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# Contributions of Computational Aeroacoustics to Jet Noise Research and Prediction

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A brief review of recent progress in the field of computational aeroacoustics (CAA) is proposed. This paper is complementary to the previous reviews of Tam [(1995a) "Computational aeroacoustics: issues and methods", *AIAA J.* **33**(10), 1788–1796], Lele [(1997) "Computational Aeroacoustics: a review", AIAA Paper 97-0018, *35th Aerospace Sciences Meeting and Exhibit*, Reno, Nevada] and Glegg [(1999) "Recent advances aeroacoustics: the influence of computational fluid dynamics", *6th International Congress on Sound and Vibration*, Copenhagen, Danemark, 5–8 July, 43–58] on advances in CAA. After a short introduction concerning the current motivations of jet noise studies, connections between computational fluid dynamics (CFD) and CAA using hybrid approaches are discussed in the first part. The most spectacular advances are probably provided by the direct computation of jet noise, and some recent results are shown in the second part.

*Keywords*: Computational aeroacoustics; Jet noise; Lighthill theory; Large eddy simulation

## INTRODUCTION

It was in the early 1950s that jet-engine noise became a new research domain, and that the noise of turbulent jets became a branch of aeroacoustics. The first commercial jet-powered aircraft, a de Havilland Comet operating between London and Johannesburg, entered into service on May 1952, and the same year, the first theory of aerodynamic noise was published by Lighthill (1952, 1982). From the beginning, the high noise levels of jet engines were deemed to be a serious environmental problem. Certain airports imposed operational restrictions or financial penalties, and aircraft noise certification appeared in the 1970s. World traffic increased regularly, and up to the 1980s, considerable jet noise reduction was achieved as a consequence of improvements of propulsive efficiency, particularly with higher bypass ratio engines.

Nowadays, society cannot tolerate additional noise pollution, and traffic growth must be compensated for by quieter aircrafts. Noise regulations are becoming more stringent (International Civil Aviation Organization chapter 2 phases out and is replaced by a new chapter 4 in a few years). Furthermore, potential solutions to reduce jet noise are now often in conflict with the optimization of engine performance. As a consequence, innovative methods must be proposed to reduce the jet noise of existing and new larger subsonic airliners. This is also a key point to consider in the planning of a second generation supersonic transport aircraft since the new subsonic noise legislation may also be applied to them. In parallel, problems specifically facing military aircrafts must be tackled.

Prediction methods for jet noise were mainly semi-empirical until about ten years ago, and were based on the power laws established by Lighthill (1952) and others later. Over the last ten years, advances in computational fluid dynamics (CFD) have made it possible to improve predictions by replacing flow parameters of these semi-empirical models by computed values. But the most spectacular aspect of this period has been the rapid development of computational aeroacoustics (CAA). Two main classes of methods have been developed in CAA. In the first one, concepts that appeared early in aeroacoustics, namely acoustic analogies or hybrid approaches, are applied to time-dependent CFD data. In the second one, the aerodynamic field and the acoustic field are simultaneously calculated by solving the compressible unsteady Navier–Stokes equations. This direct noise computation (DNC) is ambitious, and allows for a more physical investigation of noise source mechanisms, but serious numerical issues must then be addressed.

The goals of each class of methods are different, but in the end, the aeroacoustics community needs both groups

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of predictive methods to face practical engineering applications. A modern high-bypass ratio turbofan engine, such as a CFM-56 engine, has a core jet velocity $U_p \simeq 375\,\mathrm{m\,s^{-1}}$ and a temperature $T_p \simeq 890\,\mathrm{K}$ at take-off conditions, with an area $A_p \simeq 0.296\,\mathrm{m}^2$. The bypass stream or secondary jet exhaust is about $U_s \simeq 255\,\mathrm{m\,s^{-1}}$, with $T_s \simeq 330\,\mathrm{K}$ and $A_s \simeq 1.176\,\mathrm{m}^2$. Considering an equivalent jet with the same thrust, exit area and mass flow, its velocity is around $U_j \simeq 350\,\mathrm{m\,s^{-1}}$ at a temperature of $T_j \simeq 400\,\mathrm{K}$. The Mach number is $M \simeq 0.87$ and the Reynolds number is $Re_D = U_jD/\nu \simeq 3.1\times10^7$. At $\theta = 90°$, the acoustic spectrum peak, estimated with the relation $\mathrm{St} = fD/U_j \simeq 0.5$, is $f \simeq 127\,\mathrm{Hz}$ or in terms of acoustic wavelength $\lambda \simeq 2.7\,\mathrm{m}$. In the downstream direction at $\theta = 15°$, the relation $\mathrm{St} = fD/c_\infty \simeq 0.15$ provides $f \simeq 37\,\mathrm{Hz}$ and $\lambda \simeq 9.1\,\mathrm{m}$. A computation of the pressure signal up to a frequency of $f = 2\,\mathrm{kHz}$ requires a cutoff Strouhal number $\mathrm{St} \simeq 10$. This is, therefore, a high Reynolds number and broadband problem.

The areas of interest are large in aeroacoustics, and this paper is restricted to the contribution of CAA towards improving predictions and better understanding of noise source mechanisms in subsonic jets. It is not a review with an exhaustive list of references, and the reader interested by recent advances in CAA could consult the reviews of Tam (1995a), Lele (1997) and Glegg (1999) as well as proceedings of the recent jet noise workshop in Huff (2001) and other contributions in this issue. The present paper is organized as follows. In the second section, a brief history of progress concerning jet noise is presented and connections between CFD and CAA are discussed. The direct calculation of noise, as defined before, is tackled in the third section. Finally, some perspectives are outlined in the fourth section.

## PROGRESS ON JET NOISE

### Lighthill's Theory

The first formulation of an acoustic analogy was derived by Lighthill (1952). The compressible fluid dynamic equations are recasted into an inhomogeneous wave equation, which yields:

$$\left(\frac{\partial^2}{\partial t^2} - c_\infty^2\nabla^2\right)\rho'(\mathbf{x},t) = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \qquad (1)$$

where $T_{ij} = \rho u_i u_j + [(p - p_\infty) - c_\infty^2(\rho - \rho_\infty)]\delta_{ij} + \tau_{ij}$ represents a distribution of equivalent noise sources. Here, $\rho$, $u_i$, $p$ and $\tau_{ij}$ are the instantaneous density, velocity vector, pressure and viscous stress tensor. The subscript $\infty$ denotes the state of fluid at rest in the far field, $\rho' = \rho - \rho_\infty$, and $c_\infty$ is the speed of sound in the medium at rest. For jet noise, the contribution of viscous terms can be neglected in Lighthill's tensor, and to simplify the subsequent discussions, entropy fluctuations are assumed to be unimportant, and are also neglected. This last assumption could be relaxed if necessary. If the volume occupied by the turbulent velocity field is far from the observer, the acoustical density fluctuations are given by:

$$\rho'(\mathbf{x},t) \simeq \frac{1}{4\pi c_\infty^4 x}\,\frac{x_i x_j}{x^2}\int \frac{\partial^2 T_{ij}}{\partial t^2}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{c_\infty}\right)\mathrm{d}\mathbf{y}. \qquad (2)$$

From this integral solution, Lighthill obtained the celebrated $u_j^8$ law governing the acoustic power radiated by a subsonic jet. The dimensional analysis was extended by Ffowcs Williams (1963) to high-speed jets, and reviews on these subtle integral formulations can be found in Crighton (1975) and in Crighton *et al.* (1992).

