

# 湍 流

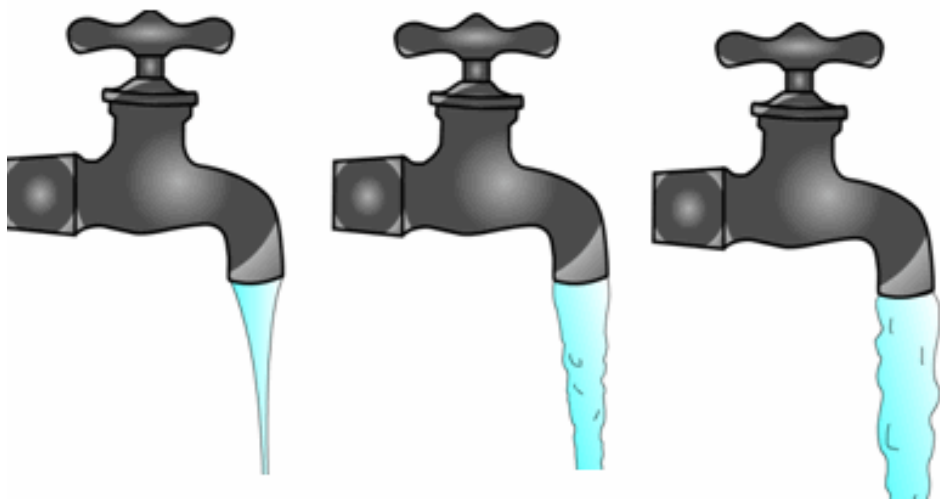
## 湍流概述

杨 越

力学与工程科学系  
北京大学工学院

# 什么是湍流 (Turbulence) ?

- 学术界并没有严格的定义
- 湍流是三维空间的**不规则非定常**运动
- 湍流会产生许多**不同长度尺度的涡**



随着流量增大（雷诺数增大），龙头下的水流逐渐由稳定的层流转变为不稳定的湍流



湍流射流流动显示  
(K. R. Sreenivasan, 1999)

# 自然界中的湍流

- 自然界中湍流无处不在，涵盖由毫米到光年的特征尺度范围



宇宙中的M100星系  
特征尺度 $\approx 10^{23}$  m



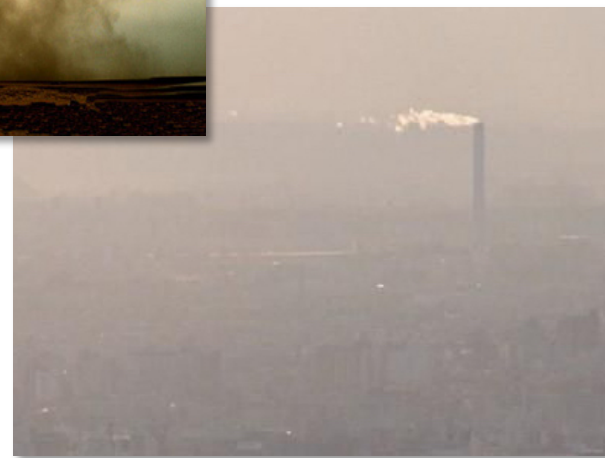
地球大气层  
特征尺度 $\approx 10^7$  m



实验室中的肥皂膜  
特征尺度 $\approx 10^{-1}$  m

# 工程应用中的湍流

- 湍流会大大**加快**流动中的**动量与能量交换**，提高混合效果
- 显著影响阻力变化，物质扩散效率，燃烧过程速率



湍流是我国航空航天**的“卡脖子”问题**。 -庄逢甘院士



# 艺术作品中的湍流

- 艺术家借湍流描绘自然中的壮观现象或精神中的情绪波动



*The Great Wave* (神奈川的巨浪)  
Katsushika Hokusai, 1830



*Starry Night* (星夜)  
Vincent van Gogh, 1889

# 娱乐作品中的湍流

- 电影与视频游戏制作人通过湍流与火焰、烟雾、沙尘、雨雪的相互作用，表现复杂的场景变化与绚丽的魔法特效



*Uncharted 3*  
©Sony Computer Entertainment, 2011



*Frozen (冰雪奇缘)*  
©Disney, 2013



# 课程简介

- 课程编号：08611830
- **先修课程：流体力学**
- 授课对象：流体力学、航空航天、能源环境等专业
- 课程学分：3
- 上课时间：双周周二3-4节，周四3-4节
- 上课地点：三教506

# 课程网站

## ■ 北京大学工学院网站>>教职员工>>杨越>>教学

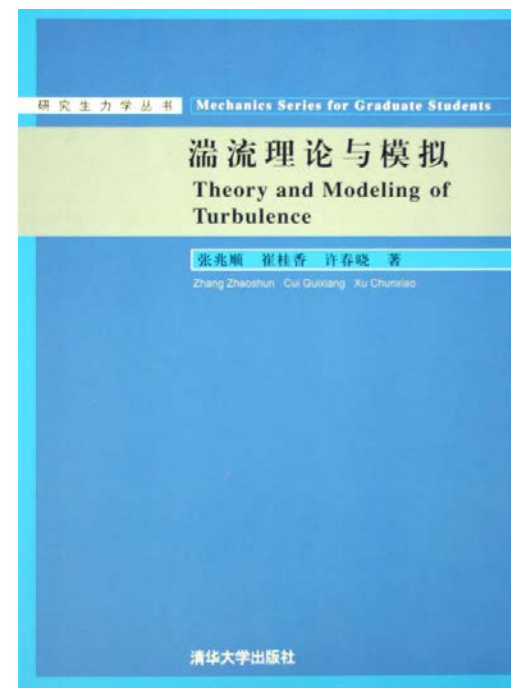
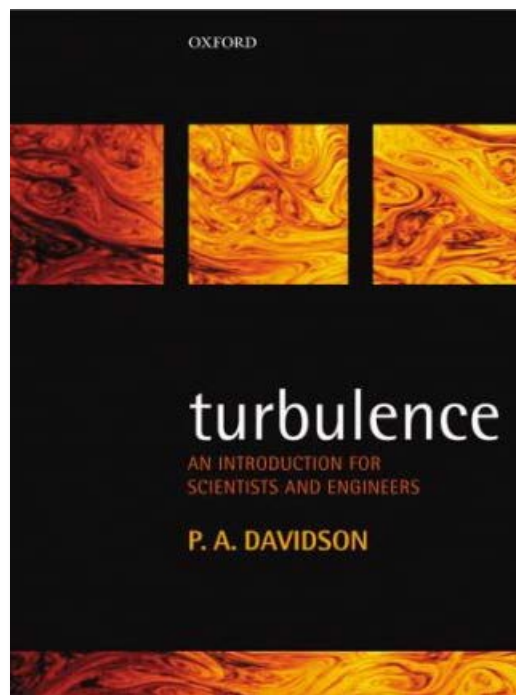
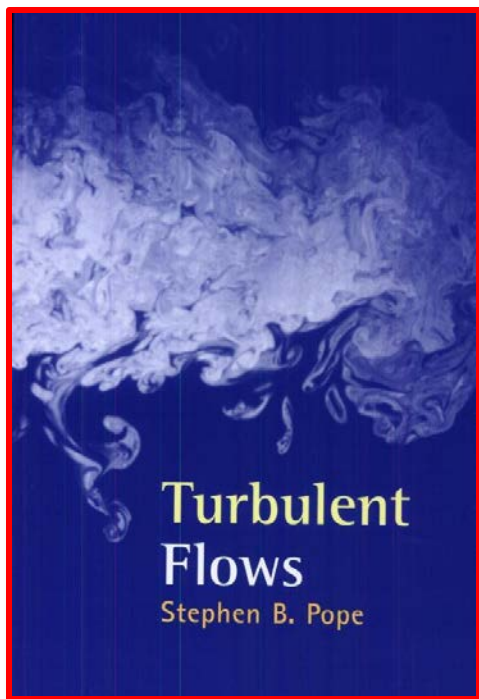
<http://www.coe.pku.edu.cn/subpaget.asp?id=500>





# 参考书目

- S. B. Pope, *Turbulent flows*, Cambridge, 2000  
主要教材，世界图书出版公司影印版，2010
- P. A. Davison, *Turbulence: An introduction for scientist and engineers*, Oxford, 2004
- 张兆顺, 崔桂香, 许春晓, 湍流理论与模拟, 清华大学出版社, 2005

