Acoustic Cloaking in a Mean Flow

Siyang Zhong* and Xun Huang†

Peking University, Beijing, 100871, People's Republic of China

Acoustic cloak is a metamaterial that makes the cloaked objects impervious towards sound waves. Existing acoustic cloak designs were originally formulated in a stationary medium. An acoustic shadow appears if those cloaking designs are located in a non-stationary flow field that discloses the concealed objects. A simple design is formulated to largely address this mean flow related issue, constituting the main contribution of the present article. The proposed new cloaking design is demonstrated in numerical simulations. The sound scattering from the cloaking design is studied using the model of linearized Euler equation. The background mean flow is calculated at low Mach numbers between 0 and 0.06, using potential and incompressible flow models, respectively. The performance of the proposed cloaking design is examined by studying both near- and far-field acoustic solutions.

I. Introduction

Acoustic cloak is a metamaterial that shields the cloaked objects from sound waves. The idea of cloaking has attracted an increasing research interest over the past years.[1–6] Potential applications mainly include anti-sonar designs. Acoustic cloak could make aircraft cabin impervious towards external aircraft noise. On the other hand, cloak materials could cover airframe to reduce undesired sound scattering. However, according to the best knowledge of the authors, existing acoustic cloaking designs were formulated with no consideration of the mean flow. The cloaking design taking account of non-stationary mean flow effects has to be developed for potential aeroacoustic applications.

The theory behind cloaking design is a conformal map that transforms a physical region with an interior cloaked hole to a virtual domain that is mathematically simply connected. The sound propagation is simply governed by the classical wave equation. When an object exists, the physical domain can be mapped to the virtual domain by a transformation function $\vec{F}$. The transformation yields the desired material parameters of the so-called cloak that guides the acoustic propagation surrounding the concealed object.[2,4]

Previous works consider the simple acoustic propagation cases in a stationary medium. It is interesting, and of practical importance, to formulate a new cloaking design with the consideration of non-stationary fluid medium. To gain some insight of the problem, we firstly suppose an idealized case that a time-harmonic plane acoustic wave incident on an object immersing in a presumably uniform mean flow.

We assume that the frequency of the sound is invariant during the propagation in this fluid medium. The wave numbers are then different in the fluid and the conventional cloak layer. A shadow appears that disclosed the existence of the cloaked object, although the acoustic wave is still perfectly guided through the object with no reflection. This drawback can be theoretically remedied by simply setting the transformed region according to the surrounding uniform flow field. As a result, the equivalent sound speed and impendence matching conditions determine the resultant parameters of the proposed cloaking design.

However, flow patterns local to the concealed object are definitely not uniform due to the object and the cloak layers. Then, the effectiveness of the proposed design is examined in this work, for those more complicated flow conditions at a relatively low Mach number between 0 and 0.06. For now, some practical issues, in particular the potential implementations of cloaking materials, limit our attention to the low Mach number fluid medium. Nevertheless, similar investigations can be conducted at high Mach number

*Graduate student, Department of Aeronautics and Astronautics, College of Engineering, Peking University. Email: zhongsy@pku.edu.cn.

†Professor. Affiliations: (1) Department of Aeronautics and Astronautics; and (2) State Key Laboratory of Turbulence and Complex Systems. AIAA Senior Member. Email: huangxun@pku.edu.cn.

conditions. It is also worthwhile to mention that flow-induced noise from the interaction between the flow and the cloak is omitted.

The outline of this work is as follows. In section II, we describe the basic idea and the associated mathematical formulation for the proposed acoustic cloaking design in a non-stationary uniform mean flow. In section III, we conduct numerical simulations to examine the performance of the proposed cloaking design at various fluid conditions. In section 4 we summarize the current research and ongoing topics.

II. Formulations

The acoustic propagation in a non-stationary mean flow is governed by[7]

$$(\frac{\partial}{\partial t} + \vec{U}_0 \cdot \nabla)^2 p' - c_0^2 \nabla^2 p' = 0, \tag{1}$$

where p' is the sound pressure, which is considered as a perturbation of the pressure of the background fluid field. $\vec{U}_0$ is the velocity of the background mean flow. The speed sound is c_0. If the fluid medium is stationary, the wave number is $k_0 = \omega/c_0$, which however depends on the mean flow velocity as well. If the incident sound wave has the form $p'(\vec{x}, t) = P(x)e^{i\omega t - ik_1 x}$ then we can have

$$(\frac{\omega}{c_0})^2 - 2M_a \frac{\omega}{c_0} k_1 - (1 - M_a^2) k_1^2 = 0,$$

where $M_a = |\vec{U}_0|/c_0$ is the Mach number of the mean flow. It is straightforward to achieve

$$k_{11,2} = \frac{k_0}{M_a \pm 1}, \tag{2}$$

where $k_{11,2}$ are the wave number of the plane wave corresponding to the sounds propagate toward the $x+$ and $x-$ direction, respectively.

The material properties of the proposed cloaking design can then be determined. Firstly, as illustrated in Fig. 1, we assume replace some region in the mean flow with an equivalent medium. The sound wave propagates in this equivalent stationary medium propagates with a same speed as in the uniform mean flow while no scattering occurred at the interface of the two mediums. Secondly, design a cloak in such a medium that shields the concealed object based on the previous theories.[2,4] Of course, the flow field local to the cloaked region is largely not uniform. Then, the proposed design is examined in numerical simulations that approach practical flow influences.