### *Statistical Approach*

When a numerical calculation of the time-dependent Lighthill tensor $T_{ij}$ is not available, an alternative approach is to estimate the autocorrelation function of the pressure defined as:

$$R(\mathbf{x},\tau) = \frac{\overline{p'(\mathbf{x},t)p'(\mathbf{x},t+\tau)}}{(\rho_\infty c_\infty)}.$$

Using the relation $p' = c_\infty^2\rho'$ in the far field, relation (2) yields, for stationary turbulence:

$$R(\mathbf{x},\tau) = \frac{1}{16\pi^2\rho_\infty c_\infty^5 x^2}\,\frac{x_i x_j x_k x_l}{x^4} \times \int\int \frac{\partial^4}{\partial\tau^4}\overline{T_{ij}[\mathbf{y}_A,t]T_{kl}[\mathbf{y}_B,t+\tau]}\,\mathrm{d}\mathbf{y}_A\,\mathrm{d}\mathbf{y}_B \qquad (3)$$

where square brackets $[\mathbf{y}, t]$ denote the quantity is to be evaluated at the retarded time $t - |\mathbf{x}-\mathbf{y}|/c_\infty$. Usually, only the main contribution $T_{ij} \simeq \overline{\rho}u_iu_j$ is retained in the Lighthill tensor, where $\overline{\rho}$ is the local mean density. Consequences of this assumption are discussed in the next paragraph. Thus, the closure problem consists of expressing the fourth-order two-point two-time correlation tensor:

$$R_{ijkl}(\mathbf{y},\boldsymbol{\eta},\tau+\tau_\eta) = \overline{T_{ij}[\mathbf{y}_A,t]T_{kl}[\mathbf{y}_B,t+\tau]} \qquad (4)$$

where $\mathbf{y} = \mathbf{y}_A$, $\boldsymbol{\eta} = \mathbf{y}_B - \mathbf{y}_A$ is the separation vector and $\tau_\eta = \mathbf{x}\cdot\boldsymbol{\eta}/(xc_\infty)$ is the variation of the retarded time. Before modelling $R_{ijkl}$ in Eq. (4), a frame moving with the energy-containing eddies at the convection velocity is introduced to separate the convective amplification from the evolution of the turbulence itself. In this frame, Ribner (1969) developed the Lagrangian velocity correlations by using analytical properties of isotropic turbulence statistics. This analysis was repeated for axisymmetric turbulence by Goldstein and Rosenbaum (1973). Without going into further details, it must be observed from Eq. (4)

that turbulence is assumed isotropic or anisotropic only locally over the $\boldsymbol{\eta}$ integration:

$$R(\mathbf{x},\tau) = \frac{1}{16\pi^2\rho_\infty c_\infty^5 x^2}\frac{x_i x_j x_k x_l}{x^4} \times \int \mathrm{d}\mathbf{y} \int \frac{\partial^4}{\partial\tau^4} R_{ijkl}(\mathbf{y},\boldsymbol{\eta},\tau+\tau_\eta)\mathrm{d}\boldsymbol{\eta}. \quad (5)$$

This expression can provide an estimate of the jet noise directivity and space-frequency distribution, by taking the Fourier transform of $R(\mathbf{x},\tau)$.

### *Compressible Part of $T_{ij}$*

In statistical models, and more generally when using hybrid methods for practical applications, one intends to get an estimate of the radiated noise when only the incompressible part of the velocity field is known. However, by using such an approximation in Lighthill's tensor, the acoustic-mean flow interactions are definitively lost. Thus, convection of noise sources can be taken into account in Eq. (2) or in Eq. (5) but not the sound waves which propagate without being affected by the presence of the mean flow.

To emphasize this important point, useful in interpreting numerical simulations, we consider the Lighthill integral (2) while still neglecting the effects of viscosity and entropy fluctuations. If the velocity is split into $u_i = \overline{u}_i + u_i'$, the Lighthill tensor becomes:

$$T_{ij} = \underbrace{\rho u_i' u_j'}_{T_{ij}^f} + \underbrace{\rho\overline{u}_i u_j' + \rho u_i'\overline{u}_j + \rho\overline{u}_i\overline{u}_j}_{T_{ij}^l}. \quad (6)$$

The first part $T_{ij}^f$ involves quadratic velocity fluctuations, and is responsible for the self-noise component in Eq. (2). The second part $T_{ij}^l$ is linear in fluctuations, and the contribution of the two first terms to Eq. (2) is called the shear-noise according to Lilley (1958). This decomposition into quadratic and linear terms is well known in the field of incompressible turbulence where the fluctuating pressure generated by a velocity field satisfies the Poisson equation:

$$-\frac{1}{\rho_\infty}\nabla^2\rho' = \frac{\partial^2}{\partial x_i \partial x_j}(u_i'u_j' - \overline{u_i'u_j'}) + 2\frac{\partial\overline{u}_i}{\partial x_j}\frac{\partial u_j'}{\partial x_i}. \quad (7)$$

The first source term in Eq. (7) is associated to the slow part of pressure while the term involving the mean velocity gradients is associated to the rapid part, which is the leading term in the rapid distorsion theory. This starting equation was used for instance by Kraichnan (1956) to derive a statistical modeling of pressure fluctuations in a turbulent boundary layer.

Reverting to the compressible case, as pointed out by Csanady (1966) and Lilley (1972) among others, the linear terms in Eq. (6) are also propagation terms. Assuming a sheared mean flow $\overline{u}_i = \overline{U}_1(x_2)\delta_{1i}$ and using the conservation of mass, Lighthill's equation takes the following equivalent form:

$$\frac{1}{c_\infty^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial^2 \rho u_i' u_j'}{\partial x_i \partial x_j} + 2\frac{\mathrm{d}\overline{U}_1}{\mathrm{d}x_2}\frac{\partial \rho u_2'}{\partial x_1} + \frac{1}{c_\infty^2}\frac{\partial^2 p}{\partial t^2} - \frac{1}{c_\infty^2}\frac{\overline{\mathrm{D}}^2 p}{\overline{\mathrm{D}}t^2} \quad (8)$$

where the differentiation following the mean flow is denoted $\overline{\mathrm{D}}/\overline{\mathrm{D}}t = \partial/\partial t + \overline{U}_1\partial/\partial x_1$. To obtain a single equation on the pressure, the linear term in $u_2'$ must be eliminated. By applying the mean flow convective operator $\overline{\mathrm{D}}/\overline{\mathrm{D}}t$ to Eq. (8), and by noting that the conservation of momentum provides

$$\frac{\partial}{\partial x_1}\frac{\overline{\mathrm{D}}\rho u_2'}{\overline{\mathrm{D}}t} = -\frac{\partial^2 p}{\partial x_1 \partial x_2} - \frac{\partial^2 \rho u_2' u_j'}{\partial x_1 \partial x_j}, \quad (9)$$

the following inhomogeneous wave equation is obtained:

$$\frac{\overline{\mathrm{D}}}{\overline{\mathrm{D}}t}\left(\frac{1}{c_\infty^2}\frac{\overline{\mathrm{D}}^2 p}{\overline{\mathrm{D}}t^2} - \nabla^2 p\right) + \underbrace{2\frac{\mathrm{d}\overline{U}_1}{\mathrm{d}x_2}\frac{\partial^2 p}{\partial x_1 \partial x_2}}_{(c)} = \underbrace{\frac{\overline{\mathrm{D}}}{\overline{\mathrm{D}}t}\frac{\partial^2 \rho u_i' u_j'}{\partial x_i \partial x_j}}_{(a)} - \underbrace{2\frac{\mathrm{d}\overline{U}_1}{\mathrm{d}x_2}\frac{\partial^2 \rho u_2' u_j'}{\partial x_1 \partial x_j}}_{(b)}. \quad (10)$$

