Penta-BCN: A New Ternary Pentagonal Monolayer with Intrinsic Piezoelectricity

Kexian Zhao, Yaguang Guo, Yiheng Shen, Qian Wang, Y. Kawazoe, and Puru Jena

ABSTRACT: Going beyond conventional hexagonal sheets, pentagonal 2D structures are of current interest due to their novel properties and broad applications. Herein, for the first time, we study a ternary pentagonal BCN monolayer, penta-BCN, which exhibits intrinsic piezoelectric properties. Based on state-of-the-art theoretical calculations, we find that penta-BCN is stable mechanically, thermally, and dynamically and has a direct band gap of 2.81 eV. Due to its unique atomic configuration with noncentrosymmetric and semiconducting features, penta-BCN displays high spontaneous polarization of \(3.17 \times 10^{-10} \text{C/m}\) and prominent piezoelectricity with \(d_{31} = 0.878 \text{pm/V}, d_{22} = -0.678 \text{pm/V},\) and \(d_{16} = 1.72 \text{pm/V},\) which are larger than those of an \(h\)-BN sheet and functionalized pentagaphene. Since B, C, and N are rich in resources, light in mass, and benign to the environment, the intrinsic polarization and piezoelectricity make the penta-BCN monolayer promising for technological applications. This study expands the family of 2D pentagonal structures with new features.

Since the theoretical prediction of pentagraphene (PG), a carbon allotrope purely composed of five-membered rings, tremendous efforts have been made in recent years to rationally design and synthesize pentagon-based 2D materials with novel properties and promising applications. Many 2D nanosheets constructed exclusively from pentagon-based structural units have been proposed, and some of them have been experimentally synthesized, including penta-PdSe, and pentasilicene nanoribbons. These materials exhibit interesting electronic properties and have broad applications. For instance, a penta-CN sheet exhibits high energy density of 4.41 kJ/g, high in-plane axial Young’s modulus of 319 N/m (even stiffer than a \(h\)-BN monolayer), and an interesting double degeneracy of its electronic band structure protected by nonsymmorphic symmetry. In addition, the penta-CN sheet was also found to have a high lattice thermal conductivity of 660.71 W m\(^{-1}\) K\(^{-1}\) due to its unique geometry, contrary to conventional wisdom that lone-pair electrons lead to low lattice thermal conductivity. Parallel to the study of a penta-CN sheet, penta-CB and penta-BN were also systematically investigated; the former can be flexibly tuned from a semiconductor to a semimetal by strain, while the latter shows a negative Poisson’s ratio and tunable electronic structure from semiconducting to metallic. These intriguing properties of pentasheets make them promising for vast applications. For example, an Ohmic contact between pentagaphene and graphene can be achieved by applying an electric field or doping the graphene sheet with N atoms. A Schottky contact built by vertically stacking pentagaphene and penta-BN can be tuned from \(n\)-type to \(p\)-type when the interlayer distance is changed. Pentagaphene and penta-BN have been found to be promising anode materials for Li-ion batteries.

However, the design of nonmetal-element-based pentagonal 2D structures so far is limited to binary systems. We note that when going from unitary pentagaphene to binary pentasheets including penta-CB, penta-BN, and penta-CN, the physical properties have been significantly enriched. Similarly, going from binary to ternary systems can offer more degrees of freedom to tune the physical properties. This motivated us to design a stable ternary pentasheet containing B, C, and N atoms, because they are neighbors in the periodic table with similar atomic radii but distinctive electronegativity. In addition, they have many advantages over other nonmetal elements. Using first-principles calculations, we found a stable ternary pentagonal sheet, penta-BCN, which exhibits novel properties, including spontaneous polarization and intrinsic piezoelectricity.
The structural and electronic properties of the penta-BCN sheet are calculated using density functional theory (DFT) as implemented in the Vienna Ab initio Simulation Package (VASP). The interactions between electrons and nuclei are treated within the projector augmented wave (PAW) method with a kinetic cutoff energy of 600 eV. The electronic exchange-correlation potential energy is incorporated using the generalized gradient approximation due to Perdew–Burke–Ernzerhof (PBE). The hybrid Heyd–Scuseria–Ernzerhof (HSE06) functional is further used to achieve more accurate results of the electronic properties. A vacuum spacing of 20 Å along the direction perpendicular to the plane is included to avoid the interactions between two neighboring images. The criteria for total energy and forces are set to 10⁻¹² eV and 10⁻² eV/Å, respectively. The first Brillouin zone is sampled by a 9 × 9 × 1 k-point grid within the Monkhorst–Pack scheme. A PHONOPY package is used to calculate phonon spectra to confirm the dynamic stability, and ab initio molecular dynamics (AIMD) simulations are performed to confirm the thermal stability.