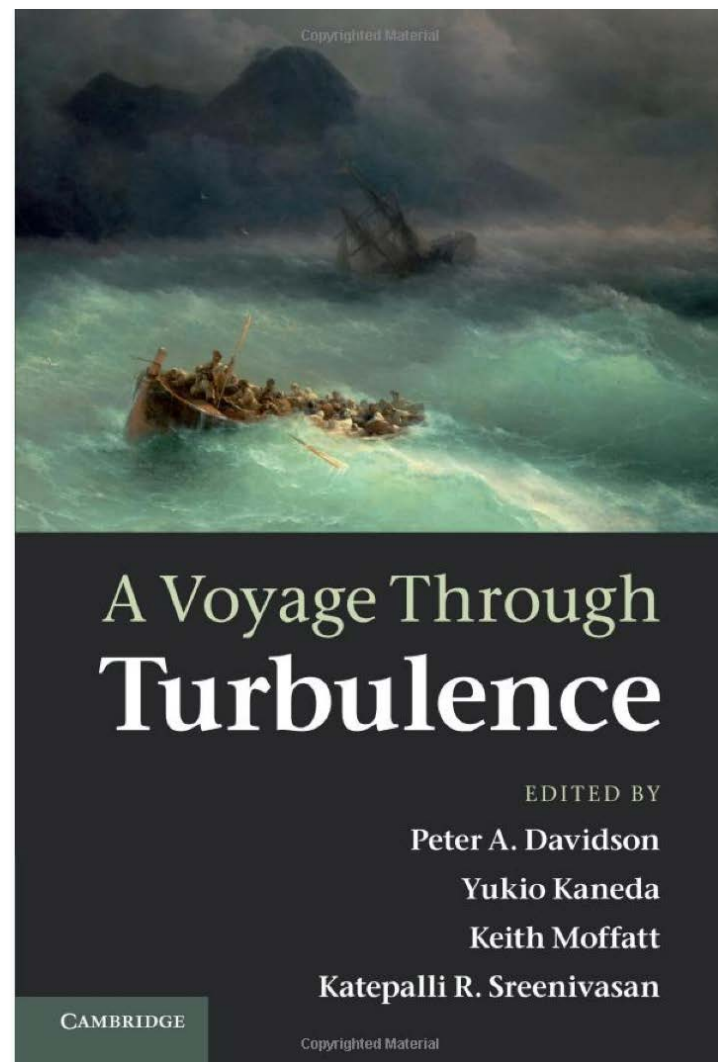


# 课程评价方法

- 作业共7次，内容包含理论推导习题与MATLAB数值编程
- 每两周布置一次，**每次权重10%，共70%**
- 可以进行2-4人小组讨论，每人必须**独立撰写作业报告**
- 逢双周**周二课前提交作业**（首次提交作业时间：3月11日）
- 周二课上会讲解作业答案
- 每次作业可提前两周在课程网站下载
- **期末考试（开卷）**，主要基于作业内容，**权重30%**

# 湍流研究回顾

- 历史跨度长，距今约一百五十年
- 名家云集，包括数学、力学、物理、工程等领域，汇集人类智慧精华
  - G. I. Taylor，英国剑桥学派
  - L. Prandtl，德国哥廷根学派
  - T. von Karman，美国加州理工
  - A. N. Kolmogorov，俄罗斯学派
  - 周培源，中国北京大学
- 各种科学理念之间的交叉与碰撞

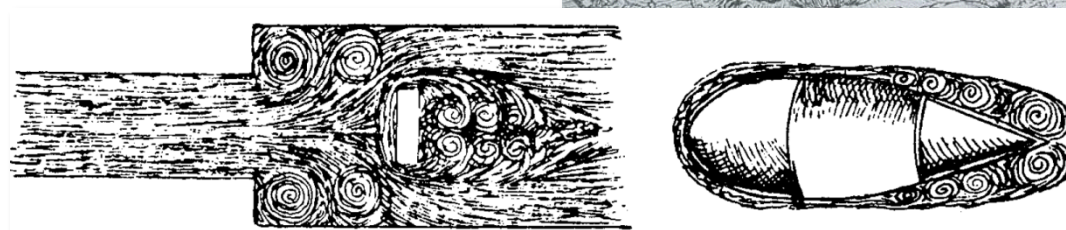


# 16世纪 湍流多尺度性质的描绘

- 达芬奇观察到出水口附近与钝体绕流后的水流由大大小小的旋涡组成，定性诠释了湍流运动的复杂性



Leonardo da Vinci  
艺术家，工程师





# 18-19世纪 流体运动的数学描述

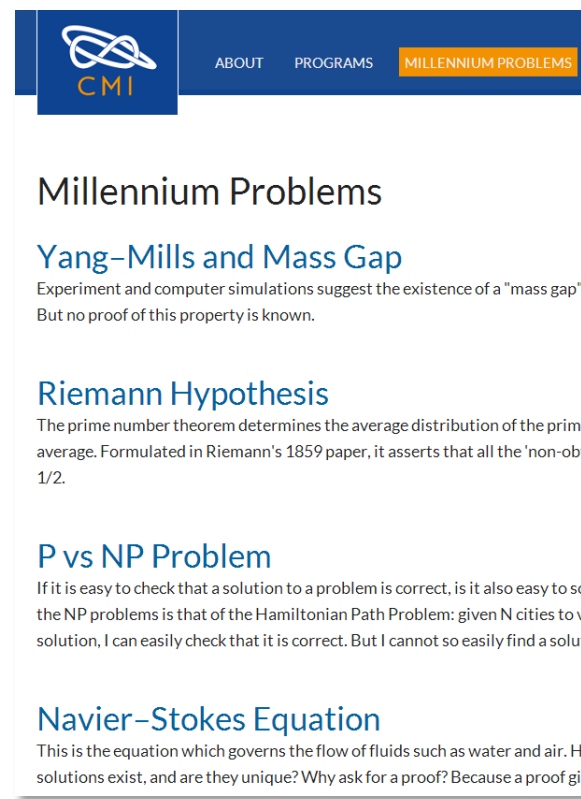
- 流体运动的微分方程描述：Euler方程与Navier-Stokes方程
- 目前Navier-Stokes方程理论解的存在性与光滑性尚未证明，被美国Clay数学研究所列为新千年七大数学难题之一



Leonhard Euler  
数学家，物理学家



George G. Stokes  
数学家，物理学家

A screenshot of the Clay Mathematics Institute (CMI) website showing the Millennium Problems. The page has a blue header with the CMI logo and navigation links: ABOUT, PROGRAMS, and MILLENNIUM PROBLEMS. The main content lists several problems: Yang-Mills and Mass Gap, Riemann Hypothesis, P vs NP Problem, and Navier-Stokes Equation. Each problem has a brief description. The Navier-Stokes Equation problem is highlighted in orange.

**Millennium Problems**

**Yang-Mills and Mass Gap**  
Experiment and computer simulations suggest the existence of a "mass gap". But no proof of this property is known.

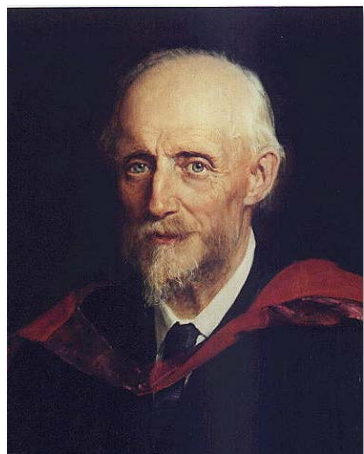
**Riemann Hypothesis**  
The prime number theorem determines the average distribution of the prime average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function lie on the critical line  $\sigma = 1/2$ .

**P vs NP Problem**  
If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? The NP problems is that of the Hamiltonian Path Problem: given N cities to visit, I can easily check that it is correct. But I cannot so easily find a solution.

**Navier-Stokes Equation**  
This is the equation which governs the flow of fluids such as water and air. How do solutions exist, and are they unique? Why ask for a proof? Because a proof gives us confidence that the equation is correct.

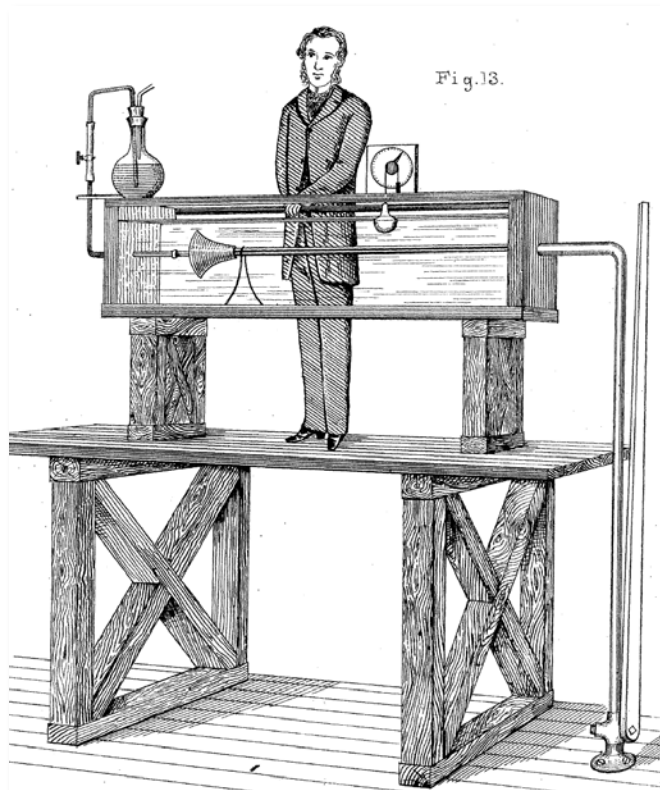
# 1883 层流-湍流转捩实验

## ■ 圆管流动中发现湍流运动，雷诺数定义，雷诺平均方程



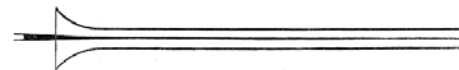
Osborne Reynolds  
物理学家

*Phil. Trans. Roy. Soc.*, 174 : 935-982, 1883



(1.) When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube, fig. 3.