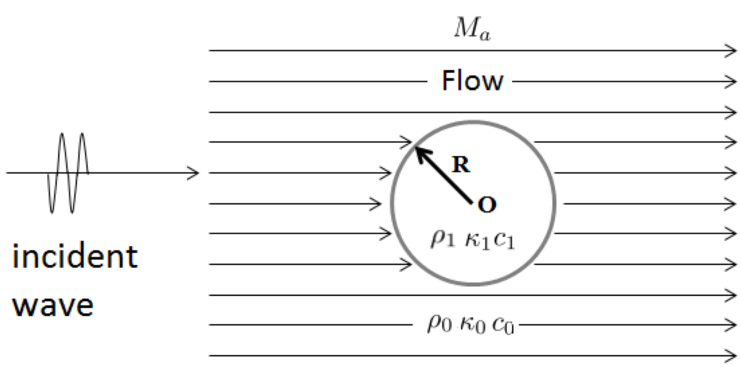

Figure 1. Schematic of the cloak design in the transformation domain. The region is of the radius R, the parameters ρ_1, κ_1 are obtained by the equivalent sound speed and continuity of acoustic impendence.

Assume the sound plane wave propagates and uniform fluid flows both in the $x+$ direction. Then, the equivalent sound speed is:

$$c_1 = \frac{\omega}{k_1} = \frac{c_0 k_0}{k_1} = (1 + M_a) c_0.$$

To ensure the same impendence remains between the two regions, the equivalent density ρ_1 satisfies

$$\rho_0 c_0 = \rho_1 c_1.$$

The equivalent density inside the cloaked region can thus be obtained, as

$$\rho_1 = \frac{\rho_0}{1+M_a}. \tag{3}$$

We represent the bulk modulus of the background flow and the equivalent medium by κ_0 and κ_1, respectively. From the relationship of material density, bulk modulus and sound speed,

$$\begin{aligned} c_0^2 &= \frac{\kappa_0}{\rho_0}, \\ c_1^2 &= \frac{\kappa_1}{\rho_1}, \end{aligned}$$

we have

$$\kappa_1 = \kappa_0(1+M_a). \tag{4}$$

The cloaking problem is thus described as follows. The object coated with the cloaking layers is located in the stationary medium. The radius of the object is denoted as R_1, which is also the inner radius of the cloak layer. The outer radius of the cloak is R_2. The outer boundary of the cloak and the equivalent stationary medium are coinciding. This means that equivalent stationary medium is just a bridge of the uniform mean flow and the cloak layers and it has no necessity to be demanded in the cloaking design. To achieve an efficient acoustic cloak, metamaterial with anisotropic density is needed. The concept of anisotropic density means that the same point, the effect of the density to the behavior of the sound is different in various directions. A practical implementation is still a challenge but is not the topic of the present research. Various implementation strategies have been proposed recently in material community.

The governing equation of the wave propagating in the cloak layer is[2,4]

$$\begin{aligned} \frac{\partial}{\partial t}p' + \kappa \nabla \cdot \vec{v}' &= 0, \\ \vec{\vec{\rho}} \frac{\partial}{\partial t}\vec{v}' + \nabla p' &= 0, \end{aligned} \tag{5}$$

where $\vec{\vec{\rho}}$ is a matrix corresponding to an anisotropic density tensor.

For simplicity, we consider the 2D axis symmetrical cloaking model in this work. The anisotropic density will be denoted as $\vec{\vec{\rho}}(r) = diag\{\rho_r, \rho_\theta\}$, which are described in the $(r-\theta)$ cylindrical coordinates. We denote the transformation function as $f(r)$ and assume it is continuous, satisfies $f(R_2) = R_2$ while $f(R_1) = R_0$. For the ideal cloak, $R_0 = 0$. According to Norris,[4] the material parameters for acoustic cloak can be written as:

$$\rho_r(r) = \frac{rf'(r)}{f(r)}\rho_1, \rho_\theta(r) = \frac{f(r)}{rf'(r)}\rho_1, \kappa(r) = \frac{r}{f(r)f'(r)}\kappa_1. \tag{6}$$

Practical implementations can be achieved by forming multi-layer structure materials.[8]

So far, the acoustic cloak has been proposed for sound propagation in the same direction as the uniform mean flow. It is easy to expand to a more general case $\vec{U}_0 = (U_0\cos\alpha, U_0\sin\alpha)$ described in the $(x-y)$ coordinate. We can introduce a new coordinate (ξ, η) in which the uniform mean flow goes along the ξ coordinate. The transformation between this two groups coordinates is

$$\begin{aligned} \xi &= x\cos\alpha + y\sin\alpha, \\ \eta &= -x\sin\alpha + y\cos\alpha. \end{aligned}$$

