Application of Compressive Sensing Based Beamforming in Aeroacoustic Experiment

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Aeroacoustic Application

Airframe noise research

4.5 m × 3.5 m × 11 m, lined wall closed-section wind tunnel, Aviation Industry Corporation of China, January 2013.

1 Bao Chen, Qingkai Wei, Xun Huang, “Aeroacoustic imaging experiments of airframe noise in lined wall closed-section wind tunnel”, Journal of Aerospace Engineering, minor revision.
Overview of Beamforming Methods in common use

- Conventional Beamforming [1];
- DAMAS (deconvolution approach for the mapping of acoustic sources), DAMAS2, DAMAS3, DAMAS-C: point spread function based on steering vector [2];
- SC-DAMAS (sparsity constrained DAMAS) and CMF (covariance matrix fitting): make use of convex optimization and sparsity to achieve comparable performance [3];
- CLEAN-SC: not assume the true steering vectors are known and attempts to estimate them from the measurements [4].

Aeroacoustic Application

Airframe noise research

40m/s: 2kHz, 3kHz, 4kHz, 5kHz.

[Images of conventional beamforming and DAMAS]

Conventional Beamforming; DAMAS.

1 Bao Chen, Qingkai Wei, Xun Huang, “Aeroacoustic imaging experiments of airframe noise in lined wall closed-section wind tunnel”, *Journal of Aerospace Engineering*, minor revision.
Compressive Sensing

Compressive sensing (CS): reduce sampling efforts extensively by conducting $L_1$ optimization [1].

Google Scholar Citation, 6307.

Reconstruction of $x$, which is sparse in some Hilbert basis $\psi \in \mathbb{C}^{N \times N}$, that is, $x = \psi s, s \in \mathbb{C}^N$.

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Compressive Sensing

\[ \arg \min ||\hat{s}||_1, \text{ subject to } ||y - \phi \psi \hat{s}||_2 = 0, \]

For those measurements polluted by some noise, as the following

\[ y = \phi \psi \hat{s} + \text{noise}. \]

\[ \arg \min ||\hat{s}||_1, \text{ subject to } ||y - \phi \psi \hat{s}||_2 \leq \delta, \delta > 0. \]
Compressive Sensing

Application criteria

- The matrix $\phi$ obeys a condition known as the restricted isometry property (RIP) as:
  For each integer $k = 1, 2, \ldots$, define the isometry constant $\delta_s$ of a matrix $\phi$ as the smallest number such that
  
  $$(1 - \delta_k) \|x\|_2^2 \leq \|\phi x\|_2^2 \leq (1 + \delta_k) \|x\|_2^2.$$ 

  holds for all $s$-sparse vectors.

- 

  $$M > C_k \cdot K \cdot \log(N/M),$$

  where $C_k$ is a universal constant that directly determines the accuracy of the optimization outcomes.

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Research of Compressive Sensing based Beamforming

Most works in the literature use

- uniformly linear array (direction-of-arrival estimation problem, or at-sea by towed-array data);
- examined only by simulation data.

In aeroacoustic applications,

- planar array with known location of microphones;
- contain strong background noise at broadband frequencies.


Wave Propagation Expression

Single signal of interest in frequency domain:

\[ Y(j\omega) = \frac{1}{4\pi r} X(j\omega)e^{-j\omega \tau} = G_v(r, j\omega)X(j\omega), \]

where \( G_v \in \mathbb{C}^{M \times 1} \) is the associated steering vector

\[ Y = G_vX. \]
Wave Propagation Expression

Multiple signals of interest in frequency domain:

\[
\begin{bmatrix}
  Y_1 \\
  \vdots \\
  Y_i \\
  \vdots \\
  Y_M
\end{bmatrix}
= 
\begin{bmatrix}
  G_{11} & \ldots & G_{1k} & \ldots & G_{1N} \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  G_{i1} & \ldots & G_{ik} & \ldots & G_{iN} \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  G_{M1} & \ldots & G_{Mk} & \ldots & G_{MN}
\end{bmatrix}
\begin{bmatrix}
  X_1 \\
  \vdots \\
  X_k \\
  \vdots \\
  X_N
\end{bmatrix}
+ 
\begin{bmatrix}
  N_1 \\
  \vdots \\
  N_i \\
  \vdots \\
  N_M
\end{bmatrix},
\]

\[Y = G_{m_l}X + N, \text{SNR}_{\text{dB}} = 10 \log_{10} \left( \frac{\sum_{i=1}^{N} |G_{ik}X_i|^2}{|N_i|^2} \right) .\]