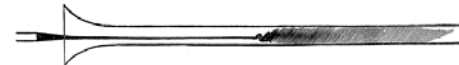
Fig. 3.



(2.) If the water in the tank had not quite settled to rest, at sufficiently low velocities, the streak would shift about the tube, but there was no appearance of sinuosity.

(3.) As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from the trumpet or intake, the colour band would all at once mix up with the surrounding water, and fill the rest of the tube with a mass of coloured water, as in fig. 4.

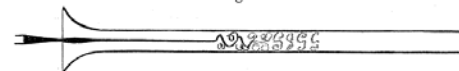
Fig. 4.



Any increase in the velocity caused the point of break down to approach the trumpet, but with no velocities that were tried did it reach this.

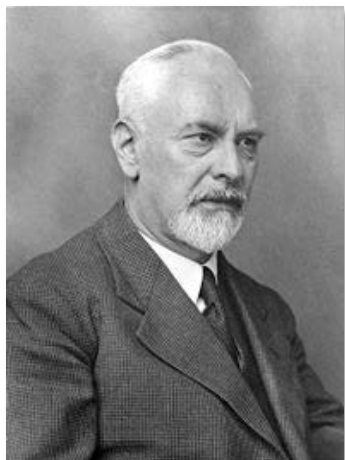
On viewing the tube by the light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies, as in fig. 5.

Fig. 5.

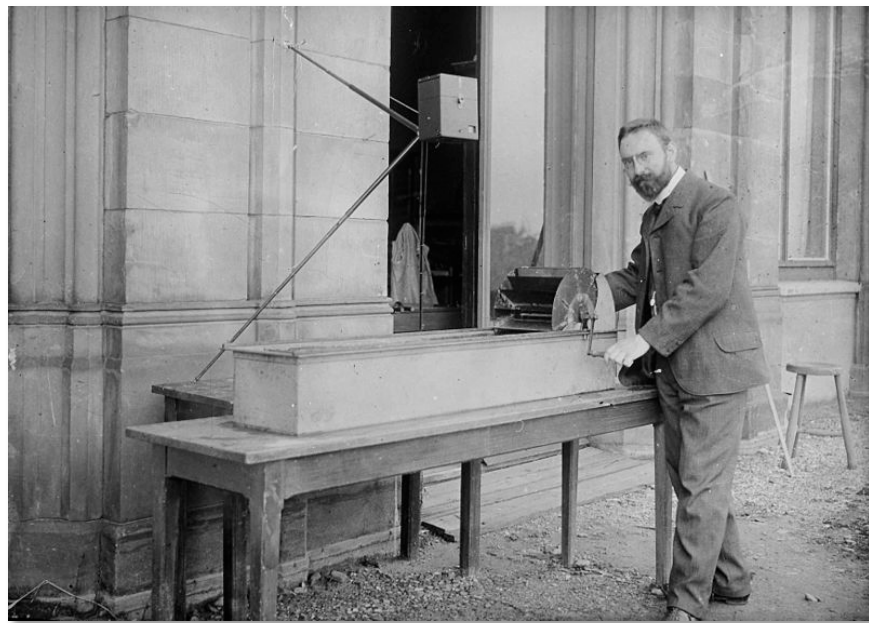


# 1904 边界层概念的提出

- 1904年Prandtl提出边界层理论
- 流体力学研究对象由早期无粘理想流动转向真实粘性流动
- 1925年Prandtl提出混合长理论，成为最早的湍流模型



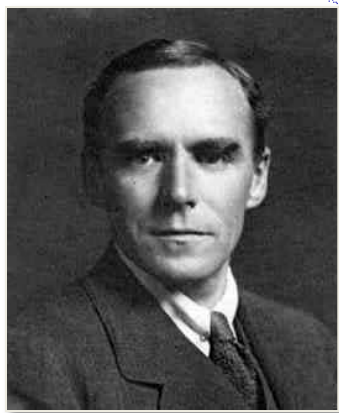
Ludwig Prandtl  
力学家，航空工程师  
近代流体力学奠基人



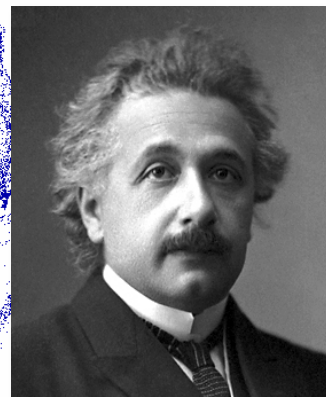
Prandtl与水洞实验台，1904

# 1921 概率统计方法引入湍流研究

- 20世纪前期，概率统计、随机过程理论引入物理与工程研究
- Taylor将湍流中流体微团运动视为随机过程，提出自相关函数



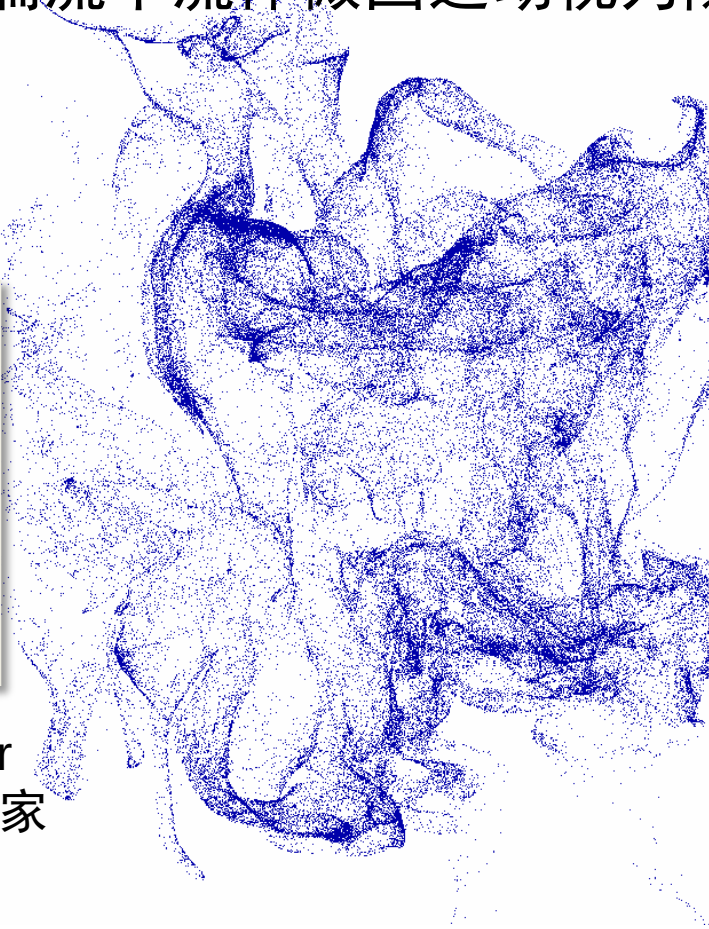
Geoffrey I. Taylor  
力学家，应用数学家



Albert Einstein  
物理学家  
布朗运动数学描述



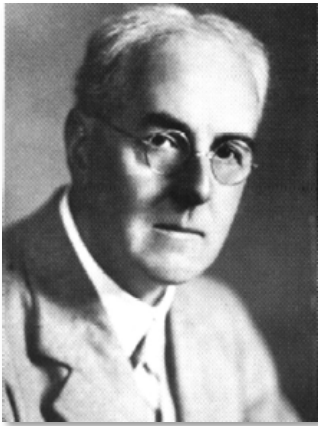
Norbert Wiener  
应用数学家  
随机过程研究先驱



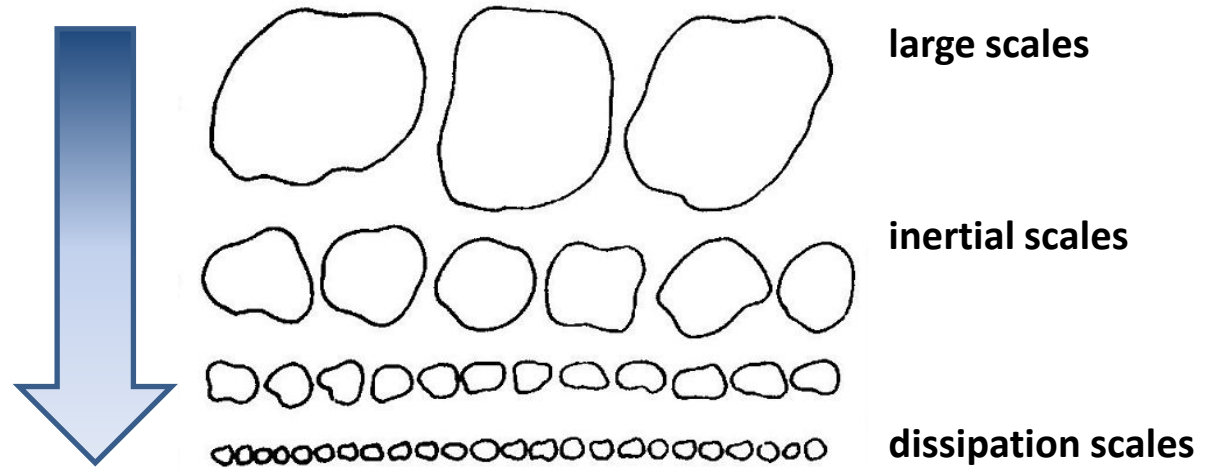


# 1922 湍流中尺度集串概念的提出

- Richardson的小诗描述了在湍流中能量由大尺度运动向小尺度运动的传递过程



Lewis Fry Richardson  
气象学家



*Big whorls have little whorls,  
Which feed on their velocity,  
And little whorls have lesser whorls,  
And so on to viscosity.*

