Problem 1: Interactions between Two Surfaces with Arbitrary Potential. Consider two semi-infinite blocks of the same material separated by a distance h. Compute the interaction energy and force between the two blocks as a function of h when:

(a) The pairwise intermolecular potential between molecules composing the blocks is $Vr) = -Cr^m$, where C is a constant.

(b) The intermolecular potential is $V(r) = Ce^{-\beta r}$, where β is a positive constant.

Problem 2: Van der Waals Interactions between Two Surfaces with a Finite Thickness. Consider two infinite plates of the same material of thickness d separated by a distance h. The molecules composing the blocks interact via pairwise van der Waals interactions $V(r) = -Cr^6$, where C is a constant.

(a) What is the interaction energy between the plates as a function of h?

(b) How does this simplify in the limits of $d/h \ll 1$ and $d/h \gg 1$?

(c) How thick do the plates have to be so that the interaction energy is practically (within 1%) of that between two semi-infinite blocks? Estimate the dimensional distance using typical C based on London-Dispersion forces.

Problem 3: Traction Separation Relationship between Two Surfaces. Consider two semi-infinite blocks. The interaction energy per unit area between the two surfaces is given by

$$U(h) = -\frac{A}{12\pi h^2} + \frac{B}{48\pi h^8},$$

where h is the distance between the plates, $A > 0$ is the Hamaker constant, and $B > 0$ is the constant associated with the short-ranged Born repulsion between molecules.

(a) Sketch the energy as a function of h.

(b) For what value $h_{\min}$ is the energy a minimum? Answer in terms of A and B.

(c) The traction-separation relationship between the two surfaces is the force per unit area between the plates as a function of separation measured from $h_{\min}$. Sketch this force per unit area versus $h - h_{\min}$.

(d) Does a force maximum or minimum occur? If yes, what is it, and Occurring at what value of h?

(e) At what value of h is the force 1% of the force at the extrema?

Problem 4: Van der Waals Forces on an AFM Tip. Consider an AFM tip attached to a cantilever spring. The AFM tip interacts with the surface according to *a vdW interaction between a sphere and a flat surface*, which should be assumed to be attractive as discussed in the class. The height of the undeflected tip above the surface is H and the height of the deflected tip is h. Such deflection offers a resistance to the attraction like that of a Hookean spring, given by $k(H-h)$, where k is the spring constant. See the schematic below.

(a) Pull-in instability: At what critical distance $d_{\min}$ does the tip touch the surface?

(b) After the tip touches and makes contact with the surface with a fixed gap g, what D is required to pull the tip off the surface?

(c) Describe how you would measure the Hamaker constant if you can easily measure the deflection.