Generally, it is assumed that \(X\) and \(N\) are of zero-mean and statistically independent.
Assumption: spatially sparse and incoherent sound sources.

\[ Y = G_{m_{1}}X + N, \]

\[ \text{arg min} \| \hat{S} \|_1, \text{subject to} \| Y - G_{m_{1}}\hat{S} \|_2 \leq 0, \]

\[ P_{CSB-I} = \| \hat{S} \|_2. \]
Assumption: spatially sparse and incoherent sound sources.

\[ Y = G_{m1}X + N, \]

\[ \text{arg min} \|\hat{S}\|_1, \text{ subject to } \|Y - G_{m1}\hat{S}\|_2 \leq \delta, \delta > 0, \]

\[ P_{CSB-I} = \|\hat{S}\|_2. \]
Compressive Sensing Based Beamforming (CSB-I and CSB-II)

CSB-II

Assumption: spatially sparse and incoherent sound sources.

\[ R_V = G_{mII} P + Q, \]

\[ \arg \min ||\hat{P}||_1, \text{ subject to } \hat{P} > 0, ||\hat{R}_V - G_{mII} \hat{P}||_2 \leq \delta, \delta > 0. \]

\[ P_{CSB-II} = \hat{P}. \]
**CSB-II**

**CSM(cross spectrum matrix)-based**

\[
\mathbf{R} = \mathbf{E}\{\mathbf{YY}^*\}, \quad \hat{\mathbf{R}} \approx \frac{1}{K} \sum_{k=1}^{K} \mathbf{YY}^*,
\]

\[
\mathbf{Y} = \mathbf{G}_m \mathbf{X} + \mathbf{N},
\]

\[
\mathbf{E}\{\mathbf{YY}^*\} = \mathbf{E}\{\mathbf{G}_m \mathbf{XX}^* \mathbf{G}_m^*\} + \mathbf{E}\{\mathbf{NN}^*\},
\]

\[
\mathbf{R} = \mathbf{E}\{\mathbf{YY}^*\} = \\
\begin{bmatrix}
\mathbf{E}\{Y_1 Y_1^*\} & \mathbf{E}\{Y_1 Y_2^*\} & \cdots & \mathbf{E}\{Y_1 Y_M^*\} \\
\mathbf{E}\{Y_2 Y_1^*\} & \mathbf{E}\{Y_2 Y_2^*\} & \cdots & \mathbf{E}\{Y_2 Y_M^*\} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{E}\{Y_M Y_1^*\} & \mathbf{E}\{Y_M Y_2^*\} & \cdots & \mathbf{E}\{Y_M Y_M^*\}
\end{bmatrix},
\]

\[
\mathbf{R}_V = (\mathbf{E}\{Y_1 Y_1^*\}, \mathbf{E}\{Y_1 Y_2^*\}, \cdots, \mathbf{E}\{Y_1 Y_M^*\}, \cdots, \mathbf{E}\{Y_M Y_1^*\}, \cdots, \mathbf{E}\{Y_M Y_M^*\})^T
\]
CSB-II

CSM(cross spectrum matrix)-based

\[
R = E\{YY^*\}, \quad \hat{R} \approx \frac{1}{K} \sum_{k=1}^{K} YY^*,
\]

\[
Y = G_{m_1} X + N,
\]

\[
E\{YY^*\} = E\{G_{m_1}XX^*G_{m_1}^*\} + E\{NN^*\},
\]

\[
R_{V[(i-1) \times M+j]} = R_{ij} = Y_i Y_j^*
\]

\[
= E \left\{ \sum_{p=1}^{N} (G_{ip}X_p) \sum_{q=1}^{N} (X_q^*G_{jq}^*) \right\} = \sum_{n=1}^{N} G_{in}G_{in}^* E\{X_nX_n^*\}. 
\]
CSB-II