The piezoelectric tensors and piezoelectric strain tensors are defined as

\[
e_{ijk} = \frac{\partial P_i}{\partial x_j} = e_{ijk}^{\text{elc}} + e_{ijk}^{\text{ion}}
\]

and

\[
d_{ijk} = \frac{\partial \sigma_{ik}}{\partial x_j} = d_{ijk}^{\text{elc}} + d_{ijk}^{\text{ion}}
\]

Here, \(P_i\) is the piezoelectric polarization, and \(\sigma_{ik}\) and \(\sigma_{jk}\) are strain and stress, respectively. The relaxed-ion piezoelectric tensors \(e_{ijk}\) and \(d_{ijk}\) are strain and stress, respectively. The relaxed-ion piezoelectric tensors \(e_{ijk}\) are obtained from the sum of ionic and electronic contributions with the fully relaxed ionic positions. Using the Voigt notation and the mapping of indices (11 → 1, 22 → 2, 33 → 3, 23 → 4, 31 → 5, and 12 → 6), the values of \(d_{ijk}\) can be derived from the equation

\[
e_{ijk} = d_{ijk} C_{ijk}
\]

where piezoelectric tensors \(e_{ijk}\) can be written as

\[
e_{ijk} = \begin{pmatrix}
e_{11} & e_{12} & \cdots & e_{16} \\
e_{21} & e_{22} & \cdots & e_{26} \\
e_{31} & e_{32} & \cdots & e_{36}
\end{pmatrix}
\]

Specifically, for 2D materials, we only need to consider in-plane strain components. Thus, the relationship among the elastic, piezoelectric stress, and strain tensors becomes

\[
\begin{pmatrix}
e_{11} & e_{12} & e_{16} \\
e_{21} & e_{22} & e_{26} \\
e_{31} & e_{32} & e_{36}
\end{pmatrix} = \begin{pmatrix}
d_{11} & d_{12} & d_{16} \\
d_{21} & d_{22} & d_{26} \\
d_{31} & d_{32} & d_{36}
\end{pmatrix} \begin{pmatrix}
C_{11} & C_{12} & C_{16} \\
C_{21} & C_{22} & C_{26} \\
C_{31} & C_{32} & C_{36}
\end{pmatrix}
\]

The elastic stiffness tensor \(C_{ij}\) and piezoelectric tensor \(e_{ijk}\) are calculated by using the finite difference method and density functional perturbation theory (DFPT) as implemented in the VASP package, respectively, which has been widely used in previous studies.

We constructed an initial ternary pentagonal structure, penta-BCN, based on the atomic configuration of pentagraphene, by alternately replacing the threefold-coordinated C atoms with B and N and keeping the positions of four-coordinated C atoms unchanged. The optimized geometry of penta-BCN is given in Figure 1(a), which shows that penta-BCN is composed of pure BCN pentagons. The relaxed 2D crystal structure possesses C\_2\_2 symmetry (space group No. 4) with the lattice parameters \(a = 3.67\) Å and \(b = 3.63\) Å and a buckling height of 1.34 Å. The unit cell contains six atoms with a C/B/N ratio of 1:1, as denoted by dashed lines in Figure 1(a). In this structure, the B–N, C–N, and C–B bond lengths are 1.41, 1.51, and 1.62 Å, respectively. We note that the B–N bond length is shorter than that in h-BN (1.44 Å)\(^{35}\) and penta-BN\(_2\) (1.55 Å),\(^{5}\) suggesting that the B–N covalent bond in penta-BCN is stronger than that in h-BN and penta-BN\(_2\). We also note that the special arrangement of B–N bonds breaks the centrosymmetry of the geometry, which in turn would induce spontaneous polarization and piezoelectric effect.

