Problem 1: Capillary force of adhesion. A glass sphere of radius R is placed on a flat glass plate and a drop of liquid, surface tension γ, spreads between them. Assuming that the liquid completely wets the glass and that the radius of the sphere is much larger than the thickness of the liquid, show that the capillary force of adhesion is $4\pi\gamma R$.

Problem 2: Condensation on a Cylinder. A liquid film on a flat surface exposed to saturated vapor will grow indefinitely if the Hamaker constant across the film A_{SLV} is negative. However, this is not the case for a film on a curved surface.

(a) Explain why this is so.

(b) Derive a relationship for the equilibrium thickness D of the film on the surface of a cylindrical fiber of radius R in terms of A_{SLV}, R and γ (the surface tension of the liquid).

(c) For a quartz fiber of radius 10 μm in contact with a saturated vapor of octane at 20 °C, calculate the equilibrium thickness of the film on the fiber, neglecting gravitational effects. Assume that $A_{\mathrm{SLV}} = -0.7 \times 10^{-20}$ J and $\gamma = 21.6$ mJ/m^2.

Problem 3: Thickness of a lens of fluid. On page 113 in *Physical Chemistry of Surfaces*, 6th ed. by Adamson and Gast, the following formula is given for the limiting thickness t_∞ of a liquid lens floating on a liquid surface:

$$t_\infty^2 = -\frac{2S\rho_{\mathrm{A}}}{g\rho_{\mathrm{B}}\Delta\rho},$$

where S is the spreading coefficient, ρ_{A} and ρ_{B} are the densities of the underlying liquid and lens, respectively. $\Delta\rho = \rho_{\mathrm{A}} - \rho_{\mathrm{B}}$. This formula was originally derived by Irving Langmuir in 1933 and applies to large lenses. Considering the effects of buoyancy and interfacial energies in the bulk liquids, but not at the three-phase contact line, derive this equation.
Hint: The lens reaches a limiting thickness due to the action of gravity which flattens the bottom and top interfaces of the lens. Place a control volume that encompasses this region and a region far from the lens, and perform a force balance. Note that you will need to account for the partial floating of the lens using Archimedes' principal.

Problem 4: Capillary Force between Two Plates. Consider two plates a distance H apart. Initially, there is a drop of fluid between the two plates with a radius R. The fluid wets the surfaces of the plates with a contact angle θ. A force must be applied to pull the plates apart.

(a) Explain the origin of the force.

(b) Do you expect the force to be smaller or larger as the separation between the plates increases?

(c) Compute the force as a function of H, the separation distance between the plates (neglecting line forces). You may compute this from an energy perspective or a mechanical perspective. Pick one.

(d) Not a question. But if you feel interested, may read this paper https://doi.org/10.1103/PhysRevFluids.4.033601 for the case in which the plates are deformable.