Since the plane wave is assumed propagates in the x-axis direction, the plane wave can be achieved in the form

$$p'(\xi, \eta, t) = Pe^{i\omega t - ik_0(\xi\cos(\alpha) - \eta\sin\alpha)/(1+M_a\cos\alpha)}$$

according to Eq. (1). From Eqs. (7) we have

$$\xi\cos\alpha - \eta\sin\alpha = x.$$

This means the plane wave can also be described in the $(x-y)$ coordinate as

$$p'(x, y, t) = Pe^{i\omega t - ik_0 x/(1+M_a\cos\alpha)}.$$

Then the wave number of the incident wave in this case is

$$k_\alpha = k_0/(1 + M_a \cos\alpha).$$

Then the material parameters of the corresponding equivalent stationary medium are given by Eqs. (6)

$$\begin{aligned} \rho_1 &= \frac{\rho_0}{1 + M_a \cos\alpha}, \\ \kappa_1 &= \kappa_0(1 + M_a \cos\alpha). \end{aligned} \tag{7}$$

For the most commonly used linear transformation function

$$f(r) = R_2(r - R_1)/(R_2 - R_1)$$

, the material parameters of the cloak layer are

$$\begin{aligned} \kappa(r) &= (\frac{R_2 - R_1}{R_2})^2 \frac{r}{r - R_1}(1 + M_a \cos\alpha)\kappa_0, \\ \rho_r(r) &= \frac{r}{r - R_1}\frac{\rho_0}{1 + M_a \cos\alpha}, \\ \rho_\theta(r) &= \frac{r - R_1}{r}\frac{\rho_0}{1 + M_a \cos\alpha}. \end{aligned} \tag{8}$$

III. Numerical Simulation

Numerical experiments are conducted to examine the performance of the proposed cloaking design in various fluid fields.

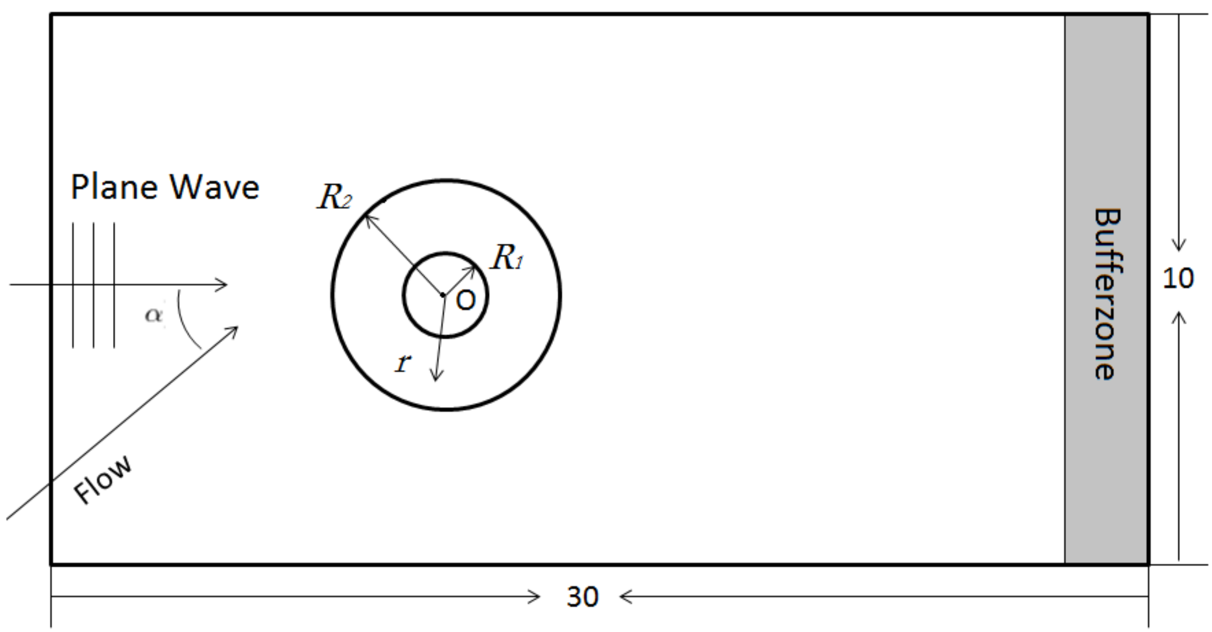

Figure 2. Schematic of the computational domain.

The computation domain is shown in Fig. 2. The plane wave propagation in the x-direction is from left to the right. The bufferzone technique[9–11] was applied to achieve the non-reflection condition. A uniform mean flow has a angle α respect with the x-direction. All the physical variables are non-dimensioned by the scales: R_1 (the length), c_0 (the velocity), R_1/c_0 (the time), ρ^* (the density), $\rho^* c_0^2$ (the pressure). Here ρ^* is the density of the background medium. The size of the computation domain is 30×10. The radius of the concealed object, which is also the inner radius of the cloak, is $R_1 = 1$. The outer radius of the cloak layer is $R_2 = 2$.