To verify the dynamic stability of penta-BCN, we calculated its phonon dispersion. As shown in Figure 1(b), there are no imaginary modes in the entire Brillouin zone, confirming that the penta-BCN structure is dynamically stable. To examine the thermal stability of penta-BCN, we further carried out AIMD simulations by using a 4 × 4 × 1 supercell for 6 ps with a time step of 1 fs for each simulation from 300 to 1200 K. We find that the structure remains intact at the end of the simulation and does not suffer significant distortion at 300 K. The total potential energy fluctuates slightly around a constant value during the simulation at 1200 K. The corresponding simulation results are plotted in Figure 1(c), which shows that penta-BCN is thermally stable even at a high temperature of 1200 K.
using the finite distortion method. The calculated elastic constants are $C_{11} = 223.56$ N/m, $C_{22} = 189.16$ N/m, $C_{12} = C_{21} = 4.90$ N/m, and $C_{66} = 104.88$ N/m. According to Born–Huang criteria, the elastic constants of penta-BCN satisfy $C_{11}C_{12} - C_{12}^2 > 0$ and $C_{66} > 0$, confirming that the penta-BCN monolayer is mechanically stable. The in-plane Young’s moduli along the (100) and (010) directions ($E_a$ and $E_b$) are derived from the formulas:

$$E_a = \frac{(C_{11}^2 - C_{12}^2)}{C_{11}} \quad \text{and} \quad E_b = \frac{(C_{22}^2 - C_{21}^2)}{C_{22}}$$

Their corresponding values are 223.45 and 189.03 N/m, respectively. Note that these values are much larger than those of the MoS$_2$ monolayer (120 N/m), suggesting that the penta-BCN sheet is even stiffer for curling. The Poisson’s ratios are calculated to be $\nu_a = 0.022$ and $\nu_b = 0.026$, along the (100) and (010) directions, respectively. The different mechanical properties in the different directions indicate a mechanical anisotropy of penta-BCN.

To further explore the dependence of mechanical properties on the crystal orientation of penta-BCN, the tensor rotation matrix $T$ is applied to calculate the stiffness tensor $C$ by using the equation

$$C' = T^{-1} \cdot C \cdot R \cdot T \cdot R^{-1}$$

where $R = \text{diag}(1, 1, 2)$ is the bridge between the engineered strain and tensor strain. The calculated results are plotted in Figure 2. One can see that the maximum in-plane Young’s modulus is located along the (100) direction (0°), and the minimum is along the (010) direction (90°). The maximum and minimum Poisson’s Ratios are located along the (010) direction (90°) and near the (110) direction (~45°) with values of 0.026 and 0.004, respectively, once again showing a high mechanical anisotropy of penta-BCN.

Figure 2. Variation of the (a) in-plane Young’s modulus and (b) Poisson’s ratio of penta-BCN with the crystal orientation of penta-BCN.

To study the electronic properties of penta-BCN, we calculated its band structure using the PBE and HSE06 functional as shown in Figure 3(a) in red and blue lines, respectively. One can see that penta-BCN is semiconducting with a direct bandgap of 1.70 eV at the PBE level, as both the valence band maximum (VBM) and conduction band minimum (CBM) are located at the same point along the $\Gamma$-$X$ path in the first Brillouin zone. At the HSE06 level, the band structure has a similar dispersion but with a larger band gap of 2.81 eV due to the well-known fact that the PBE functional underestimates the band gaps of materials. To further analyze the electronic properties of penta-BCN, the total density of states (DOS) and partial DOS (PDOS) are calculated and plotted in Figure 3(b). From the PDOS, one can see that the VBM is mainly contributed by both B and N atoms, while the CBM dominantly comes from the B atoms. The contributions to the electronic states can also be seen from the band-decomposed charge density distributions, as shown in Figure 3(c−f). In addition, we note that the charge densities are asymmetrically distributed, implying that spontaneous polarization can exist in penta-BCN, which may lead to potential application in piezoelectrics.

