Problem 1: Microspheres at an interface. Interestingly, it is energetically favorable for spheres to sit across the interface between fluids, α and β, provided the contact angle for the three phases is non-zero. (That is, neither fluid would perfectly wet the flat solid material in the presence of the other fluid.) Let us examine this system to understand it better.

(a) Show that for a non-zero contact angle θ, a sphere located at the interface is mechanically stable. (Hint: Compute the energy of a sphere sitting at the interface, and show that small deviations in position increase the energy.)

(b) What is the energy difference between a sphere straddling an interface and a sphere in solution relative to the background thermal energy $k_B T$ at room temperature? Assume the contact angle is $\pi/2$ and that interfacial tension is 30 mJ/m^2. Compute the energy difference for spheres of radii 10 nm, 100 nm, 1 μm and 10 μm.

Problem 2: Shape of a 2D sessile droplet. In class we have derived the equation for determining the shape of a 2D sessile droplet. The aim of this homework is to compute the shape of the droplet for a known dimensional drop area, A, three-phase contact angle θ, and surface tension γ.

(a) Re-derive the equation for the shape of the 2D droplet. The purpose here is to ensure you understand the mechanics of the derivation.

(b) Non-dimensionalize the equations using $A^{1/2}$ as the characteristic length-scale. What is the Bond number in the equation?

(c) What are the boundary conditions for this problem?

(d) What is the numerical strategy you would employ to determine the shape of the droplet for a given Bond number and contact angle?

(e) Solve the dimensionless shape of the droplet for a contact angle of $\pi/4$ for Bond numbers of $-1/2$, 0, $1/2$, 1, 2, and 10. Are the shapes reasonable and why? You may use Matlab, Mathematica, or any other software to determine the shapes.

Problem 3: Shape of a droplet sitting on an interface. Consider a droplet of liquid 1 of volume V straddling an interface between two other liquids 2 and 3. The interfacial tensions between liquids 1 and 2, liquids 2 and 3, liquids 1 and 3 are γ_{12}, γ_{23}, γ_{13}, respectively. Assume $\gamma_{12} = \gamma_{13} = \gamma$ and that the liquids all have the same density or the drop is sufficiently small that the gravitational forces do not matter. Under certain circumstances, the droplet will 'flatten' from a sphere to an oblate spheroid straddling the interface.

(a) Why might one expect an oblate spheroid shape? Draw a sketch and explain the physics.

(b) Suppose you wanted to determine the shape of this droplet. Starting from the Young-Laplace equation of $\gamma \nabla \cdot \mathbf{n} = \Delta p$, derive the equation and boundary conditions that determine the shape of the droplet.

(c) How would you solve this equation? You do not need to do so, but sketch out the procedure.

(d) Use energy arguments on a simple approximation of the droplet as a cylinder to determine the radius of the flattened drop.

(e) Discuss your solution. Is it reasonable? Where might it fail and why?