The wave operator appearing on the left-hand side of Eq. (10) is identical to that derived from the linearized Euler equations (see for instance Goldstein (1976)). As a result, all the mean flow–acoustic interactions are included in this propagation operator for a sheared mean flow. The two source terms (a) and (b) on the right-hand side are now quadratic in velocity fluctuations. The first one comes directly from the non-linear term in Lighthill's Eq. (8). The second comes from the splitting of the shearnoise term of Eq. (8) into a propagation term (c) and the source term (b) by using Eq. (9). Lighthill's analogy consists in a reformulation of fluid motion equations, in which convection and refraction effects as well as noise generation are included in $T_{ij}$. When an accurate estimation of the compressible part of Lighthill's tensor is available, mean flow effects are correctly calculated, but in this case the acoustic field, namely the solution to the problem, is already known.

### Account of Mean Flow Effects in Hybrid Methods

The basic principle of a hybrid approach is to separate the noise generation from the linear acoustic propagation by recasting the equations of motion into an inhomogeneous wave equation $\mathcal{L}[p] = \Lambda$. Lilley (1972) showed that for a sheared mean flow $\overline{u}_i = \overline{U}_1(x_2,x_3)\delta_{1i}$, the effective noise sources can be found in this way, since the wave operator $\mathcal{L}$ governing the sound propagation is known in this case:

$$\mathcal{L} = \frac{1}{\overline{c}^2}\frac{\overline{\mathrm{D}}}{\overline{\mathrm{D}}t}\left[\frac{\overline{\mathrm{D}}^2}{\overline{\mathrm{D}}t^2} - \nabla\cdot(\overline{c}^2\nabla)\right] + 2\frac{\partial \overline{U}_1}{\partial x_i}\frac{\partial^2}{\partial x_1 \partial x_i} \quad i = 2, 3. \qquad (11)$$

The formulation of the source terms associated with Eq. (11) raises some discussions (see for example Lilley (1972) or Goldstein (1976)). One of the latest advances on the topic is the work of Goldstein (2001), who showed that at leading order, the source term is given by:

$$\Lambda = -\frac{\overline{\mathrm{D}}}{\overline{\mathrm{D}}t}\frac{\partial s_i}{\partial x_i} + 2\frac{\partial \overline{U}_1}{\partial x_i}\frac{\partial s_i}{\partial x_1}$$

with

$$s_i = -\overline{\rho}\frac{\partial u_i' u_j'}{\partial x_j}. \qquad (12)$$

Expression (11) is a generalization of the operator appearing in Eq. (10) for a stratified mean flow, for which $\overline{c} = \overline{c}(x_2, x_3)$, $\overline{\rho} = \overline{\rho}(x_2, x_3)$ and $\overline{p} = p_\infty$ is constant.

Since sound propagation is governed by the linearized Euler equations (LEE), Bailly *et al.* (1995) and Bogey and Bailly (2002a) derived an acoustic analogy directly using the LEE as a wave operator. In this hybrid approach, the following equations are solved:

$$\begin{cases} \frac{\partial \rho'}{\partial t} + \nabla\cdot(\rho'\overline{\mathbf{u}} + \overline{\rho}\mathbf{u}') = 0 \\ \frac{\partial(\overline{\rho}\mathbf{u}')}{\partial t} + \nabla\cdot(\overline{\rho}\,\overline{\mathbf{u}}\,\mathbf{u}') + \nabla p' + (\overline{\rho}\mathbf{u}' + \rho'\overline{\mathbf{u}})\cdot\nabla\overline{\mathbf{u}} = \mathbf{s} \\ \frac{\partial p'}{\partial t} + \nabla\cdot[p'\overline{\mathbf{u}} + \gamma\overline{p}\mathbf{u}'] + (\gamma-1)p'\nabla\cdot\overline{\mathbf{u}} - (\gamma-1)\mathbf{u}'\cdot\nabla\overline{p} = 0 \end{cases} \qquad (13)$$

where the source term is again given by Eq. (12), having noted that $s_i = -\partial\overline{\rho}u_i'u_j'/\partial x_j$ when there is no mean temperature gradient. The two analogies based on Eqs. (11)–(13) are equivalent in the sense that the inhomogeneous wave equation $\mathcal{L}[p'] = \Lambda$ derived from Eq. (13) yields exactly Eqs. (11) and (12).

However, even in the case of a stratified mean flow, the problem is not so well posed for noise generation. The homogeneous equation $\mathcal{L}[p'] = 0$ is a generalization of the Rayleigh equation to a compressible perturbation in the space–time domain. As a consequence, the fluctuating pressure obtained in solving Eqs. (11) and (12) is not necessarily of acoustic nature, and can also be associated to instability waves, likely to generate noise. This mechanism is for example the dominant noise source for supersonic jet noise. In other words, there is now a potential generation term on the left-hand side of $\mathcal{L}[p'] = \Lambda$. In other respects, these instability waves can overwhelm the acoustic solution of Eq. (13). High-frequency approximations can be used to remove them, as proposed in Bogey and Bailly (2002a) for instance. For a time-harmonic response, Agarwal *et al.* (2003) have proposed to filter out the instability waves by solving LEE in the frequency domain.

Just as for Lighthill's analogy, statistical models based on Lilley's equation have been developed, such as those derived by Goldstein and Howes (1973) or by Balsa and Gliebe (1977). Tam and Auriault (1999) recently developed another approach from the system (13) in which the source term is taken to be $\mathbf{s} = -\nabla(2\overline{\rho}k_s/3)$, where $k_s$ is the time-dependent part of the turbulent kinetic energy. Unlike in previous semi-empirical models, the source correlations are estimated in a fixed frame and the Green function is obtained from the adjoint solution of Eq. (13) by following the method elaborated in Tam and Auriault (1998). This approach was extended by Tam *et al.* (2000a) to take into account effects of forward flight on jet noise.

## Role of Instability Waves in Supersonic Jet Noise

It is now well established both experimentally and theoretically that instability waves play a very important role in supersonic jet-noise generation. Responsible for the principle analytical and numerical results in this area, Tam (1995b) and Morris (2001) have provided two complementary reviews.

