Observer-Based Beamforming Algorithm for Real-Time Acoustic Array Signal Processing

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Outline

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4 Summary
Conventional Beamforming Formulations

Noise source

Given $x(t) \in \mathbb{R}^1$ or $X(j\omega) \in \mathbb{C}^1$

Array outputs

- Time domain: $y(t) = \frac{1}{4\pi r} x(t - \tau), \tau = \frac{r}{C}$.
- Frequency domain: $Y(j\omega) = \frac{1}{4\pi r} X(j\omega) e^{-j\omega \tau} = G(r, j\omega) X(j\omega)$. 
Conventional Beamforming Formulations

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**Conventional beamforming**

$X(j\omega) = (G^*G)^{-1} G^* Y(j\omega)$, following Moore-Penrose pseudoinverse.
Performance of conventional beamforming
Numerical case—results of conventional beamforming

(a) two coherent sources, (b) removal of one source
Numerical case—coherent sound sources

\[ A_{Y_B} \approx \frac{1}{K} \sum_{k=1}^{K} Y_{B|k} Y_{B|k}^*, \]
Problems in conventional beamforming

Background noise and solutions

- Array outputs $Y_B(j\omega) = G(r, j\omega)X_B$, and $Y_{BS}(j\omega) = G(r, j\omega)X_{BS}$.
- If $X_{BS} = X_B + X_S$, sound satisfies superposition.
- Define $A_B = Y_B Y_B^*$, $A_{BS} = Y_{BS} Y_{BS}^*$, $\langle A_S \rangle \approx \langle A_{BS} \rangle - \langle A_B \rangle$.
- Open problem: cases with coherent sources, i.e. $\langle X_B X_S^* \rangle \neq 0$. 
Problems in conventional beamforming (Cont.)

Real-time computation

- The correlation matrix should be obtained by averaging the sampled data—not valid for real-time computation.

\[
A_Y \approx \frac{1}{K} \sum_{k=1}^{K} Y|_k Y|_k^* .
\]

- A possible solution?

\[
A_Y|_k = \frac{k-1}{k} A_Y|_{k-1} + \frac{1}{k} (Y_S|_k Y_S|_k^*) , \quad k = 2, 3, ...
\]
Numerical case—moving sources

The sound source moves around one circle in 8.53s.
Numerical case—results of conventional beamforming

(a) 0.085 sec,  (b) 2.56 sec,  (c) 5.12 sec. (d) 8.53 sec
Numerical case—results of conventional beamforming & observer-based beamforming
State observer

Signal model

\[ \dot{x}(t) = Ax(t), \text{ state equation, a new equation.} \]
\[ y(t) = Gx(t), \text{ measurement equation.} \]
State observer

**Signal model**

\[
\dot{x}(t) = Ax(t), \ \text{state equation, a new equation.}
\]
\[
y(t) = Gx(t), \ \text{measurement equation.}
\]

**Observer**

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + L(y - \hat{y}).
\]
\[
\hat{y}(t) = G\hat{x}(t), \ \text{where } \hat{x} \ \text{is the estimation of } x.
\]
State observer

Signal model
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Estimation error
\[ \dot{e} = (A - LG)e, e \text{ approaches 0 as long as the eigenvalue(s) of } (A - LG) < 0. L \text{ can be accordingly designed use MATLAB tool.} \]
Observer in frequency domain

Time domain to frequency domain

\[ y(t) = \sum_{m=-\infty}^{\infty} Y_m e^{jmt}, \quad x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jmt}. \]

Signal model in frequency domain

\[ \dot{X}_m = AX_m, \quad Y_m = GX_m = \frac{1}{4\pi r} X_m e^{-jm\tau}. \]
Observer in frequency domain

Time domain to frequency domain

\[ y(t) = \sum_{m=-\infty}^{\infty} Y_m e^{jmt}, \quad x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jmt}. \]

