Inverted pendulum systems: rotary and arm-driven - a mechatronic system design case study

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$$\mathrm{d}t\,\partial\phi \quad \partial\phi \quad \partial\phi$$

$$\left[m_1\ell_{11}^2 + \bar{I}_{1_{z_1}} + m_2\ell_1^2 + m_2\ell_{21}^2\cos^2\phi + \bar{I}_{2_{z_2}}\cos^2\phi + \bar{I}_{2_{y_2}}\sin^2\phi\right]\ddot{\theta} - [m_2\ell_1\ell_{21}\sin\phi]\ddot{\phi} - [m_2\ell_1\ell_{21}\cos\phi]\dot{\phi}^2 + \left[\bar{I}_{2_{y_2}} - m_2\ell_{21}^2 - \bar{I}_{2_{z_2}}\right] \times (2\cos\phi\sin\phi)\dot{\phi}\dot{\theta} = T - \left[B_\theta\dot{\theta} + T_{\mathrm{f}\theta}\mathrm{sgn}(\dot{\theta})\right], \tag{11}$$

$$\left[m_2\ell_{21}^2 + \bar{I}_{2_{x_2}}\right]\ddot{\phi} - [m_2\ell_1\ell_{21}\sin\phi]\ddot{\theta} + \left[m_2\ell_{21}^2 - \bar{I}_{2_{y_2}} + \bar{I}_{2_{z_2}}\right](\cos\phi\sin\phi)\dot{\theta}^2 + m_2 g\ell_{21}\cos\phi = -\left[B_\phi\dot{\phi} + T_{\mathrm{f}\phi}\mathrm{sgn}(\dot{\phi})\right]. \tag{12}$$

Define

$$\alpha = \frac{\pi}{2} - \phi. \tag{13}$$

When the pendulum is balanced and the driven link is centered, both θ and α are zero. The nonlinear equations of motion are

$$\left[m_1\ell_{11}^2 + \bar{I}_{1_{z_1}} + m_2\ell_1^2 + m_2\ell_{21}^2\sin^2\alpha + \bar{I}_{2_{z_2}}\sin^2\alpha + \bar{I}_{2_{y_2}}\cos^2\alpha\right]\ddot{\theta} + [m_2\ell_1\ell_{21}\cos\alpha]\ddot{\alpha} + [m_2\ell_1\ell_{21}\sin\alpha]\dot{\alpha}^2 + \left[\bar{I}_{2_{z_2}} + m_2\ell_{21}^2 - \bar{I}_{2_{y_2}}\right] \boxed{-}(2\cos\alpha\sin\alpha)\dot{\alpha}\dot{\theta} = T - \left[B_\theta\dot{\theta} + T_{\mathrm{f}\theta}\mathrm{sgn}(\dot{\theta})\right], \tag{14}$$

error

Should be

$$\left[m_1\ell_{11}^2 + \bar{I}_{1_{z_1}} + m_2\ell_1^2 + m_2\ell_{21}^2\sin^2\alpha + \bar{I}_{2_{z_2}}\sin^2\alpha + \bar{I}_{2_{y_2}}\cos^2\alpha\right]\ddot{\theta} + [m_2\ell_1\ell_{21}\cos\alpha]\ddot{\alpha} + [m_2\ell_1\ell_{21}\sin\alpha]\dot{\alpha}^2 \boxed{+} \left[\bar{I}_{2_{z_2}} + m_2\ell_{21}^2 - \bar{I}_{2_{y_2}}\right] \boxed{-}(2\cos\alpha\sin\alpha)\dot{\alpha}\dot{\theta} = T - \left[B_\theta\dot{\theta} + T_{\mathrm{f}\theta}\mathrm{sgn}(\dot{\theta})\right], \tag{14}$$

Change to $\boxed{-}$

Change to $\boxed{\times}$