The acoustic propagates inside the cloak region is governed by the Eqs. (5). Anisotropic density materials are demanded and the parameters of such materials can be obtained according to Eqs. (8). Outside the cloak region, the acoustic propagation is governed by the linearized Euler equations (LEE)[7,10] which reads

$$\begin{aligned} \frac{\partial \rho'}{\partial t} + \nabla \cdot (\vec{v}_0 \rho' + \vec{v}' \rho_0) &= 0, \\ (\frac{\partial}{\partial t} + \vec{v}_0 \cdot \nabla)\vec{v}' + \vec{v}' \cdot \nabla \vec{v}_0 + \vec{v}_0 \cdot \nabla \vec{v}_0 \frac{\rho'}{\rho_0} + \frac{\nabla p'}{\rho_0} &= 0, \\ p' &= \rho' c_0^2, \end{aligned} \tag{9}$$

where $\rho_0, c_0, p_0, \vec{v}_0$ are the density, sound speed, pressure and velocity of the background flow. The non-dimensional value of the variables are $p_0 \sim 1$, $\rho_0 \sim 1$, $|\vec{v_0}| \sim M_a$, $c_0 \sim 1$, $p' \sim 10^{-5}$, $\rho' \sim 10^{-5}$, $|\vec{v}'| \sim 10^{-5}$, $\omega = k$. In our simulation, the amplitude of the incident plane wave is $P = 10^{-5}$. The corresponding effective pressure of the incident sound wave is $\overline{p_i'} = \frac{\sqrt{2}}{2}P$.

Figure 3 shows the result when the wave number is $k = 2$ and the Mach number of the uniform mean flow is $M_a = 0.06$.

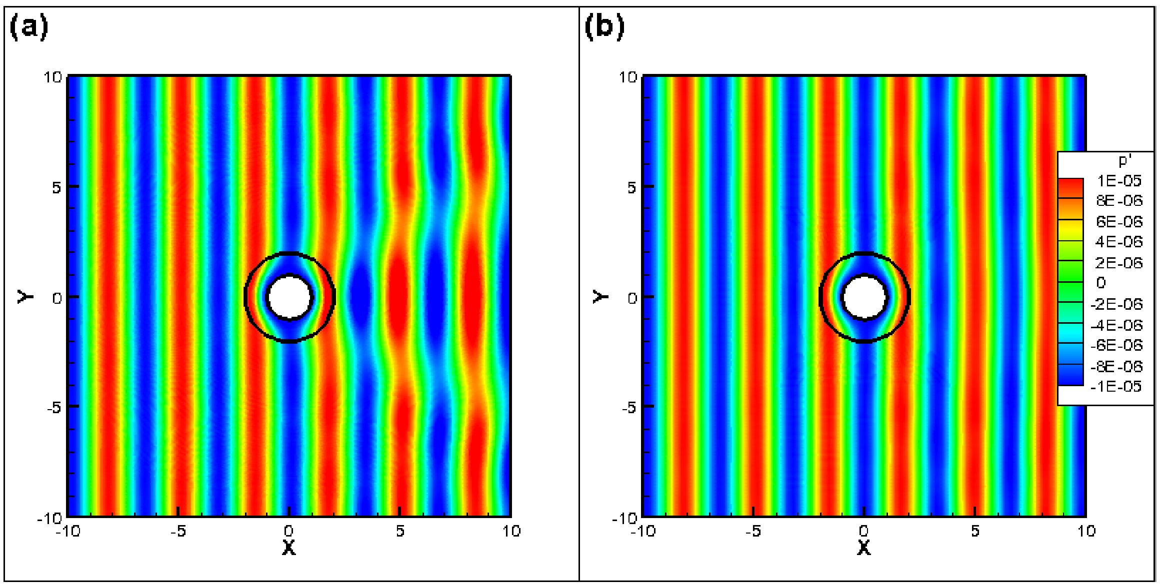

Figure 3. The sound field distribution in the mean flow at a specific time with different cloaks. (a) the classical cloak; (b) the proposed cloak.

According to Fig. 3(b), the proposed works well in the presence of the idealized uniform mean flow, suppressing the shadows in the case of the classical cloak (see Fig. 3(a)). For the classical cloaking design, the sound wave propagates through the cloak region with a time delay that leads to the shadows. The proposed cloaking design compensates the delay.

However, the real background flow is affected by the cloak and is not uniform in the spatial domain. Numerical simulations are conducted to examine the performance of the proposed method in the presence of the real flow field.

Figure 4 shows the schematic of the numerical experiments. The entire numerical setup is similar to that in the mean flow. A measurement surface is introduced to estimate the overall shadow at far field. The surface is located at $10R_1$ from the origin (the center of the cloaked region). The measurement angles are from -60deg to 60deg to estimate the acoustic shadow. At very low Reynolds numbers, the flow can be simply regarded as a potential flow with analytical solutions.

$$\begin{aligned} u_{0x}(x,y) &= |\vec{U}_0|[(x^2+y^2)^2 - (x^2-y^2)R_3^2]/(x^2+y^2)^2, \\ u_{0y}(x,y) &= -|\vec{U}_0|2xyR_3^2/(x^2+y^2)^2, \\ p_0(x,y) &= p_\infty + \rho_0(|\vec{U}_0|^2 - |\vec{u}_0|^2)/2, \end{aligned} \tag{10}$$

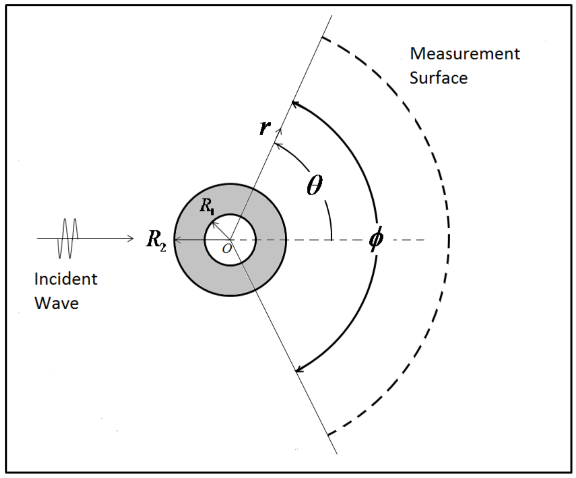

Figure 4. Schematic of the numerical experiments to study the effectiveness of the cloaks in presence of real fluid fields (not to scale).