To possess spontaneous electric polarization in 2D materials, it is necessary to have polarized bonds arranged in a noncentrosymmetric configuration. For pentagonal structures, such as PG and penta-CN$_p$, the biggest challenge in inducing polar states is that the $sp^2$ hybridized atoms are equivalent,
where the positive and negative charge centers cannot be effectively separated due to the nonpolar bonds. To this end, we proposed that a small tilt of Si dimers can lead to ferroelectricity in a pentasilicene sheet, in which the polar bonds result from the inequivalent sp² Si atoms. However, a more intuitive way is to use two different elements to replace the dimers. This is the case with penta-BCN, where one can see that the B–N bond is intrinsically polarized, differing from the sp² hybridized C–C in PG and N–N in penta-CN₂. Therefore, electric polarization can survive in this penta-BCN sheet. To quantify the spontaneous polarization of penta-BCN, we use the finite electric field method based on the Berry phase theory¹¹,¹² to calculate the dipole moment. The total polarization \( P_s \) is determined by the sum of the ionic contribution \( P_{s}^{\text{ion}} \) and electronic contribution \( P_{s}^{\text{ele}} \):

\[
P_s = P_{s}^{\text{ion}} + P_{s}^{\text{ele}}
\]  

(7)

The ionic contribution can be simply described as

\[
P_{s}^{\text{ion}} = \frac{e_i}{\Omega} \sum_{i=1}^{N_{\text{atom}}} z_i^N \Delta r_i
\]  

(8)

where \( e_i, \Omega, N_{\text{atom}} \) are the atomic charge, volume of the unit cell, number of atoms in the unit cell, valence charge of the \( i \)-th atom, and the displacement of \( i \)-th atom, respectively. The electronic contribution \( P_{s}^{\text{ele}} \) can be derived using the Berry phase approach. We find that the polarization \( P_s \) of penta-BCN is along the (010) direction, namely the direction parallel to the B–N bonds, while that along the (100) direction cancels each other out. The value of \( P_s \) is calculated to be \( 3.17 \times 10^{-10} \text{C/m} \), which is larger than that of h-BN (2.174 \times 10^{-10} \text{C/m})²³ because penta-BCN has a stronger B–N bond. Here, we find that the dipole in penta-BCN is permanent, i.e., it is difficult to change the direction of the dipole moment. Thus, penta-BCN can be considered as a pyroelectric material instead of a ferroelectric one.

We next study the piezoelectricity of penta-BCN as a pyroelectric material, which also exhibits a piezoelectric effect. The piezoelectric tensors for this 2D structure with C₂ᵥ symmetry can be written as

\[
e_{ij} = \begin{pmatrix}
0 & 0 & 0 & e_{14} & 0 & e_{16} \\
e_{21} & e_{22} & 0 & 0 & e_{25} & 0 \\
0 & 0 & e_{34} & 0 & e_{36}
\end{pmatrix}
\]  

(9)

According to eq 5, the relationship among the elastic, piezoelectric stress, and strain tensors becomes

\[
\begin{pmatrix}
0 & 0 & e_{16} \\
e_{21} & e_{22} & 0 \\
0 & 0 & e_{36}
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & d_{16} \\
d_{21} & d_{22} & 0 \\
0 & 0 & d_{36}
\end{pmatrix}
\begin{pmatrix}
C_{11} & C_{12} & C_{16} \\
C_{21} & C_{22} & C_{26} \\
C_{31} & C_{32} & C_{36}
\end{pmatrix}
\]  

(10)

The \( d_{16}, d_{21}, d_{22}, \) and \( d_{36} \) are derived as

\[
d_{16} = \frac{e_{16}}{C_{66}}
\]  

(11)

\[
d_{21} = \frac{C_{12}e_{21} - C_{11}e_{22}}{C_{21}C_{12} - C_{11}C_{22}}
\]  

(12)

\[
d_{22} = \frac{C_{12}e_{21} - C_{11}e_{22}}{C_{21}C_{12} - C_{11}C_{22}}
\]  

