14.3 Proof of shift theorem for the moment of a set of bound vectors

To establish the validity of equation (5), consider a set S of bound vectors $\mathbf{F}_1, \ldots, \mathbf{F}_{\mathrm{n}}$, whose lines of action pass through points Q_i ($i = 1, \ldots, n$), respectively. The moment of S about a point P is defined in terms of $\mathbf{r}^{Q_i/P}$ (Q_i's position vector from P) as

$$\mathbf{M}^{S/P} \underset{(2)}{\triangleq} \sum_{i=1}^{n} \mathbf{r}^{Q_i/P} \times \mathbf{F}^{Q_i} \tag{7}$$

Q_i's position vector from P may be written as

$$\mathbf{r}^{Q_i/P} = \mathbf{r}^{O/P} + \mathbf{r}^{Q_i/O} \tag{8}$$

Substituting equation (8) into equation (7), distributing the summation, and rearrangement yields

$$\begin{aligned} \mathbf{M}^{S/P} &\underset{(7,8)}{=} \sum_{i=1}^{n} \mathbf{r}^{O/P} \times \mathbf{F}^{Q_i} + \sum_{i=1}^{n} \mathbf{r}^{Q_i/O} \times \mathbf{F}^{Q_i} \\ &= \mathbf{r}^{O/P} \times \sum_{i=1}^{n} \mathbf{F}^{Q_i} + \sum_{i=1}^{n} \mathbf{r}^{Q_i/O} \times \mathbf{F}^{Q_i} \end{aligned} \tag{9}$$

The first summation in equation (9) is the definition of $\mathbf{F}^S$ (the resultant of S). The second summation in equation (9) is the definition of $\mathbf{M}^{S/O}$ (the moment of S about O). Combining these two facts produces equation (5), the shift theorem for the moment of a set of bound vectors, namely

$$\mathbf{M}^{S/P} = \mathbf{r}^{O/P} \times \mathbf{F}^S + \mathbf{M}^{S/O}$$