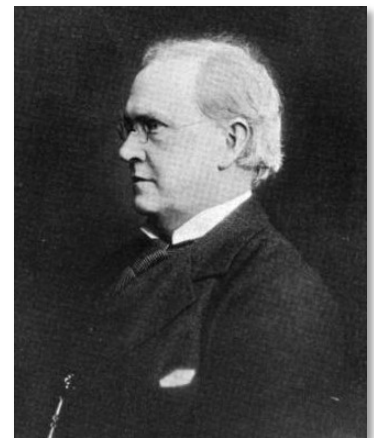
# 1930s-1940s 湍流理论研究的兴起

It was at a meeting of the British Association in London in 1932 that I remember that Lamb remarked “I am an old man now, and when I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am really rather optimistic.” (I have quoted from memory, so do not guarantee all the actual words. But the sense is correct. I have heard a similar story since repeated with other names than Lamb and other times and places.) Lamb was correct on two scores. All who knew him agreed that it was Heaven that he would go to, and he was right to be more optimistic about quantum electrodynamics than turbulence.

S. Goldstein, *Annu. Rev. Fluid Mech.*, 1:1-29, 1969

“我死后希望在天堂能得到两个问题的启示，一个是量子电动力学，另一个是流体湍流。我对前一个问题（量子电动力学）相当乐观。”

Horace Lamb  
物理学家，应用数学家



# 1930 壁湍流中的对数律

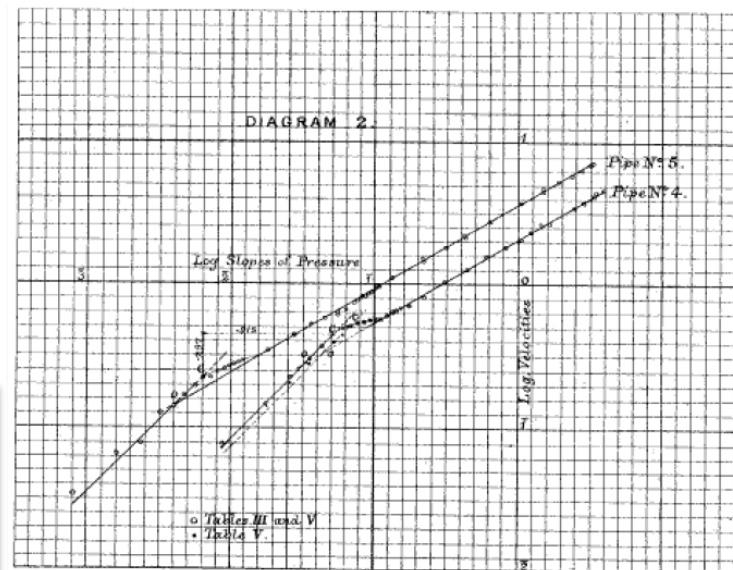
## ■ 二十世纪前期航空工业的发展推动湍流研究



Theodore von Kármán  
力学家，航空工程师  
获美国首届国家科学奖章



空气动力学师徒三代合影  
Prandtl, von Kármán, 钱学森



发现壁湍流平均速度剖面分布律

# 1935 湍流统计理论

- 1935-1938年G. I. Taylor发表数篇原创性论文
- 引入关联函数与傅里叶分析等研究方法
- 开创湍流统计理论，将湍流研究由定性研究提升为**定量研究**

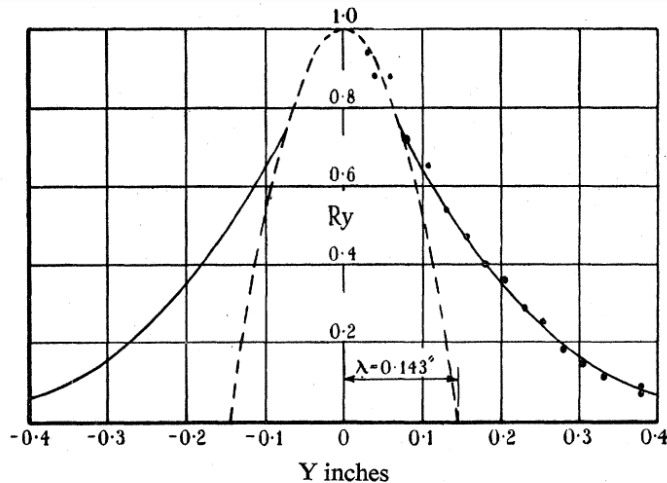


FIG. 1—Measured values of  $R_y = \overline{u_0 u_y} / \overline{u^2}$  behind 0.9 inch by 0.9 inch honeycomb;  $\overline{u^2} = 0.1015 \text{ (ft sec)}^2$

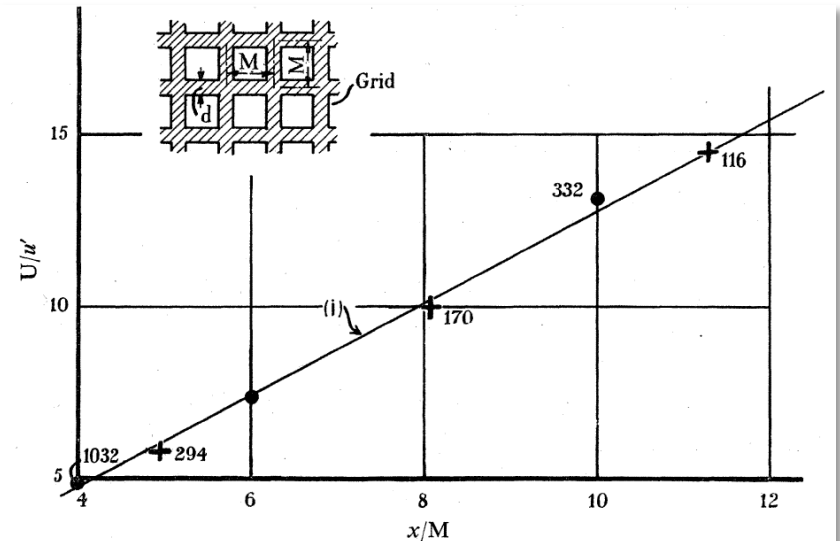


FIG. 2—Decay turbulence behind grid of flat strips; ● = 1.5 inches;  $d = 0.5$  inches; +  $M = 0.62$  inches; (i)  $U/u' = -0.7 + 1.32 x/M$

湍流风洞实验数据与统计理论预测对比  
G. I. Taylor, *Proc. Roy. Soc. Lond. A*, 873:444-454, 1935



# 1939 湍流封闭问题

- 雷诺平均方程的封闭问题
- 准高斯假设与湍流模式理论
- 后被广泛用于湍流计算模型



M. D. Millionschikov  
物理学家



周培源  
物理学家，力学家

“这一代人中至少有四位来自四个国家的流体力学巨匠。我想到的有美国的von Karman，英国的G. I. Taylor，苏联的Kolmogorov，还有中国的周培源。”

J. L. Lumley, *Annu. Rev. Fluid Mech.*, 27:1-15, 1995

At the beginning of Chou's career, Chinese science was hamstrung by the complexity of the Chinese language and the problems it created for typing, printing, and ultimately for computation and electronic communication. It was Chou who saw the tremendous significance of the Computerized Editing and Laser Typesetting System for Chinese Characters, and gave from the very beginning the strongest possible support to the research team led by Professor Xuan Wang at Peking University, paving the way for the success of this research project—a project that has finally brought about a revolution in today's printing industry in China.

This short introduction gives just a hint of the many phases of Chou's long and productive career. We do not have space here to do more than mention that he was also responsible for research and critical evaluation of resource exploitation and ecological protection in southwestern China. He hosted vast numbers of foreign guests, and on many occasions represented China at international conferences and activities for world peace.