$$\mathbf{R}_V = \begin{bmatrix} G_{11}G_{11}^* & G_{12}G_{12}^* & \cdots & G_{1N}G_{1N}^* \\ G_{11}G_{21}^* & G_{12}G_{22}^* & \cdots & G_{1N}G_{2N}^* \\ \vdots & \vdots & \ddots & \vdots \\ G_{M1}G_{M1}^* & G_{M2}G_{M2}^* & \cdots & G_{MN}G_{MN}^* \end{bmatrix}$$

$$\mathbf{P} + \mathbf{Q} = \mathbf{G}_{mII} \mathbf{P} + \mathbf{Q},$$

$$\mathbf{R}_V = (\mathbf{E}[Y_1Y_1^*], \mathbf{E}[Y_1Y_2^*], \cdots, \mathbf{E}[Y_MY_M^*])^T \in \mathbb{C}^{M^2 \times 1} \text{ by reshaping CSM } \hat{\mathbf{R}};$$

$$\mathbf{P} = (\mathbf{E}[X_1X_1^*], \mathbf{E}[X_2X_2^*], \cdots, \mathbf{E}[X_NX_N^*])^T \in \mathbb{R}^{N \times 1};$$

$$\mathbf{Q} \text{ is the vertical vector form of } \mathbf{E}[\mathbf{NN}^*] \in \mathbb{C}^{M^2 \times 1}.$$  

$$\arg \min \| \hat{\mathbf{P}} \|_1, \text{ subject to } \hat{\mathbf{P}} > 0, \| \hat{\mathbf{R}}_V - \mathbf{G}_{mII} \hat{\mathbf{P}} \|_2 \leq \delta, \delta > 0.$$  

$$\mathbf{P}_{CSB-II} = \hat{\mathbf{P}}.$$
Revisit of Algorithms

Step 0: Data acquisition and obtain $Y$ by performing Fourier transform.

CSB-I

Step 1: Prepare $G_{m_I}$.
Step 2:
\[
\text{arg min } \|\hat{S}\|_1, \text{ subject to } \|Y - G_{m_I}\hat{S}\|_2 \leq \delta, \delta > 0,
\]
\[
P_{CSB-I} = \|\hat{S}\|_2.
\]

CSB-II

Step 1: Prepare $G_{m_{II}}, R_V$ by reshaping $\hat{R}$.
Step 2:
\[
\text{arg min } \|\hat{P}\|_1, \text{ subject to } \hat{P} > 0, \|\hat{R}_V - G_{m_{II}}\hat{P}\|_2 \leq \delta, \delta > 0.
\]
\[
P_{CSB-II} = \hat{P}.
\]
Simulation

Beamforming results of the monopole case at 5 kHz, with CSB-I, CSB-II method. where: (a) $\text{SNR} = \infty$, CSB-I; (b) $\text{SNR} = -10 \text{ dB}$, CSB-I; (c) $\text{SNR} = -10 \text{ dB}$, CSB-II.

CSB-I algorithm is quite sensitive to the sensing noise, while CSB-II is OK.

Experiment Setup

Wind tunnel

The test section of the wind tunnel is 4.5 m × 3.5 m × 11 m (width × height × length), Aviation Industry Corporation of China, Jan. 2013.
Experiment Setup

Facility

1/12 MA-60 model, in the lined wall closed-section wind tunnel.

Validation of CSB-I and CSB-II Method

Experiment Setup

Microphone array

The performance of the array consisting of 110 channels of Brüel and Kjær 4954 microphones.
Validation of CSB-I and CSB-II Method

Experiment Results

Acoustic images at 4 kHz, 40 m/s, (a) using conventional beamforming algorithm; (b) using CSB-II algorithm.

CSB-II algorithm has a good performance in terms of resolution and sidelobe rejection. Consumption time for $20 \times 20$ case: 3s to 370s.
Experiment Results

Comparison of SPL on the line shown in figure in previous page.

CSB-II algorithm has a good performance in terms of resolution and sidelobe rejection.
Summary and Discussion

Summary

- The two algorithms are examined using both simulation and aeroacoustic experiments;
- CSB-I algorithm is quite sensitive to the sensing noise;
- CSB-II algorithm has a good performance in terms of resolution and sidelobe rejection.

Limitation

- There is an assumption of spatially sparse sound sources, if not, a transform matrix is needed;
- Verification of the Restricted Isometry Property for random matrices is easy, but for the specific matrix?
- CSB-II is still time-consuming, improvement in convex optimization step?
Thanks!

Questions?