Transition to turbulence and mixing in free shear flows are driven by large-scale or instability wave structures even at high Reynolds numbers. These structures were identified experimentally in the seventies by Crow and Champagne (1971) among others. From an analytical point of view, features of these large-scale structures are well described by the in-viscid linear instability theory. The linear evolution of small compressible perturbations is actually governed locally by the homogeneous equation $\mathcal{L}[p'] = 0$ where $\mathcal{L}$ is defined by Eq. (11) in a parallel mean flow approximation. The pressure perturbation is assumed to take the form:

$$p'(x_1, r, \theta, t) = Re\left[\hat{p}(r)e^{i(kx_1 + n\theta - \omega t)}\right] \qquad (14)$$

and satisfies the compressible form of Rayleigh's equation:

$$\frac{\mathrm{d}^2\hat{p}}{\mathrm{d}r^2} + \left[\frac{1}{r} - \frac{1}{\overline{\rho}}\frac{\mathrm{d}\overline{\rho}}{\mathrm{d}r} - \frac{2k}{k\overline{U}_1 - \omega}\frac{\mathrm{d}\overline{U}_1}{\mathrm{d}r}\right]\frac{\mathrm{d}\hat{p}}{\mathrm{d}r} - \left[k^2 + \frac{n^2}{r^2} - \frac{(k\overline{U}_1 - \omega)^2}{\overline{c}^2}\right]\hat{p} = 0 \qquad (15)$$

written in cylindrical coordinates, with, in this case, $\overline{U}_i = \overline{u}_1(r)\delta_{1i}$. In the spatial theory, well suited to treating convective instabilities, Eq. (15) with appropriate boundary conditions at $r = 0$ and $r \to \infty$ defines an eigenvalue problem for the complex axial wavenumber $k(\omega) = k_r + ik_i$.

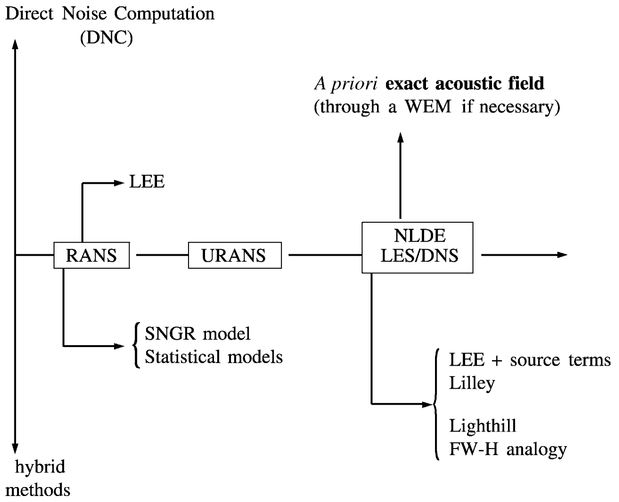



FIGURE 1 Computational aeroacoustics: some methods used for predicting aerodynamic noise. In the direct noise calculation, the aerodynamic field and the acoustic field are obtained in the same computation from DNS, LES, NLDE or URANS, and a wave extrapolation method (WEM) may be used to obtain the far field. In hybrid methods, generation and propagation are split into two distinct steps.

For supersonic jets, the size of the large-scale structures is of the same order as the acoustic wavelength. Thus, a fraction of the near pressure field generated by instability waves is expected to radiate into the far field. To determine this acoustic field, a global solution in which the instability wave amplitude grows and decays spatially is necessary, and the expression obtained for the pressure fluctuations associated with the instability waves must be valid far from the jet flow. The method was presented in Tam and Morris (1980) and in Tam and Burton (1984). Considering a locally parallel flow approximation in which the characteristics of instability waves with angular frequency $\omega$ are governed by the local wavenumber $k(\omega, x_1)$, the lowest-order pressure perturbation outside the jet associated with the $n$th azimuthal mode takes the form:

$$p'(x_1, r, \theta, t) = \int_{-\infty}^{+\infty} \hat{g}(\xi)\, H_n^{(1)}(i\lambda_\xi r)\, e^{i(\xi x_1 + n\theta - \omega t)} \mathrm{d}\xi \quad (16)$$

where $\lambda_\xi = \sqrt{(\xi^2 - \omega^2)/\bar{c}^2}$, $H_n^{(1)}$ is the $n$th-order Hankel function of the first kind, and the Fourier transform of $g$ is given by:

$$\hat{g}(\xi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} A_0 e^{\left\{ i\int_0^{x_1} k(\omega, x_1')\mathrm{d}x_1' \right\}} e^{-i\xi x_1} \mathrm{d}x_1$$

where $A_0$ is the initial amplitude of the perturbation at the nozzle exit. The acoustic far field is then obtained with the stationary phase method. Only wavenumbers lower than $k_c \simeq (\rho_\infty/\rho_j)^{1/2}\omega/c_j$ can contribute to the radiated acoustic field, which corresponds to the contribution of components with phase velocities that are supersonic compared to the relative ambient speed of sound. The angle of maximum emission occurs at $\cos\theta \simeq k_p/k_c$ where $k_p$ corresponds to the maximum of $|\hat{g}|$. In this approach, a direct link is established between the large-scale structures of the flow and the radiated sound field. Since the initial amplitude $A_0$ is unknown for each angular frequency $\omega$, only relative levels are obtained, but comparisons with experimental data by Tam and Burton (1984) for supersonic jets and by Dahl and Morris (1997) for supersonic coaxial jets are favorable. For more complex mean flows, a direct numerical approach based on linearized Euler's equations is also possible (see Mankbadi *et al.* (1998) for instance).

## Connections with Numerical Simulations: from CFD to CAA

It is worthwhile to review the different strategies available to connect the acoustic modelings with CFD. The CFD methods fall usually into one of the three following categories: direct numerical simulation (DNS), large eddy simulation (LES) or Reynolds averaged Navier–Stokes (RANS). In DNS, the Navier–Stokes equations are solved for all the scales of the flow, without any turbulence model. In LES, the governing equations are the Navier–Stokes equations explicitly filtered in space, and a turbulence model is used to represent the subgrid-scale stress tensor. In the last category, the RANS, equations are solved to obtain the mean flow and some statistical quantities such as the turbulent kinetic energy and the rate of dissipation. Moreover, unsteady RANS simulation or semi-deterministic modeling can provide a weakly time-dependent solution in the sense that only the largest scales are calculated. For aeroacousticians, another kind of classification is the compressible nature of time-dependent simulations, which can directly capture the noise associated to the resolved part of turbulence if precautions are taken in the resolution to preserve the sound waves. This approach is highlighted in the next part of the paper, and corresponds to the upper part of the sketch in Fig. 1. The RANS simulations are now an engineering tool in CFD, and a first reasonable idea is to introduce some data provided by a $k - \epsilon$ turbulence closure in statistical models. Béchara *et al.* (1995), Bailly *et al.* (1996, 1997) or Khavaran (1999) developed such applications to jet noise. An interesting analysis was developed in Morris and Farassat (2002) to compare these methods based on Lighthill or Lilley's acoustic analogy with the approach developed by Tam and Auriault (1999) which gives better predictions. Morris and Farassat showed that, at 90° to the jet axis, the two kinds of statistical methods yield identical predictions if the same assumptions are made in the statistical description of turbulent sources. Again from a RANS calculation, a stochastic space–time turbulent field can be synthesized to calculate the source term $\mathbf{s}$ in the LEE (13), e.g. see Bailly *et al.* (1995, 1999) but the generation of a suitable random inhomogeneous turbulent field is a difficult task.