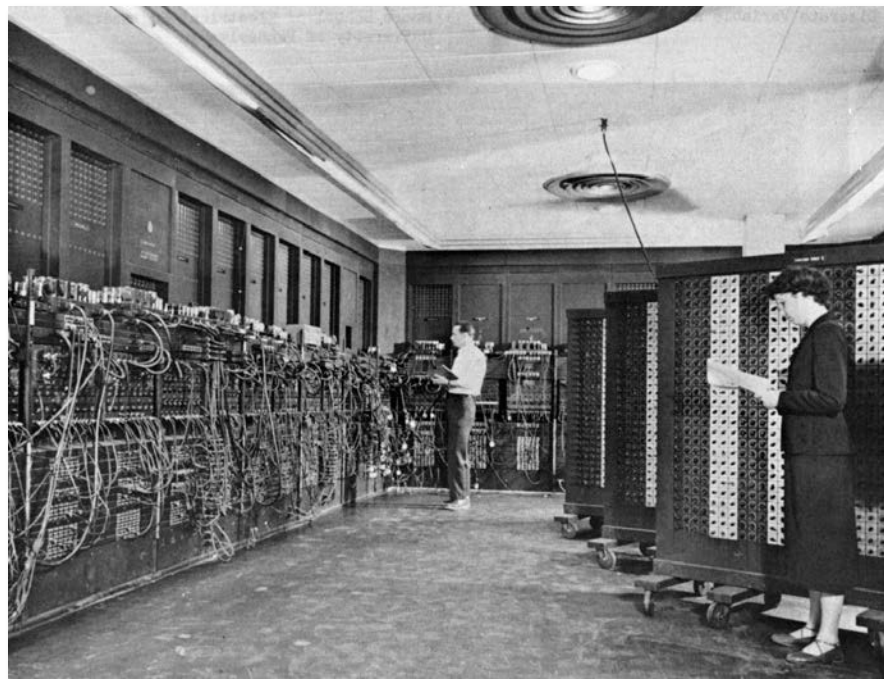
In this generation there were at least four giants in fluid mechanics from four countries; men who had, each in his own way, tremendous influence inside and outside his country, certainly by contributing to the development of areas of fluid mechanics, but also by providing intellectual and personal leadership; men of such stature that substantial numbers of distinguished workers in fluid mechanics in each country could trace their academic lineage to that great man. I have in mind von Karman in the United States, Kolmogorov in the (then) Soviet Union, G. I. Taylor in the United Kingdom, and Pei-Yuan Chou of China.

# 1940s 电子计算机的发展

- 由于三维Navier-Stokes方程的数值求解的可能性，冯诺依曼预言数字计算机会带来湍流研究的革命



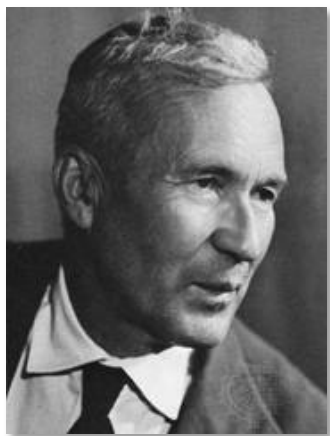
John von Neumann  
数学家，现代计算机创始人之一



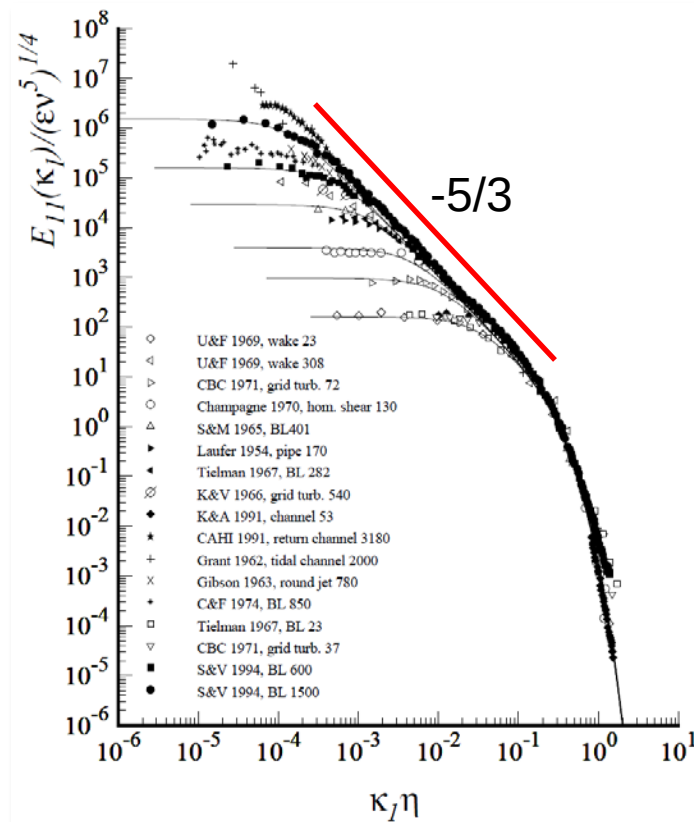
ENIAC，世界上第一台通用电子计算机  
宾西法尼亚大学，1946

# 1941 湍流能谱标度律

- 尺度假设和量纲分析
- 湍流能量谱与小尺度运动的普适规律
- 最重要的湍流理论成果之一（K41）
- 主要结论后期为大量实验结果验证



A. N. Kolmogorov  
数学家，建立概率公理化体系

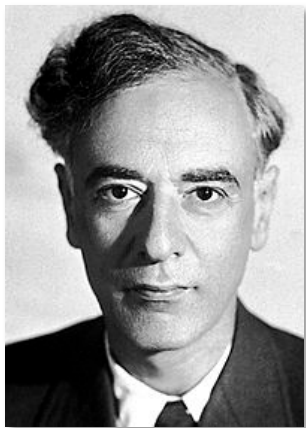


K41理论中-5/3能谱的实验验证  
S. B. Pope, *Turbulent flows*  
Cambridge, 2000

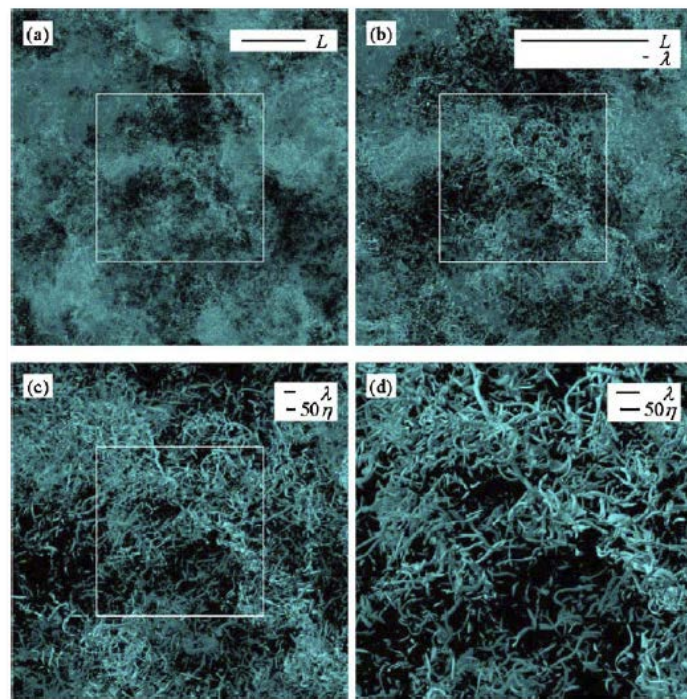


# 1944 K41理论的质疑

- K41理论假设与普适性受到Landau等学者质疑
- Landau认为瞬时湍动能耗散在时空中分布极不均匀



Lev D. Landau  
物理学家，诺贝尔奖得主



数值模拟发现湍流中分布不均的涡结构  
(Kaneda & Ishihara, 2006)



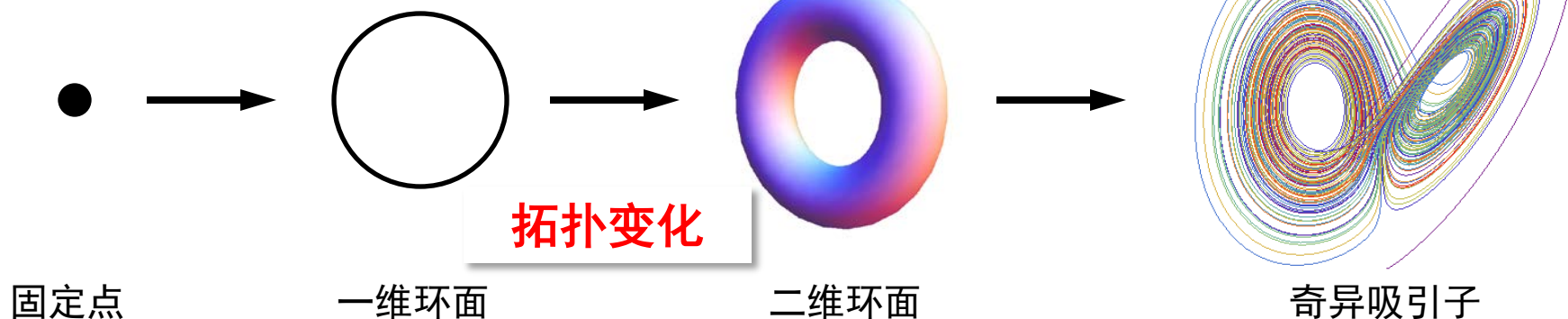
# 1944 分叉理论

- Landau(1944), Hopf(1948), Ruelle-Takens-Newhouse(1978)
- Landau-Hopf 理论给出层流-湍流转捩一种可能的动力学路径



Heinz Hopf  
数学家

动力系统观点下的层流-湍流转捩过程



# 1947 K41理论在西方国家的传播

- Batchelor由英译俄文文献发现Kolmogorov在1941年的理论
- 同时代Heisenberg、Weizsacker、Onsager得出类似结论

No. 4024 December 14, 1946 NATURE 883

**DOUBLE VELOCITY CORRELATION FUNCTION IN TURBULENT MOTION\***

By G. K. BATCHELOR  
Trinity College, Cambridge

dissipation per unit mass of the fluid ( $\epsilon$ ). The second asserts that the statistical characteristics of the motion due to the larger of the eddies contained within