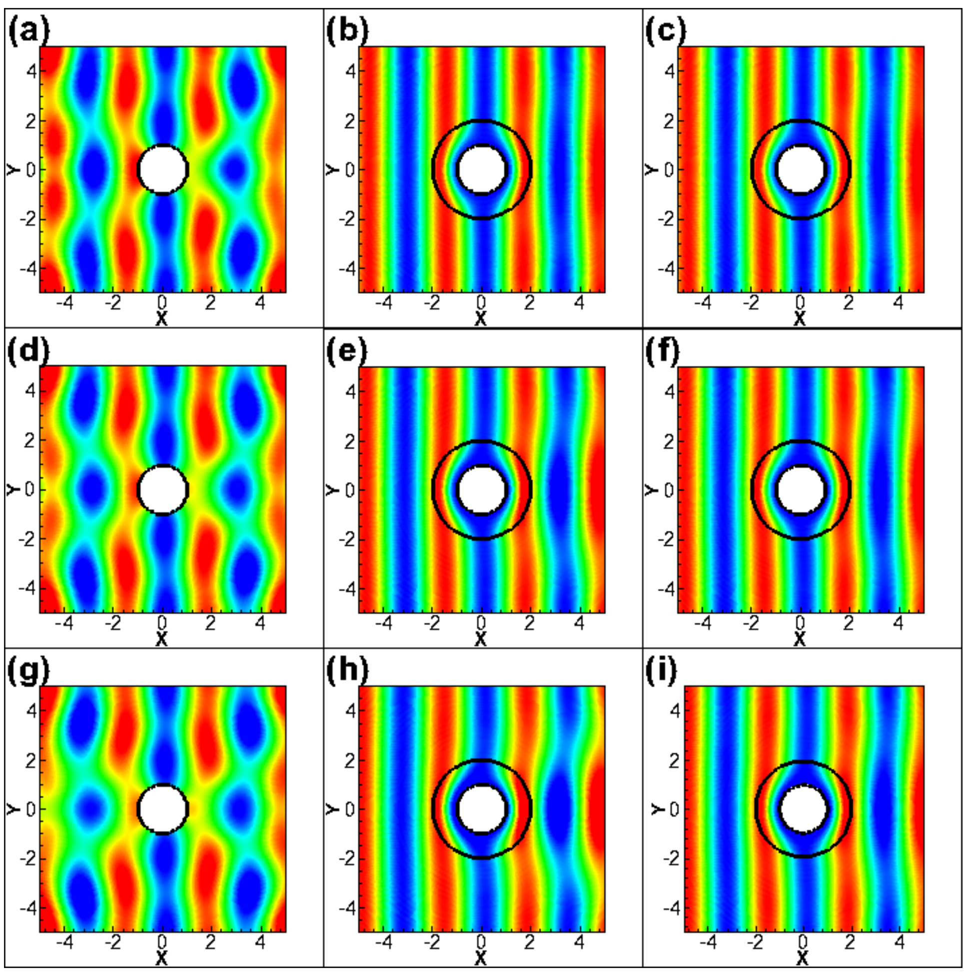

Figure 5. The near field sound pressure distribution displayed in 11 contour levels between -10^{-5} to 10^{-5}. (a-c),$M_a = 0$; (d-f), $M_a = 0.03$; and (g-i), $M_a = 0.06$. (a,d,g) no cloak; (b,e,h) the classical cloak; and (c,f,i) the proposed new cloak.

where R_3 is the radius of the cylinder. $R_3 = R_2$ if a cloak is deployed and otherwise $R_3 = R_1$.

Figure 5 shows the near field sound pressure distribution at the same specific time. The Mach number is 0, 0.03, 0.06, respectively. The background fluid fields local to the cylindrical object and the cloaked region are given by using theoretical solutions of potential flow. Numerical results of sound fields without cloak, with the classical cloak and the proposed new cloak are compared. It can be seen that the both cloaking designs completely suppress the scattering in Figs. 5(a,d,g). In addition, the proposed new cloaking design largely reduces the acoustic shadows in the downstream regions.

The following criterion is defined for those test points at the far field measurement surface

$$\mathrm{err} = 20\log_{10}\left(\frac{\overline{p'}}{\overline{p'_i}}\right) \tag{11}$$

to quantitatively examine the acoustic shadow. It can demonstrate the performance of the proposed new cloak. This function shows the difference between the effective pressure of the real sound field ($\overline{p'}$) and the incident wave ($\overline{p'}_i$) at each point on the surface. If no obstacle exits, the value is identically equal to 0 dB at each measurement point. In addition, err > 0 means that the effective pressure of the real pressure field is stronger than that of the incident wave and the vice versa. Figure 6 shows this error function of the cloaks in different flows. When there is no flow ($M_a = 0$), it is understandable that the effects of the two cloaks are identical. When fluid fields exist (see Figs. 6(b-c)), it can be seen that the proposed new cloaking design helps to reduce the acoustic shadow.

