variations in the models (i.e. uncertainty), due to the expected physical variability of the hardware that is being simulated.

If we could eliminate non-linearities completely, then the flight control system design, implementation and flight clearance tasks would be greatly simplified, as the number of design cases to be assessed would be reduced. It is therefore, prudent to minimise the non-linearities in the basic unaugmented aircraft by:

- Ensuring that the vehicle's control surfaces and sensors provide sufficient control power and range of measurement, respectively.
- Ensuring that the actuation systems have sufficient displacement, rate and acceleration capability, and that their performance is not adversely affected under loading conditions.
- Designing the airframe to have aerodynamic characteristics that are as linear as can be reasonably expected across the operating envelope, particularly with respect to angles of attack and sideslip, and Mach number.

We will then have a set of the 'parasitic'[1] non-linearities that must be taken into account in the design. Then, before beginning any control systems design, it is important to identify and fully understand these non-linearities and how they are likely to affect the aircraft's control characteristics, as its operating condition varies. It is possible to take the following actions:

- Directly compensate for the non-linearity within the control laws, if its characteristics are known to within acceptable engineering limits, and appropriate sensor measurements are available to enable reliable and robust compensation.
- Minimise the effects of non-linearities by careful design. This might be achieved by optimising the feedback through the non-linearity, by limiting the command authority of the signal that is input to the non-linearity, or simply by accepting a design compromise, whereby a fixed controller is used to cover a range of non-linear characteristics.

In the worst case, if functional compensation cannot be satisfactorily achieved, it might be necessary to modify the airframe or its hardware but this should only be considered as a last resort, owing to the considerable cost that is likely to be involved. As a compromise, it might be acceptable to avoid any problematic operating points (e.g. extreme corners of the flight envelope) by imposing operational restrictions on the aircraft, to be either manually observed or automatically controlled.

Having dealt with the parasitic non-linearities, it is then possible for the control system designer to carefully introduce the 'intended' non-linearities such as a small dead-zone on the command signal, or the hysteresis function that prevents limit-cycle switching between states. Further examples of non-linearities in flight control systems can be found in the literature[12], where designers have taken advantage of physical knowledge to minimise the uncertainty and effort associated with the linear design task, by creating the appropriate non-linear functions and control law structures, as an important first step in the design.

Finally, it is noted that the flight clearance of complex non-linear systems is highly dependent on using vast amounts of numerical integration to simulate the many possible cases of non-linear behaviour. With the increase in desktop computing capabilities, the authors wonder if there is an alternative to supporting the many non-linear simulations with an even larger number of local linearisations. This is seen as an interesting challenge for the research community.

ACKNOWLEDGEMENTS

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