\* A contribution to the Turbulence Symposium at the Sixth International Congress for Applied Mechanics held in Paris during September 22-29.

correlations between velocity differences. The second



Werner Heisenberg  
物理学家，诺贝尔奖得主



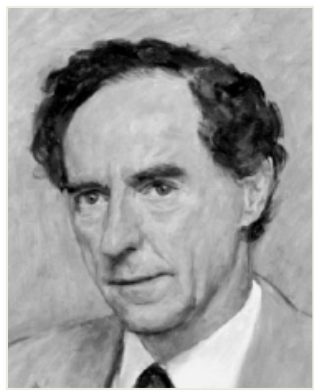
C. von Weizsacker  
物理学家



Lars Onsager  
物理化学家，诺贝尔奖得主

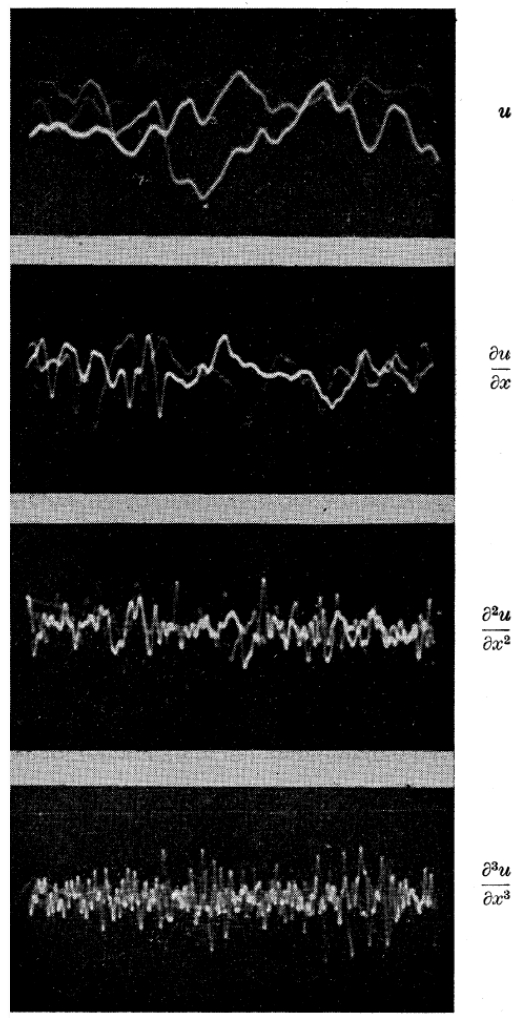
# 1949 湍流小尺度间歇性

- 发现湍流速度信号中的间歇性，指出K41理论在高阶统计量预测的缺陷



George K. Batchelor  
力学家，创办流体力学杂志*JFM*

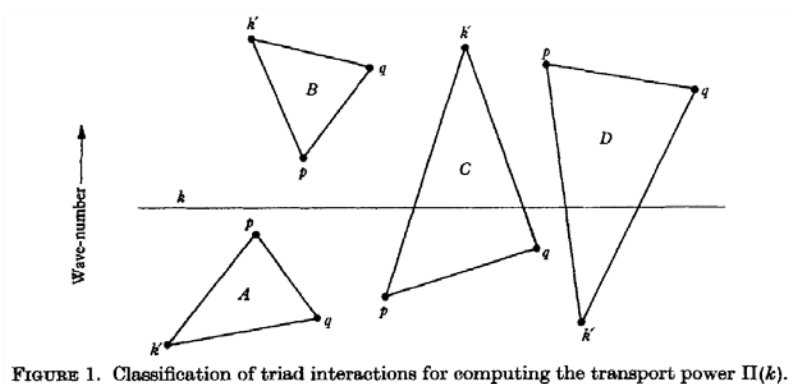
剑桥学派师徒合影  
G. I. Taylor与Batchelor



实验发现湍流速度信号中的强间歇性  
(G. K. Batchelor & A. A. Townsend, 1949)

# 1959 基于场论的湍流解析理论

- Kraichnan于1959年和1965年提出基于理论物理场论的湍流理论，由NS方程出发可解析推导出K41理论中的-5/3能谱
- 由于其理论中复杂的数学技巧与较晦涩的物理解释在当时学术界存在一定争议，质疑者如P. G. Saffman



谱空间中相互作用的Fourier模态  
*J. Fluid Mech.*, 5:498-543, 1959



Robert H. Kraichnan  
物理学家，爱因斯坦晚年助手



Philip G. Saffman  
应用数学家



# 1961 湍流研究的分水岭

- 1961年法国马赛湍流会议，一时湍流学术明星云集
- 组织者：Favre, Batchelor, Kovasznay
- 参会人：G. I. Taylor, von Karman, Kolmogorov, Obukhov, Yaglom, Millionschikov, Liepmann, Corrsin, Lumley, Roshko, Saffman, Kraichnan等
- Kolmogorov修正原先提出的K41标度律
- Kraichnan提出革命性的统计理论
- Batchelor对湍流统计纯理论研究前景忧虑



会后Batchelor开始撰写流体力学教材  
H. K. Moffat, *J. Fluid Mech.*, 663:2-27, 2010

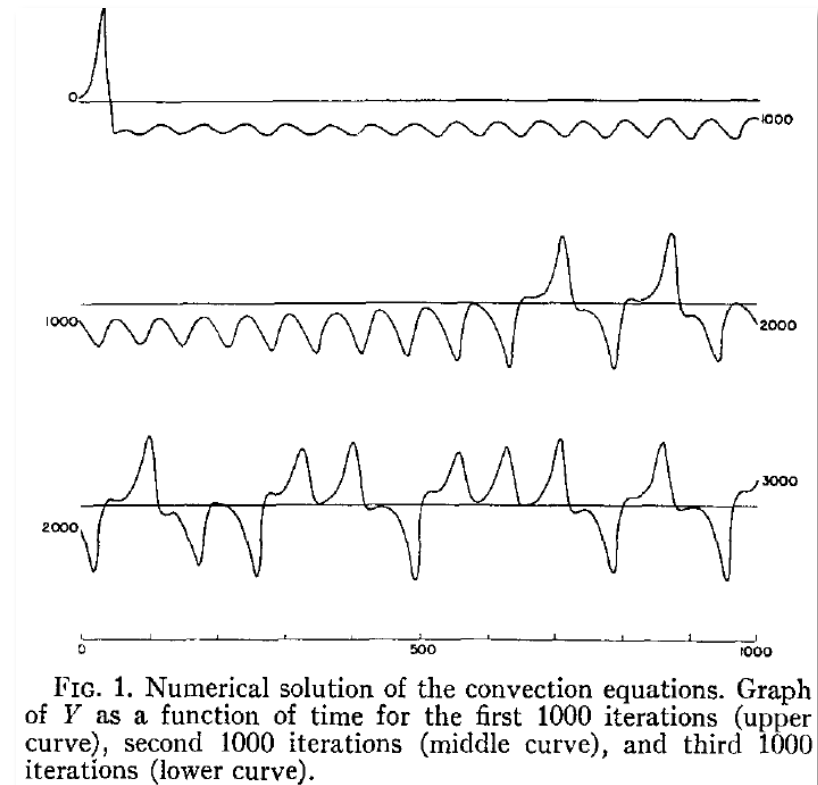
# 1963 决定性混沌理论

■ 混沌理论引发思考：湍流是**确定性的**，还是**随机性的**？



Edward Norton Lorenz  
数学家，气象学家

“一只蝴蝶在巴西轻拍翅膀，可以导致一个月后德克萨斯州的一场龙卷风。”



微分方程数值解的初值敏感性

E. N. Lorenz, *J. Atmos. Sci.*, 20:130-141, 1963

# 1963 大涡模拟

- 大涡模拟：直接求解流体大尺度运动，使用模型模拟小尺度运动对大尺度运动的影响
- Smagorinsky最早提出大涡模拟模型用于大气科学研究
- 目前被认为是未来工程应用中最具前途的湍流计算方法



Joseph Smagorinsky  
气象学家



剪切湍流大涡模拟中速度脉动分布  
J. W. Deardorff, *J. Fluid Mech.*, 41:453-480, 1970

# 1972 直接数值模拟

## ■ 均匀各向同性湍流

### Numerical Simulation of Three-Dimensional Homogeneous Isotropic Turbulence

Steven A. Orszag\*

*Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

and

G. S. Patterson, Jr.†

*Department of Engineering, Swarthmore College, Swarthmore, Pennsylvania 19081*

(Received 6 December 1971)