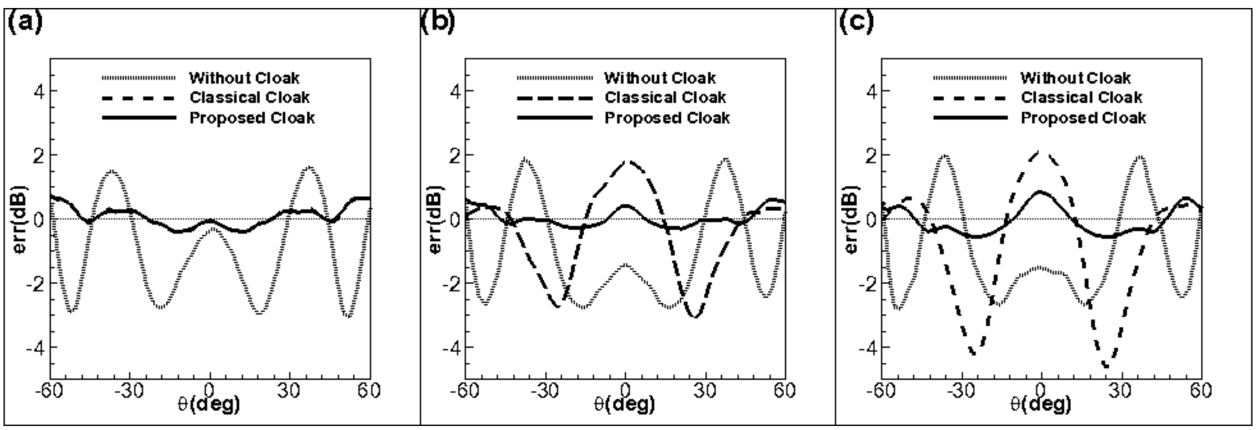

Figure 6. Err of the potential flows at various Mach numbers. (a), $M_a = 0$; **(b),** $M_a = 0.03$; **(c),** $M_a = 0.06$.

To further evaluate the performance of the two cloaking designs at different flow conditions, we define

$$\mathrm{err_T} = 100\frac{\sqrt{\sum_i^N(\overline{p'} - \overline{p'_i})^2/N}}{\overline{p'_i}}. \tag{12}$$

Where N is the number of the measurement points on the surface. This definition function examines the entire sound field difference of the cloak. Figure 7 shows that the proposed new cloaking design can reduce the acoustic shadow at various flow speeds up to $M_a = 0.1$.

The above potential flow is still a simplified approximation of the real flow field. We then calculated the flow field using computational fluid dynamic (CFD) tool with an incompressible laminar flow model. Details of the computational setup are omitted here for brevity. The frequency of the acoustic wave is generally much larger than flow fluctuation frequencies. Then, various instantaneous flow fields are calculated and then taken as the background flow field in the following acoustic calculation. Figure 8 shows some instantaneous flow fields.

Then, we can repeat the same analysis as the potential flow case. Figure 9 is the near field of the sound pressure distribution with different cloaks and Mach numbers. And the corresponding err and $\mathrm{err_T}$ are shown in Figs. 10-11, respectively. It can be seen that the proposed new cloaking design still works in terms of shadows suppression.