When a time-dependent solution is available, velocity fluctuations can directly be introduced into Lighthill's integral (2) to estimate the radiated acoustic field.

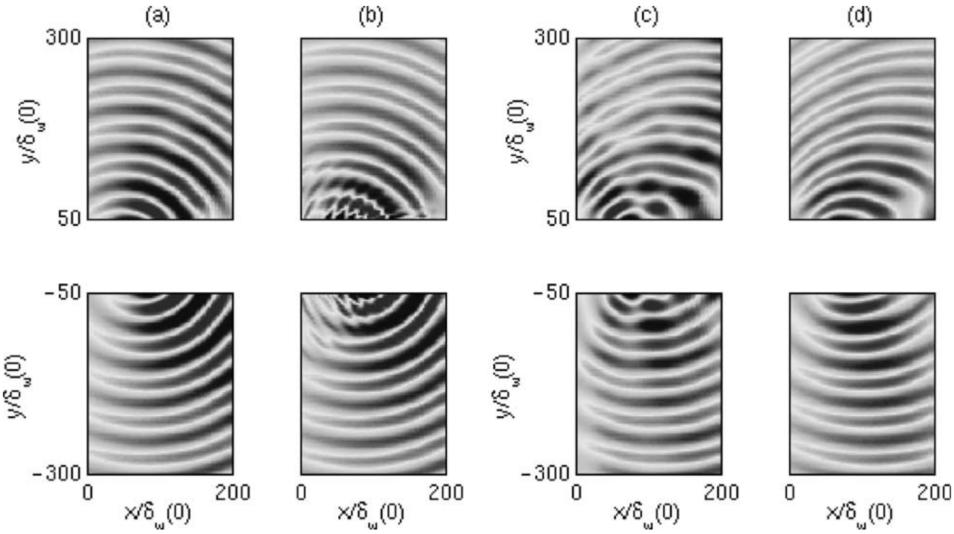



FIGURE 2 Dilatation fields obtained from Lighthill's integral with (a) $T_{ij}^f$ and (c) $T_{ij}$ as source terms, (b) from the LEE without mean flow, (d) directly from the Navier–Stokes equations. All the calculations are 2-D, and quantitative comparisons can be found in Bogey *et al.* (2003b).

One of the first applications to jet noise was performed by Bastin *et al.* (1997). Application of an integral formulation requires some numerical precautions, and 3-D calculations are a costly way of testing different assumptions and formulations. CAA needs simpler test cases and should take its inspiration from the evolution of CFD with reference configurations such as the periodic channel flow. For aerodynamic noise, sound generated by a plane mixing layer has turned out to be a very useful model problem. Colonius *et al.* (1997) computed the noise produced by a mixing layer using a DNS with a computational domain including a portion of the acoustic field. They also applied Lilley's analogy with the acoustic source terms (12) calculated from the aerodynamic field, and obtained a good agreement with their DNS.

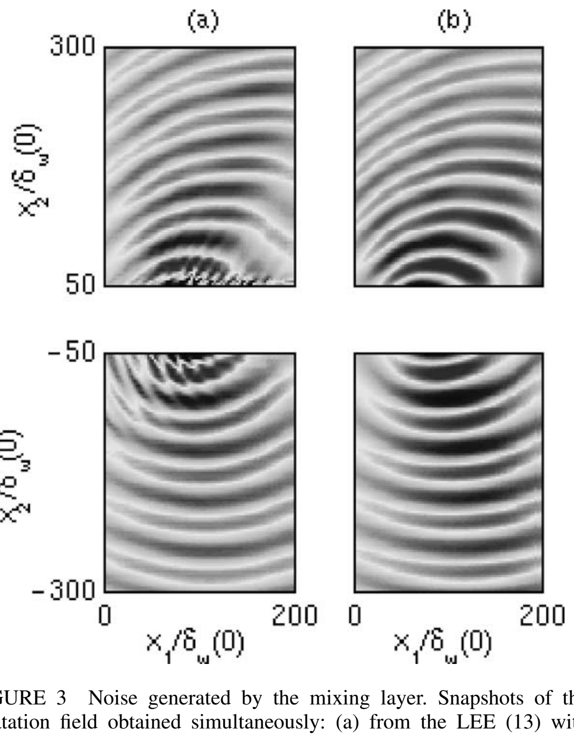



FIGURE 3 Noise generated by the mixing layer. Snapshots of the dilatation field obtained simultaneously: (a) from the LEE (13) with source terms (12), (b) directly from the Navier–Stokes equations. All the calculations are 2-D and quantitative comparisons can be found in Bogey *et al.* (2002a).

As an illustration of CAA's contribution towards better understanding the previous discussion about Lighthill's source term (6), we consider the noise generated by a plane mixing layer formed by two isothermal streams at Mach $M_1 = 0.12$ and $M_2 = 0.48$ in the lower and upper parts respectively. Referring to Colonius, a direct calculation based on a compressible LES was performed by Bogey *et al.* (2000) to obtain the aerodynamic field and a portion of the acoustic field. The flow development is driven by forcing the mixing layer at discrete frequencies so that only the sound generated by the first vortex pairings is observed in the computational domain. Vortex pairings occur at $x \simeq 70\delta_\omega(0)$ and the frequency of the radiated field is $\lambda \simeq 51\delta_\omega(0)$ where $\delta_\omega(0)$ is the initial vorticity thickness. Figure 2(d) shows a snapshot of the dilatation field $\Theta = \nabla\cdot\mathbf{u}$, which is directly linked to the fluctuating pressure field in the present case. Mean flow effects on propagation are well marked especially in the rapid stream region. For the sake of discussion, the decomposition of Lighthill's tensor (6) is repeated here:

$$T_{ij} \simeq \rho u'_i u'_j + \rho \bar{u}_i u'_j + \rho u'_i \bar{u}_j + \rho \bar{u}_i \bar{u}_j. \qquad (17)$$

The acoustic field predicted by Lighthill's analogy with the quadratic term $T_{ij}^f = \rho u'_i u'_j$ is shown in Fig. 2(a), and is compared in Fig. 2(b) with the solution obtained by LEE (13) with a free mean flow $\bar{\mathbf{u}} = 0$ and with the quadratic source term $\mathbf{s}$ given by Eq. (12). This is the self-noise component. The acoustic field predicted by Lighthill with the full tensor $T_{ij}$ is displayed in Fig. 2(c). This source term $T_{ij}$ including all interactions between the flow and the acoustic waves, the solution is in agreement with the direct