Steven A. Orszag  
应用数学家

S. A. Orszag and G. S. Patterson, *Phys. Rev. Lett.*, 12:76-79, 1972

## ■ 槽道湍流

*J. Fluid Mech.* (1987), vol. 177, pp. 133–166

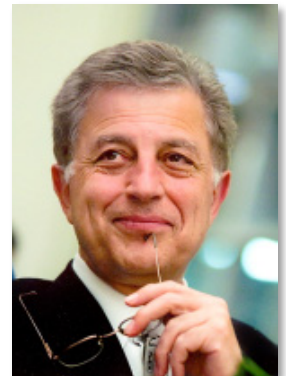
133

*Printed in Great Britain*

### Turbulence statistics in fully developed channel flow at low Reynolds number

By JOHN KIM, PARVIZ MOIN AND ROBERT MOSER

NASA Ames Research Center, Moffett Field, CA 94035, USA



Parviz Moin  
创办斯坦福大学CTR

J. Kim, P. Moin and R. Moser, *J. Fluid Mech.*, 177:133-166, 1987



# 1974 分形与湍流结构的自相似性

- 分形为具有自相似性质的几何结构
- 湍流结构亦存在一定程度上的自相似性

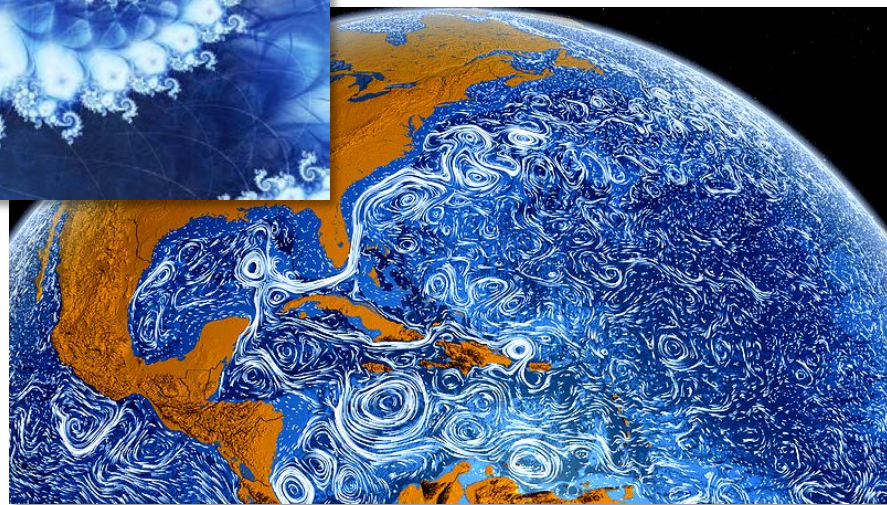


Benoit B. Mandelbrot  
应用数学家



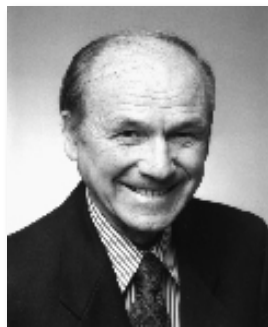
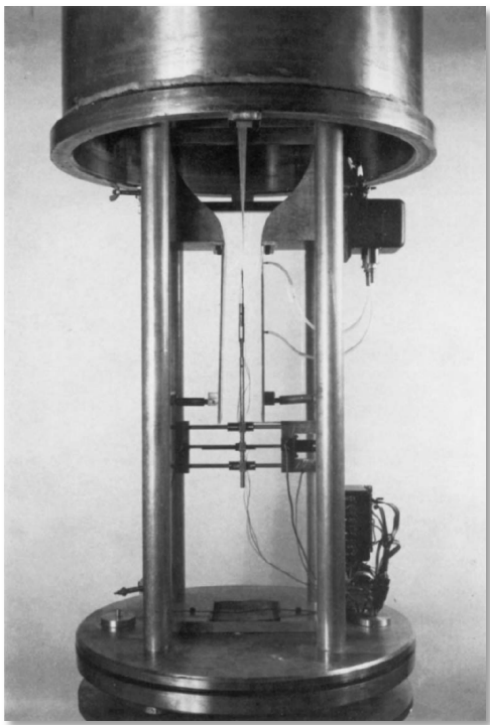
分形几何艺术作品  
(Tobias Wallin, 2010)

海洋洋流中的旋涡  
(NASA, 2012)

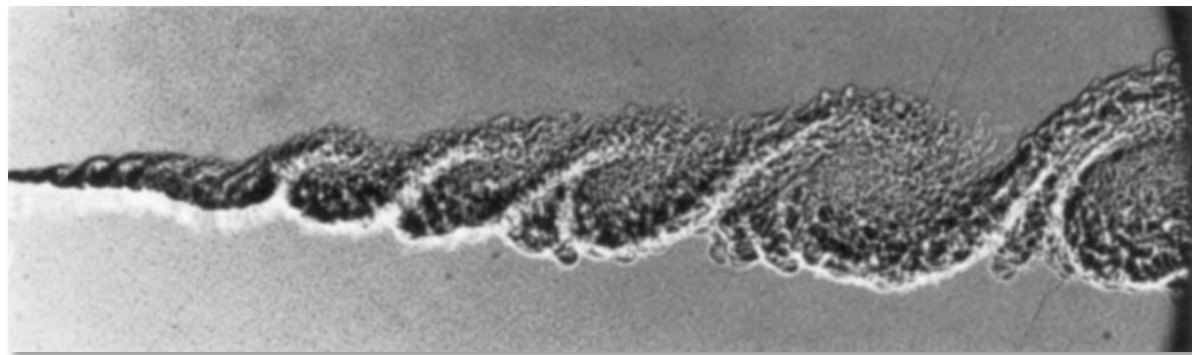


# 1974 湍流拟序结构

- 湍流拟序结构揭示湍流运动并非杂乱无章，而是乱中有序
- 二十世纪后期剪切湍流中各类拟序结构的发现与定性刻画



Anatol Roshko  
流体力学家



湍流混合层实验与拟序结构

G. L. Brown and A. Roshko, *J. Fluid Mech.*, 64:775-816, 1974

# 1982 重整化群理论

- 重整化群为处理含多种尺度与标度行为的理论方法
- 1986年Orszag利用重整化群理论构造湍流模型中的经验系数



Kenneth G. Wilson  
物理学家，诺贝尔奖得主

In this last category are the problems of fully developed turbulent fluid flow, critical phenomena, and elementary particle physics. The problem of magnetic impurities in non-magnetic metals (the Kondo problem) turns out also to be in this category.

In fully developed turbulence in the atmosphere, global air circulation becomes unstable, leading to eddies on a scale of thousands of miles. These eddies break down into smaller eddies, which in turn break down, until chaotic motions on all length scales down to millimeters have been excited. On the scale of millimeters, viscosity damps the turbulent fluctuations and no smaller scales are important until atomic scales are reached.<sup>2</sup>

In quantum field theory, “elementary” particles like electrons, photons, protons and neutrons turn out to have composite internal structure on all size scales down to 0. At least this is the prediction of quantum field theory. It is hard to make observations of this small distance structure directly; instead the particle scattering cross sections that experimentalists measure must be interpreted

Wilson在诺贝尔奖颁奖大会讲稿中解释了自然界中充分发展湍流、相变中的临界现象、基本粒子物理中具有共性的多尺度问题

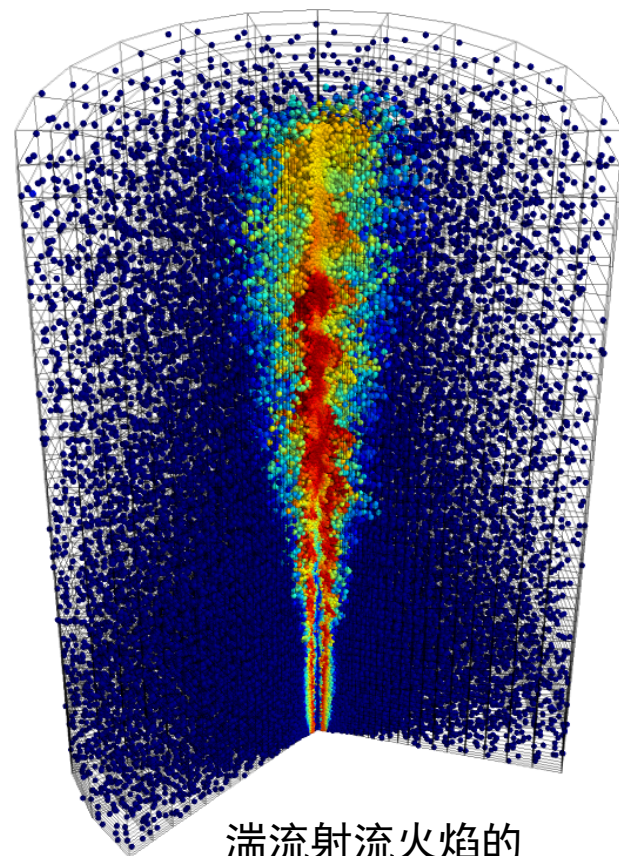
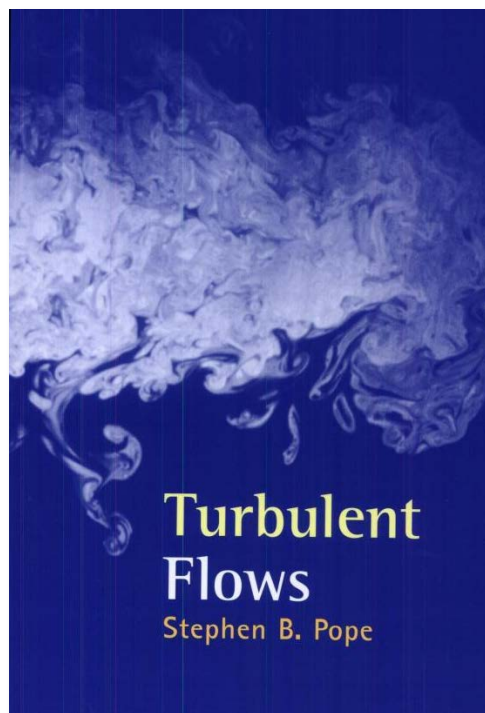