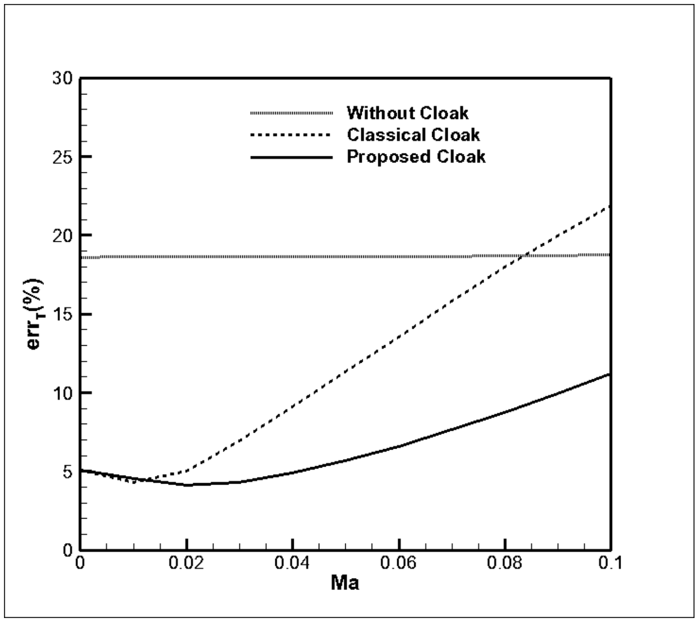

Figure 7. $\mathrm{Err_T}$ **of the potential flows at various Mach numbers.**

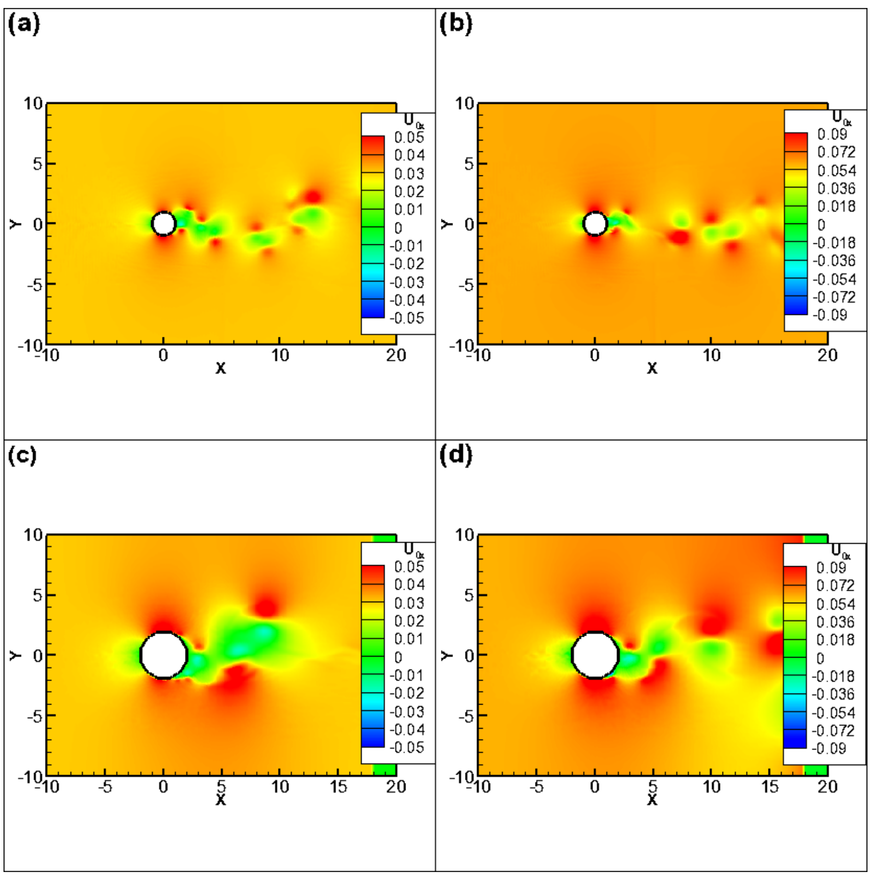

Figure 8. The velocity field in x-direction. (a), $M_a = 0.03, R_1 = 1$, flow around the obstacle; (b), $M_a = 0.06, R_1 = 1$, flow around the obstacle; (c), $M_a = 0.03, R_1 = 2$, flow around the cloak; (d), $M_a = 0.06, R_2 = 2$, flow around the cloak.