Signal model in frequency domain

\[ \dot{X}_m = AX_m, \quad Y_m = GX_m = \frac{1}{4\pi r} X_m e^{-jmt}. \]

Observer in frequency domain

\[ \dot{\hat{X}} = A\hat{X} + K(Y - \hat{Y}), \quad \hat{Y} = G\hat{X}, \] which is recursive over sampling blocks, in other words, the algorithm holds real-time capability.
Observer In wind tunnel

\[
\begin{pmatrix}
Y_B \\
Y_{BS}
\end{pmatrix} =
\begin{pmatrix}
G & 0 \\
Ge^{i\phi} & G
\end{pmatrix}
\begin{pmatrix}
X_B \\
X_S
\end{pmatrix},
\]

where \( \phi \) is the phase difference between the measurements of \( Y_B \) and \( Y_{BS} \).
Observer-Based Beamforming Method

The linear state model of the sound propagation is:

\[
\begin{pmatrix}
X_{B|k+1} \\
X_{S|k+1}
\end{pmatrix}
= \begin{pmatrix}
A & 0 \\
0 & A
\end{pmatrix}
\begin{pmatrix}
X_{B|k} \\
X_{S|k}
\end{pmatrix},
\]

\[
\begin{pmatrix}
Y_{B|k} \\
Y_{BS|k}
\end{pmatrix}
= \begin{pmatrix}
G & 0 \\
Ge^{i\phi} & G
\end{pmatrix}
\begin{pmatrix}
X_{B|k} \\
X_{S|k}
\end{pmatrix}.
\]

The corresponding state observer are:

\[
\begin{pmatrix}
\hat{X}_{B|k+1} \\
\hat{X}_{S|k+1}
\end{pmatrix}
= \begin{pmatrix}
A & 0 \\
0 & A
\end{pmatrix}
\begin{pmatrix}
\hat{X}_{B|k} \\
\hat{X}_{S|k}
\end{pmatrix} + L \left[ \begin{pmatrix}
Y_{B|k} \\
Y_{BS|k}
\end{pmatrix} - \begin{pmatrix}
\hat{Y}_{B|k} \\
\hat{Y}_{BS|k}
\end{pmatrix} \right],
\]
Observer-Based Beamforming

Acoustic image results of the observer-based beamforming for the coherent noise case, where (a)-(d) are results of the 1st, 10th, 20th and 100th sampling blocks, respectively.
Real-Time Localization of Moving Sources

Acoustic images for the numerical case with a moving monopole, (a)–(d) are instantaneous results of the 1st, 30th, 60th and 100th blocks, respectively. Observer-based beamformer was adopted.
Remarks of the Method

$A = ?$

Given a **stationary** and **ergodic** signal process, $A = 1$ for a single scanned point and $A$ is an identity matrix for scanned plane.
Remarks of the Method

\[ A = ? \]

Given a stationary and ergodic signal process, \( A = 1 \) for a single scanned point and \( A \) is an identity matrix for scanned plane.

\[ \phi = ? \]

\[ \hat{\phi}|_{k+1} = \hat{\phi}|_k + mH^*(Y|_k - \hat{Y}|_k), \]

where \( H \triangleq \partial Y / \partial \phi \). Details refer to our oncoming Journal publications.
Dynamical Tracing of $\phi$

The asymptotic tracing of the phase changes in the numerical case, where (a) $\phi$ and $\hat{\phi}$, and (b) $|\hat{\phi} - \phi|$. 
Setup

The microphone array with 28 mics.
Result of observer-based beamforming

(a) ![Image](image1)

(b) ![Image](image2)

(c) ![Image](image3)

(d) ![Image](image4)
Result of observer-based beamforming
A new algorithm—observer-based method is presented, with its ideas, equations and performances.
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Main advantage of the algorithm: removal of coherent background noise & real-time localization of moving sources.
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Main advantage of the algorithm: removal of coherent background noise & real-time localization of moving sources.

The resolution of the algorithm should be improved—further work.
Thank You!
Questions?