TABLE I Parameters of some recent simulations dealing with the direct calculation of jet noise in 3-D

| Authors | Method | $M$ | $Re_D$ | Mesh points |
|---|---|---|---|---|
| Freund *et al.*(1998, 2000) | DNS | 1.92 | $2.0 \times 10^3$ | $22.1 \times 10^6$ |
| Choi *et al.* (1999) | LES | 1.39 | $2.0 \times 10^6$ | $2.16 \times 10^6$ |
| Morris *et al.*(1999) | LES | 2.1 | $\sim 10^6$ | $1.03 \times 10^6$ |
| Freund (1999, 2001) | DNS | 0.9 | $3.6 \times 10^3$ | $25.6 \times 10^6$ |
| Bogey *et al.*(2000, 2003a) | LES | 0.9 | $6.5 \times 10^4$ | $6.05 \times 10^6$ |
| Zhao and Frankel (2000) | LES | 0.9 | $3.6 \times 10^3$ | $2.05 \times 10^6$ |
| Constantinescu and Lele (2001) | LES | 0.9 | $7.2 \times 10^4$ | $3.9 \times 10^6$ |
| Shen and Tam (2002) | URANS | 1.1–1.6 | – | – |
| Lupoglazoff *et al.* (2002) | LES | 0.7 | $1.2 \times 10^6$ | $2.4 \times 10^6$ |
| Bogey and Bailly (2002a,b) | LES | 0.9 | $4.0 \times 10^5$ | $16.6 \times 10^6$ |
| Uzun *et al.* (2003) | LES | 0.9 | $1.0 \times 10^5$ | $12 \times 10^6$ |
| Bogey and Bailly (2003a,b) | LES | 0.9 | $4.0 \times 10^5$ | $12.5 \times 10^6$ |

This is absolutely not an exhaustive list of references on the topic

noise solution 2(d). As pointed out before, this problem is academic since the evaluation of $T_{ij}$ requires the calculation of the acoustic field. The result obtained by applying the LEE (13) with the source term **s** given by Eq. (12) is shown in Fig. 3, and is in good agreement with the reference solution. Coupling with instability waves is discussed in more detail in Bogey *et al.* (2002a). In this case, we have an acoustic analogy of practical interest since only the aerodynamic velocity fluctuations are needed to evaluate the source term, and mean flow effects are fairly well calculated by the resolution of the LEE.

## DIRECT COMPUTATION OF JET NOISE

### Specific Numerical Issues for Jet Noise

As illustrated in the previous section, turbulence and acoustics are inextricably linked. Some strategies connecting CFD results to different acoustic theories represented in Fig. 1 have been briefly described in the previous section. An alternative to these methods is the DNC, where the aerodynamic field and the sound field are determined in the same computation using a mesh including a part of the acoustic field. This approach was introduced by the Stanford group with the work of Colonius *et al.* (1997). The DNC solution can be used as a reference solution to support the validity of hybrid methods as shown previously. But this kind of calculation also allows for the improvement of our knowledge of noise generation mechanisms by showing links between turbulence dynamics and the acoustic waves.

There are some important key issues specific to DNC, reviewed by Tam (1995a) and Lele (1997). Aerodynamic noise is characterized by small amplitude fluctuations, typically $u'_{\text{acous}}/u'_{\text{aero}} \sim 10^{-3}$ to $10^{-4}$ and $p'_{\text{acous}}/p'_{\text{aero}} \sim 10^{-2}$ for a jet at $M = 0.9$, and by large length scales with an acoustic wavelength $\lambda/\delta_\theta \sim 10^2$, where $\delta_\theta$ is the shear-layer momentum thickness of the velocity profile at the nozzle exit. Low dispersive and low dissipative numerical schemes must be used to preserve acoustic waves propagating in the computational domain. Inflow boundary conditions must seed the transition of the jet flow without producing spurious noise, and what is more, non-reflecting and outflow boundary conditions must be implemented. The exit of subsonic vortical disturbances from the computational domain is also a crucial point, and there is no exact way to deal with this problem but only practical approaches involving sponge or buffer zones, as explained in Colonius *et al.* (1993), in Freund (2001) or in Bogey *et al.* (2002b, 2003a) for instance.

Numerical constraints intrinsic to aerodynamic simulations must also be satisfied. The ratio between the integral longitudinal length scale $L \sim D$ and the acoustic length scale $\lambda$ is $L/\lambda \sim \text{St} \times M$. For a DNS, the number of points in one direction is $n_x \sim \lambda/l_\eta \sim Re_L^{3/4}/(\text{St}\,M)$ using the relation $L/l_\eta \sim Re_L^{3/4}$ for isotropic turbulence where $Re_L = u'L/\nu$ and $l_\eta$ is the Kolmogorov scale. The number of mesh points is $N \sim n_x^3$, the time step is $\Delta t \sim l_\eta/c$ and the number of time steps is linked to $n_t \sim (L/U_j)/\Delta t$, which yields a cost of $N \times n_t \sim Re_L^3/(M^4\,\text{St}^3)$. For an LES, the number of points in one direction is given by $n_x \sim \lambda/\lambda_g \sim Re_L^{1/2}/(\text{St}\,M)$ where $\lambda_g$ is the lateral Taylor microscale and $L/\lambda_g \sim Re_L^{1/2}$ for isotropic turbulence. A similar reasoning leads to a cost of $N \times n_t \sim Re_L^2/(M^4\,\text{St}^3)$.

Thus, the grid requirements of both LES and DNS are difficult to achieve, even just to simulate laboratory experiments with typical jet exit Reynolds numbers of about $10^6$. The parameters of some recent three-dimensional simulations of jet noise are provided in Table I. They involve jets at low Reynolds number, $Re_D \sim 10^3$, for DNS and up to about $Re_D \sim 10^6$ for LES, using grids of several million points. Note that in supersonic, only very LES can be done since large-scale structures are the essential contribution to the radiated noise.

An alternative implementation of LES for noise calculation has been proposed by Morris *et al.* (2002). The resolved variables are split into a time-independent mean value and a perturbation, and the assumed mean flow can be based on a steady RANS solution. An increased accuracy is expected with the nonlinear disturbances equations (NLDE) in working on perturbations about a mean flow.

### Direct Calculation of Subsonic Jet Noise

The choice of simulation parameters for a direct calculation of subsonic jet noise is more ambiguous than for supersonic

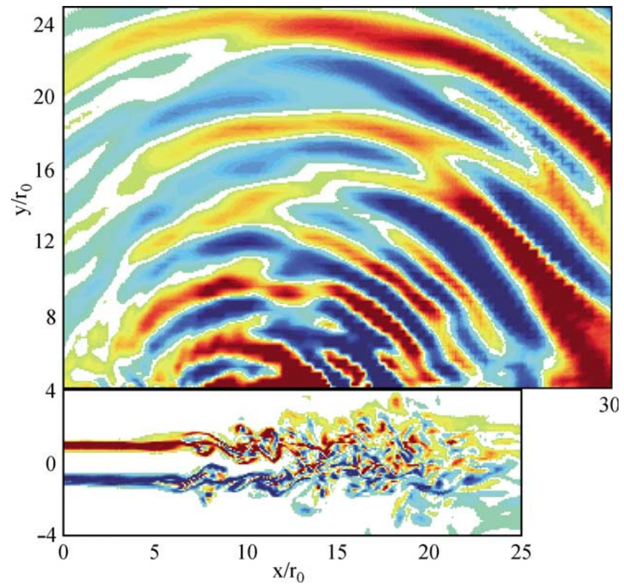
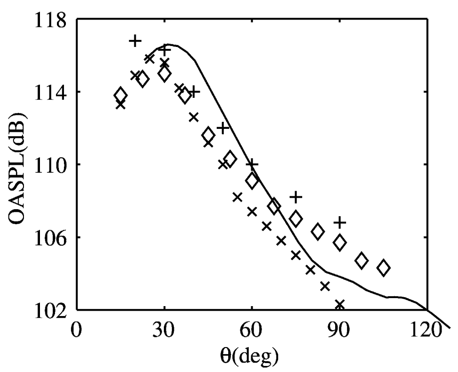