# 1985 概率密度函数方法

- Pope提出基于概率密度函数（PDF）的湍流模型方程
- 利用湍流统计理论结果确定模型系数
- 广泛用于化学反应流及湍流燃烧



Stephen B. Pope  
PDF方法主要开创者

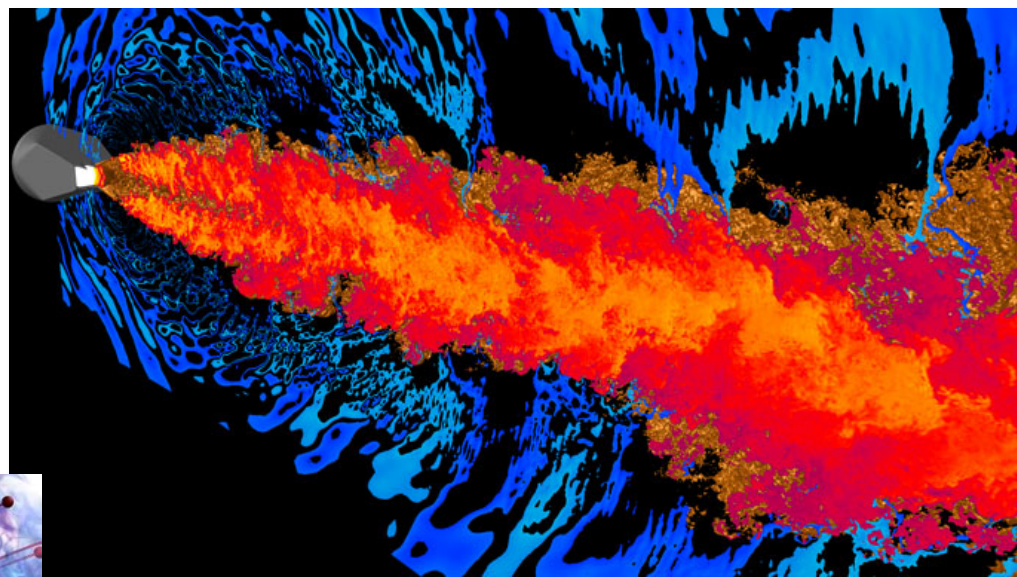


湍流射流火焰的  
大涡模拟/PDF方法计算  
(Hiremath & Pope, 2012)

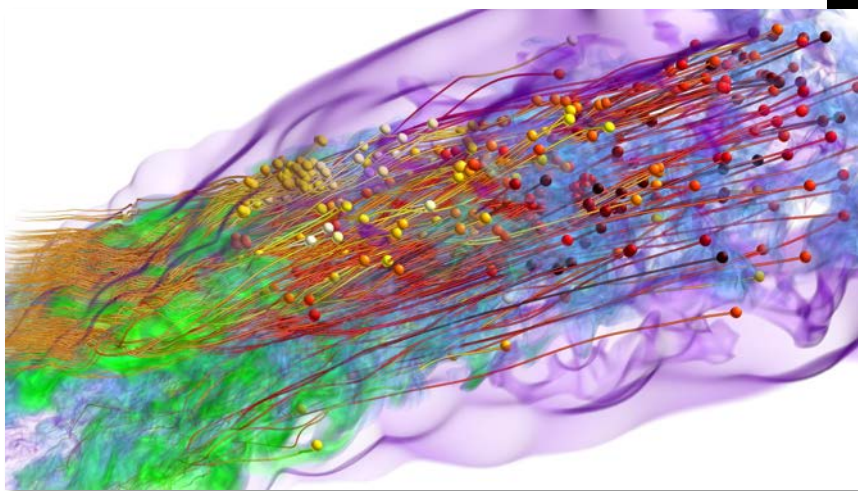


# 20世纪末至今 计算流体力学

- 大规模并行计算
- 湍流大涡模拟
- 多物理场耦合
- 面向实际工程问题



气动噪声大涡模拟(Stanford, 2013)



射流火焰直接数值模拟(Sandia, 2011)



并行计算机集群“Titan” (ORNL)

# 湍流研究特点

## ■ 研究方法多样，各类新方法的练兵场

□ 微分方程、统计、场论、信号处理等

□ 先进实验方法

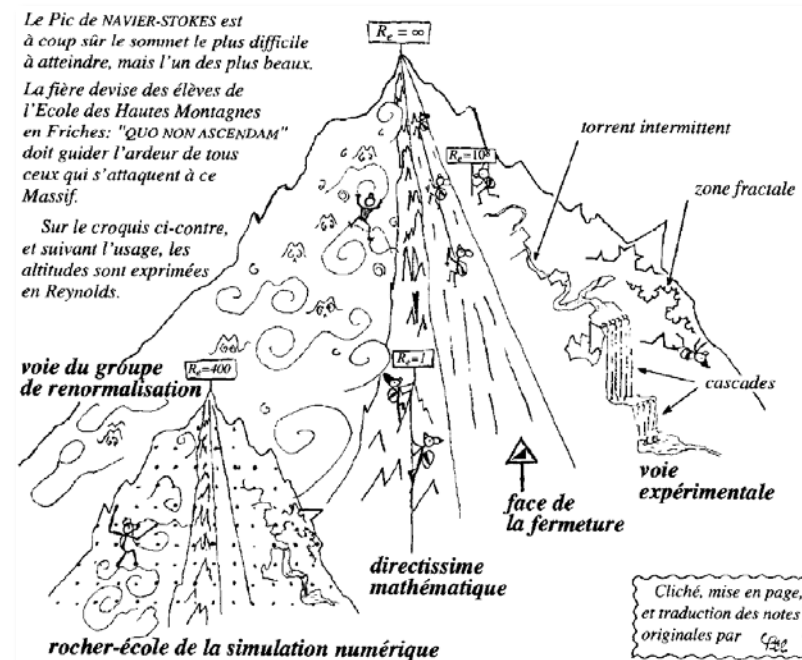
□ 高精度数值模拟

## ■ 难度大，理论问题目前尚未解决



Turbulence is described as  
"the **most important unsolved**  
problem of classical physics."

Richard Feynman  
物理学家，诺贝尔奖得主



持不同研究方法的探险队试图征服湍流研究巅峰  
U. Frisch, *Turbulence*, Cambridge, 1995

# 课程主要内容

- 湍流的基础理论与模型，以及当前主要的数值模拟方法
  - 湍流统计性质与唯象模型
  - 雷诺平均方程，涡粘模型与雷诺应力模型
  - 剪切湍流、均匀各向同性湍流、壁湍流
  - 直接数值模拟与大涡模拟
  - 标量湍流等研究专题
- 结合具体实例使学生利用数值手段研究工程中的湍流问题

# 课程教学目标

- 掌握湍流基础理论与建模方法
- 通过MATLAB编程完成如下内容
  - 湍流数据分析：统计量（速度场概率密度函数、相关函数等），速度能谱，涡量场
  - 数值模拟方法：直接数值模拟（数据获取与分析），大涡模拟（滤波运算与亚格子模型），雷诺平均模拟（边值问题求解，涡粘模型，雷诺平均模型）
- 科学报告撰写，科学图表绘制



# 课程安排

| 周次 | 周二 (3-4节)                   | 周四 (3-4节)       |
|----|-----------------------------|-----------------|
| 1  |                             | 02/20 湍流概述      |
| 2  | 02/25 湍流统计性质                | 02/27 湍流统计性质    |
| 3  |                             | 03/06 雷诺平均方程    |
| 4  | <b>03/11 提交作业1</b> 剪切湍流     | 03/13 剪切湍流      |
| 5  |                             | 03/20 均匀各向同性湍流  |
| 6  | <b>03/25 提交作业2</b> 均匀各向同性湍流 | 03/27 均匀各向同性湍流  |
| 7  |                             | 04/03 直接数值模拟    |
| 8  | <b>04/08 提交作业3</b> 壁湍流      | 04/10 涡粘模型      |
| 9  |                             | 04/17 涡粘模型      |
| 10 | <b>04/22 提交作业4</b> 雷诺平均模型   | 04/24 大涡模拟      |
| 11 |                             | 05/01 放假        |
| 12 | <b>05/06 提交作业5</b> 大涡模拟     | 05/08 大涡模拟      |
| 13 |                             | 05/15 湍流与非线性动力学 |
| 14 | <b>05/20 提交作业6</b> 湍流中的涡动力学 | 05/22 湍流中的涡动力学  |
| 15 |                             | 05/29 标量湍流      |
| 16 | <b>06/03 提交作业7</b> 拉格朗日湍流   | 06/05 概率密度函数方法  |
| 17 | 期末考试                        |                 |

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