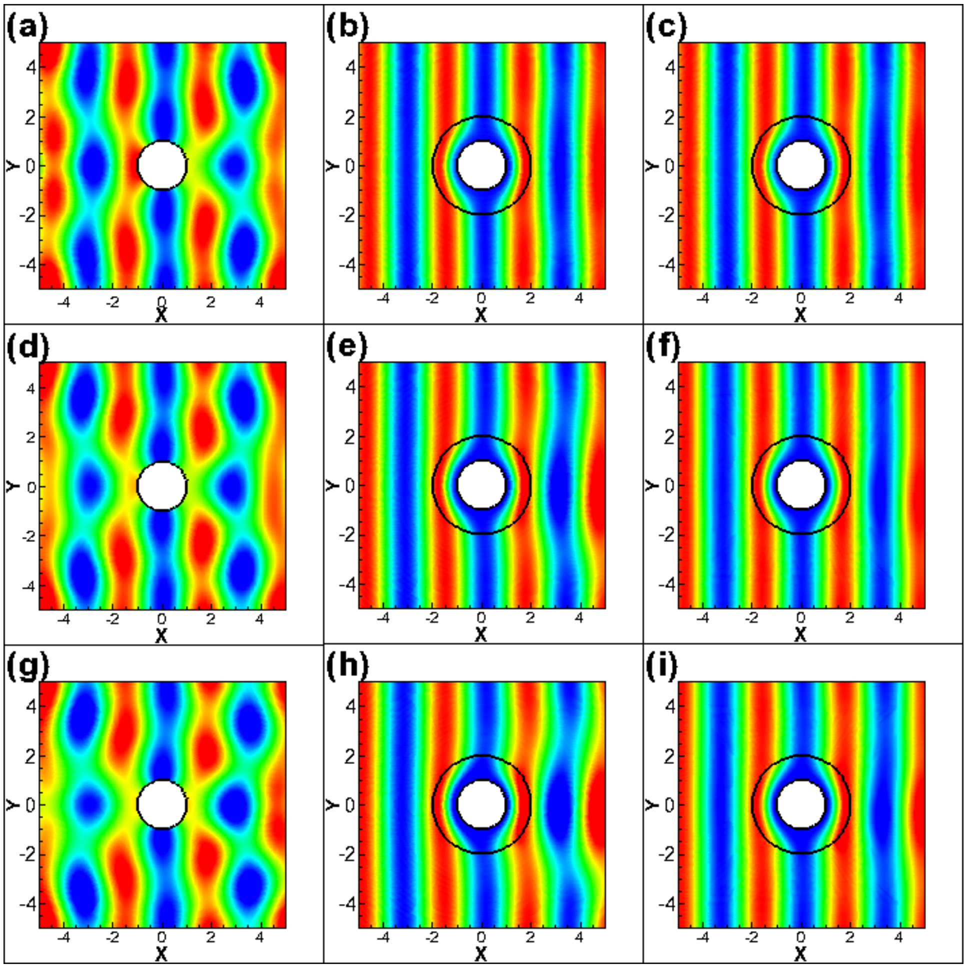

Figure 9. The near field sound pressure distribution displayed in 11 contour levels between -10^{-5} to 10^{-5} in the incompressible flow cases. (a-c),$M_a = 0$; (d-f), $M_a = 0.03$; and (g-i), $M_a = 0.06$. (a,d,g) no cloak; (b,e,h) the classical cloak; and (c,f,i) the proposed new cloak.

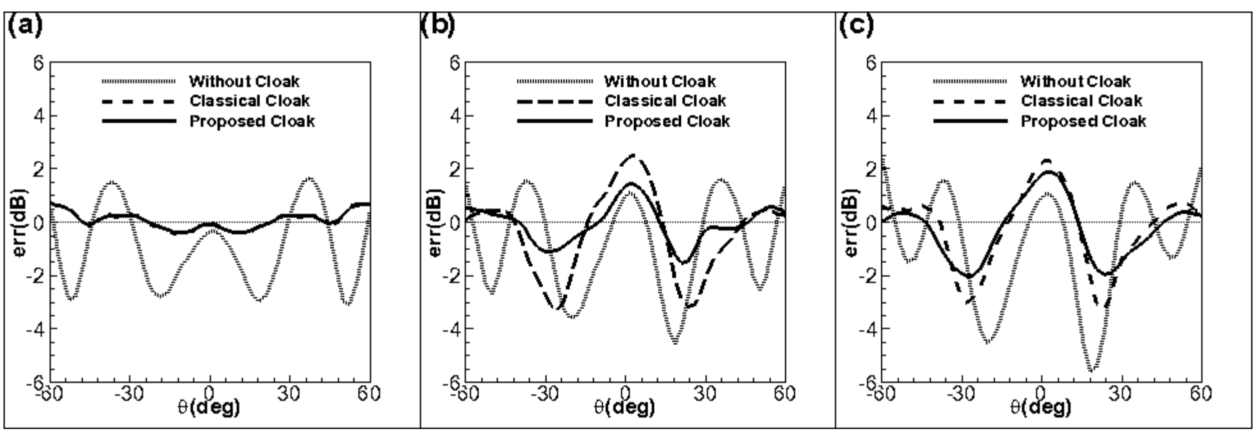

Figure 10. Err of the incompressible flows at various Mach numbers. (a), $M_a = 0$; (b), $M_a = 0.03$; (c), $M_a = 0.06$.

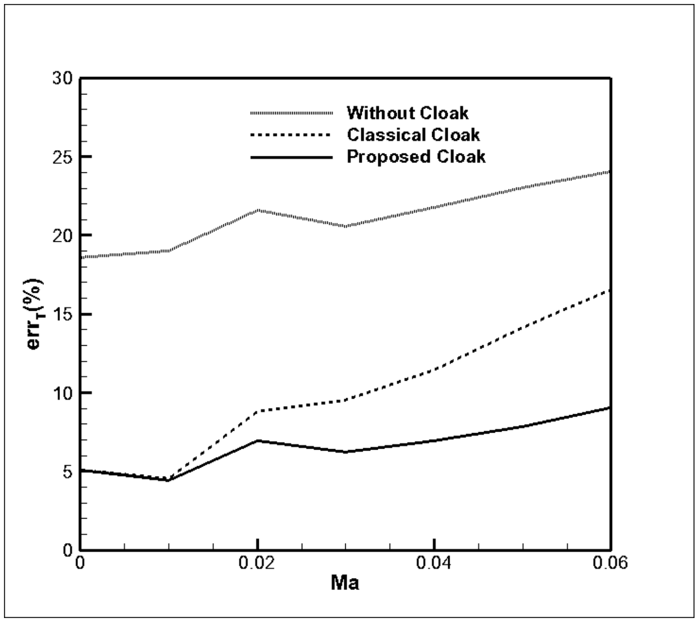

Figure 11. $\mathrm{Err_T}$ **of the potential flows at various Mach numbers.**

IV. Summary

A new design of an acoustic cloak has been proposed in this work to take account of the uniform mean flow field effect. The design details have been presented and the performance has been examined in various numerical studies. The formulation has been developed for the idealized uniform mean flow. However, the numerical studies suggest that the proposed cloak is still effective for potential flow and incompressible flow cases. For now the Mach number is low. We are still working on numerical cases with higher Mach number. The work is ongoing and will be presented in the follow-up paper. It is worthwhile to admit that the cloaking structure might induce turbulence in a flow and subsequently generate flow-induced sound. This element is not considered in the present studies.

ACKNOWLEDGMENTS

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