FIGURE 4 Direct computation of noise from LES, circular jet at $M = 0.9$ and $Re_D = 6.5 \times 10^4$, see Bogey *et al.* (2003a) for details. Snapshot of the dilatation field $\Theta = \nabla\cdot\mathbf{u}$ in the acoustic region, and of the vorticity field $\omega_z$ in the aerodynamic region, in the $x - y$ plane at $z = 0$. Note that $D = 2r_0$.

jet noise. The dependence of jet noise with respect to the Reynolds number is indeed controversial. Measurements show a directivity well marked even at low Reynolds numbers, but characteristic broadband acoustic spectra are only obtained for large Reynolds numbers about $Re_D \geq 10^5$, where a rapid transition of the shear layer occurs at the nozzle exit. Such a Reynolds number corresponds also to a Reynolds number based on the Taylor scale $\lambda_g$ of $Re_{\lambda g} \geq 500$, which is required to establish an inertial subrange after Corrsin (see Gibson, 1963). Thus, convincing discussions about noise mechanisms and noise predictions will be achieved only by increasing the simulated Reynolds




| | M | $Re_D$ | Reference |
|---|---|---|---|
| + | 0.9 | $5.4 \times 10^5$ | Mollo-Christensen *et al.* (1964) |
| ◇ | 0.88 | $5 \times 10^5$ | Lush (1971) |
| × | 0.9 | $3.6 \times 10^3$ | Stromberg *et al.* (1980) |
| - | 0.9 | $6.5 \times 10^4$ | Bogey *et al.* (2003a) |

FIGURE 5 Direct computation of noise from LES, circular jet at $M = 0.9$ and $Re_D = 6.5 \times 10^4$, see Bogey *et al.* (2003a) for details. Overall sound pressure level as a function of angle $\theta$ measured from the jet axis, at $60r_0$ from the jet nozzle, and comparison with experimental data for different Reynolds numbers.

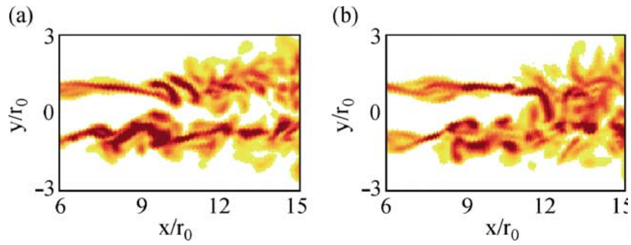



FIGURE 6 Circular jet at $M = 0.9$ and $Re_D = 6.5 \times 10^4$. Snapshot of the vorticity norm $|\omega|$ in the plane $z = 0$ (a), at $t^* = 6.2$; (b), at $t^* = 7.5$.

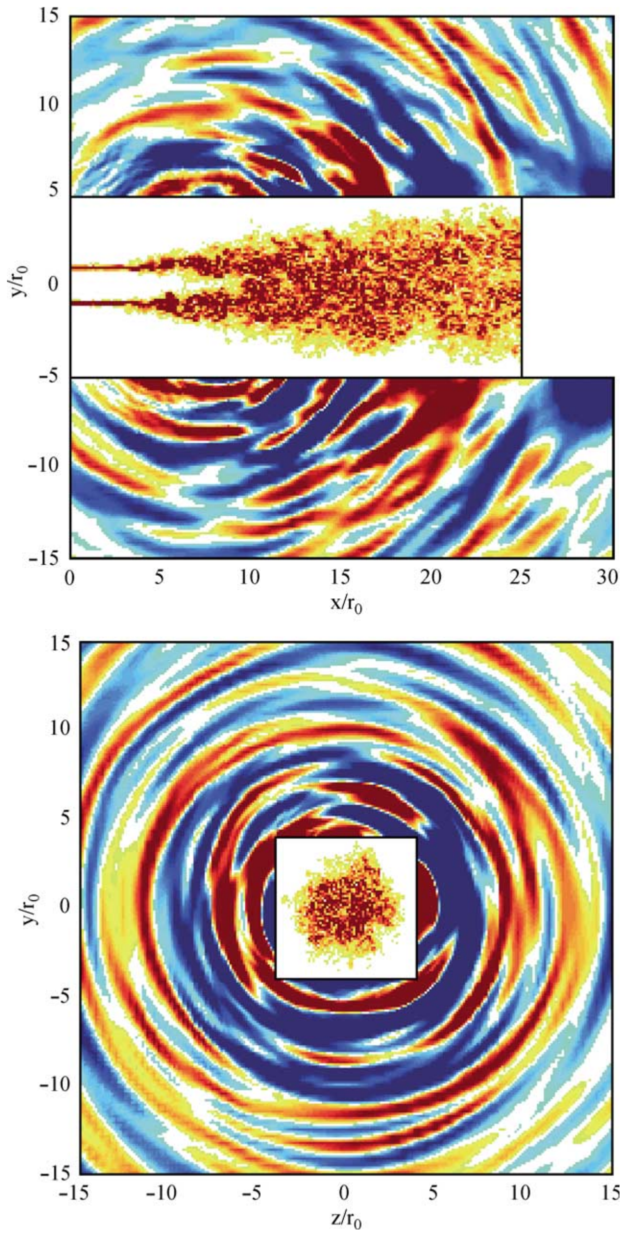



FIGURE 7 LES of a circular jet at $M = 0.9$ and $Re_D = 4 \times 10^5$. Snapshot of the vorticity $|\omega|$ in the flow and of the fluctuating pressure $p'$ outside. Left: in the $x - y$ plane at $z = 0$. Right: in the $y - z$ plane at $x = 11r_0$. The color scales are from 0 to 8 $\times$ $10^4\,s^{-1}$ for the vorticity and from $-70$ to 70 Pa for the pressure. From Bogey and Bailly (2002c).

number. LES should be the appropriate way, but the effects of turbulence modeling on aeroacoustics must be carefully evaluated beforehand.

In what follows, two configurations at different Reynolds numbers involving a circular subsonic jet with a Mach number $M = 0.9$ and initially transitional shear layers are shown. The noise generated by the jet is obtained directly by LES in both cases.

### *Moderate Reynolds Number Jet*

The first three-dimensional flow simulated by LES in our group was a circular jet with a Mach number $M = 0.9$ and a moderate Reynolds number $Re_D = 6.5 \times 10^4$. The inflow shear layer was forced with random velocity disturbances to seed the turbulence and the Smagorinsky model was used. The dimensionless simulation time was $TU_j/D = 300$, which is long enough to achieve statistical convergence both for the flow and the sound field. All the parameters of the simulation, and also validations by comparison with measurements, can be found in Bogey *et al.* (2003a). Figure 4 displays, in the plane $z = 0$, the vorticity field $\omega_z$ in the flow and the dilatation field $\Theta = \nabla \cdot \mathbf{u}$. outside, both directly obtained from the simulation. The acoustic wave fronts generated by the jet are clearly visible, and they originate mainly from the region where the mixing layers merge, around $x = 11_{r_0}(D = 2r_0)$.