(13)

The calculated elastic stiffness coefficients \( C_{ij} \) along the polarization direction (010) and piezoelectric stress tensors \( e_{ij} \) are listed in Table 1. According to eqs 11–14, the piezoelectric strain tensors \( d_{ij} \) are derived from \( C_{ij} \) and \( e_{ij} \) resulting in \( d_{11} = 0.878 \text{pm/V}, d_{12} = -0.678 \text{pm/V}, d_{16} = 1.72 \text{pm/V}, \) and \( d_{36} = -0.076 \text{pm/V} \). For comparison, we list piezoelectric coefficients \( \epsilon \) and \( d \) for h-BN²⁷ and H-PG-F⁴⁴ in Table 2.

Table 1. Elastic Stiffness Coefficients \( C_{ij} \), Piezoelectric Stress Tensors \( e_{ij} \), and Piezoelectric Strain Tensors \( d_{ij} \) of Penta-BCN

<table>
<thead>
<tr>
<th>Material</th>
<th>( e_{ij} ) (10⁻¹⁰ C/m)</th>
<th>( d_{ij} ) (pm/V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penta-BCN</td>
<td>( e_{11} ) 0.93, ( e_{16} ) 1.93</td>
<td>( d_{11} ) 0.878, ( d_{16} ) 1.72</td>
</tr>
<tr>
<td>H-PG-F</td>
<td>( e_{31} ) 1.38, ( e_{32} ) 0.60</td>
<td>( d_{31} ) 0.69</td>
</tr>
</tbody>
</table>

We note that the \( d_{11}, d_{16} \) of penta-BCN are higher than the \( d_{11}, d_{16} \) of h-BN and \( d_{31} \) of H-PG-F. Usually, the softer the material, the larger the \( d \) coefficient.⁴⁵ Though the piezoelectric stress tensor \( \epsilon \) of penta-BCN is comparable to that of h-BN, its piezoelectric strain tensor \( d \) is larger, because penta-BCN possesses smaller elastic constants. While in the functionalized pentagaphene sheet,⁴⁵ the polarization comes from the inequivalence of the H- and F-terminated surfaces. However, the small thickness of such a 2D sheet inhibits the full spatial separation of positive and negative charges, so the in-plane strain can hardly cause a significant change in polarization. This is why penta-BCN has a larger piezoelectric stress tensor \( \epsilon \) than that of H-PG-F. In addition, it is worth mentioning that the graphene-like BCN (h-BCN) monolayer has been synthesized recently,⁴⁶,⁴⁷ where the \( B_2C_N \) (BCNBN) hexagons are arranged antiparallel in a trigonal configuration. Thus, the preserved inversion symmetry hinders the form of electric dipole and piezoelectricity in such a structure. From above, one can see that penta-BCN shows advantages in piezoelectricity as compared to some other nonmetal-element-based 2D materials, suggesting that penta-BCN could be an ideal candidate for piezoelectric materials with robust electromagnetic coupling.

In summary, we have proposed a new ternary pentagonal sheet, penta-BCN, which is purely composed of BCN pentagons. By performing first-principles calculations combined with AIMD simulations, we have shown that penta-BCN is dynamically, thermally, and mechanically stable and is semiconducting with a direct band gap of 2.81 eV. The calculated Young’s modulus and Poisson’s ratio reveal that penta-BCN is mechanically anisotropic. The special arrangement of B–N bonds in this structure induces spontaneous polarization along the (010) direction with the magnitude of \( P_s \).
= 3.17 × 10−10 C/m, which is larger than that of h-BN because of the shorter B–N bond length. With noncentrosymmetric and semiconducting characteristics, penta-BCN exhibits piezoelectricity with d31 = 0.878 pm/V, d32 = −0.678 pm/V, and d16 = 1.72 pm/V, which are of the same order of magnitude as TMD materials and are larger than those in h-BN and hydrofluorinrgated PG. The spontaneous polarization and intrinsic piezoelectricity provide penta-BCN with great potential for a piezoelectric material at the nanoscale.

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**Notes**
The authors declare no competing financial interest.

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