The sound pressure levels, calculated by integrating the sound spectra, are presented in Fig. 5. They are in good agreement, for all observation angles, with experimental data from jets with similar Mach numbers but varying Reynolds numbers. As expected, the acoustic levels reach a peak for an angle of about $\theta = 30°$. For higher angles, the levels of the computed $Re_D = 6.5 \times 10^4$ jet stand between the levels measured for the $Re_D = 3.6 \times 10^3$ jet and those measured for the $Re_D \simeq 5 \times 10^5$ jets. This can be attributed to a Reynolds number effect, since the fine-scale turbulence, which becomes important at high Reynolds numbers, generates sound which may be predominant at large observation angles.

Animations of visualizations of both the flow and the acoustic radiation suggested that the noise observed for an angle of 30° comes principally from the region where the shear layers merge, at the end of the potential core. Thus, the downstream noise generation mechanism appears to be associated with the sudden accelerations of turbulent structures as they periodically enter the high-speed jet core. To support this, looking at the dynamics of vortical structures through the vorticity field, the shear layers can be distinguished up to $x = 15r_0$ in Fig. 6(a), but only up to $x = 11r_0$ in Fig. 6(b). Turbulent structures originating from the shear layers have penetrated into the jet core near $x = 10r_0$, and they have been suddenly accelerated by the higher flow velocity. It can be shown that this periodic phenomenon is correlated with the pressure signal captured at $\theta = 30°$ for the simulated moderate Reynolds number jet. For larger emission angles, the acoustic field appears rather associated with more random flow events, generating broadband frequency noise. Among these sources radiating in the sideline direction, some are likely to be located in the turbulent shear layers where vortical structures interact as suggested in Zaman (1986).

### *High Reynolds Number Jet*

Subsequently to the simulation reported in the previous section, the LES of a jet at the same Mach number but at a higher Reynolds number $Re_D = 4 \times 10^5$ yielding a jet diameter of $D = 2$ cm was carried out in Bogey and Bailly (2002c). An explicit spectral-like filtering has been used for modeling the dissipative effects of the unresolved scales. Two simulations using mesh grids of different sizes, one extending far away downstream, referred to as LESaero, and one including a part of the acoustic field, referred to as LESac, were performed to show that the flow development is not dependent on the location of the grid boundaries. Flow and sound features are in good agreement with the measurements available in the literature for a high Reynolds number, particularly regarding the changes in the acoustic field according to the observation angle. A view of the vorticity field in the jet and of the pressure field outside is displayed in Fig. 7. The vorticity fields show a large range of vortical scales, with the presence of a fine turbulence in accordance with the high Reynolds number. Two kinds of acoustic waves are again visible: a low-frequency component with high amplitude propagating in the downstream direction, and a higher frequency component for large angles appearing to come from the turbulent axisymmetric shear layer around $x \simeq 8r_0$. Effects of the inflow conditions as well as of the subgrid-scale modelings on the jet properties have been investigated recently in Bogey and Bailly (2003a,b). To exhibit again links between the turbulence and the noise radiated downstream, the sound pressure spectrum at $\theta \simeq 30°$ and the centerline profile of the axial velocity fluctuations are represented in Fig. 8. In addition to the LESac results, results corresponding to three other simulations with different inflow conditions are shown. With respect to LESac, the amplitude of the forcing is divided by two in the LESampl simulation, the shear layer is thinner in the LESshear simulation and the first four azimuthal modes in the forcing are removed in the LESmode simulation. Despite the various in-flow conditions, the four acoustic spectra are quite similar with a peak around St $\simeq 0.3$ in agreement with experimental observations. This suggests the mechanism involved for noise generation is the same for the different simulations and is identical to the one described in the preceding section. For the four simulations, the $u'_{\mathrm{rms}}$ peak values are arranged in the same order as the downstream noise levels. A smaller forcing amplitude enhances the $u'_{\mathrm{rms}}$ peak value while a thinner shear layer reduces this peak in accordance with the DNS results of Stanley and Sarkar (2000) for a low Reynolds number plane jet. As reported by Bogey

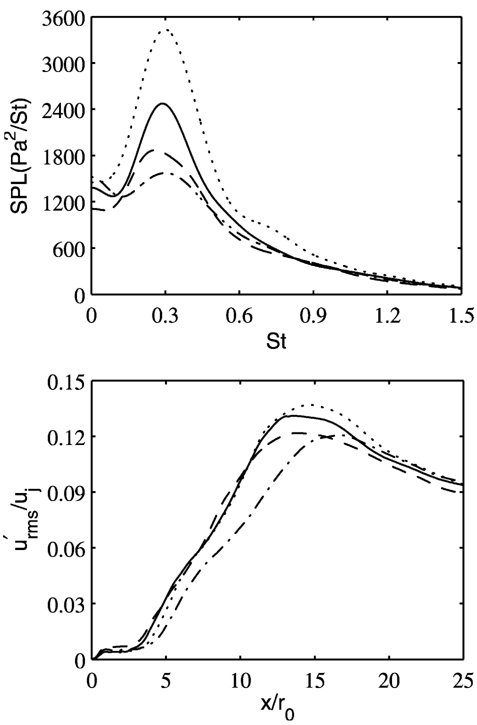



FIGURE 8 LES of a circular jet at $M = 0.9$ and $Re_D = 4 \times 10^5$. Left: sound pressure spectra at $x = 29r_0$, $r = 12r_0$ in linear scales as a function of the Strouhal number. Right: centerline profiles of the rms-values of the axial fluctuating velocity. LESac ———, LESampl ........, LESshear – – – and LESmode –·–·–. From Bogey and Bailly (2003a).

and Bailly (2003a) in a more detailed analysis, the most important changes in the flow and sound properties are obtained by removing the first azimuthal modes in building the forcing. In this case, the jet develops later and slowly, and turbulence and noise levels are significantly reduced.

## CONCLUSIONS

The direct noise calculation (DNC) is an outstanding method to correlate turbulence events with the sound far field. Such an analysis is necessary to improve our knowledge of jet noise mechanisms and to understand control of the optimizations of the turbulent flow with the aim of reducing noise.

In conclusion, we have chosen to emphasize three points among many others. First, turbulence is neither just large-scale structures, nor just fine-scale structures. Experiments indeed show that turbulence in jet flows has broadband continuous spectra including coherent structures. This behavior is retrieved in the recent empirical model of Tam and Zaman (2000b) involving similarity spectra. The model is based on two spectral components. One is associated to large-scale structures and dominates for angles close to the jet axis with a sharp peak at relatively low frequency, and the other one corresponds to fine-scale structures with a broad spectral peak. Experimental data are well approximated by the model. Second, in the context of DNC, LES seems the better tool to clarify Reynolds number effects on subsonic jet noise. But subgrid-scale modelings must be seriously discussed for LES, as suggested in Pruett (2001) for instance. LES for high Reynolds flows raised some questions about the influence of the turbulence closure on aeroacoustics. The recent work of Bogey and Bailly (2002c), for example, shows that the exit Reynolds number of the simulated flow is recovered by using an optimized solver with high-order explicit filters and no subgrid-scale model. Third, numerical simulations need experimental data in which turbulence and acoustics are measured together. High Reynolds number flows must be characterized in order to validate turbulence closures of LES. Also, DNC must be used to guide experimentally investigatations of noise source mechanisms. This synergy between computation and experiments is a key point if rapid advances are to be hoped for.

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