Problem 1: Lateral forces between two unlike surfaces. Consider two slightly buoyant rectangular particles sitting on an air-liquid interface with their nearest interfaces separated by a distance d, similar to the ones we discussed in class. This time, assume the particles are made of different materials so that the contact angle on one is θ_1 and on the other is θ_2.

(a) For the case when d is "small" and $\theta_1 + \theta_2 = \pi$, what is the shape of the interface between the two particles?

(b) What is the force between the two particles? Is it attractive or repulsive?

(c) Assume the contact angles are around $\pi/2$ and that the slope of the interface between the two particles is small. What are the equation and boundary conditions that describe the interface?

(d) Again assume $\theta_1 + \theta_2 = \pi$, plot the dimensionless interface shape versus the dimensionless lateral position x/ℓ_c, where $\ell_c = (\gamma/\rho g)^{1/2}$, ρ is the density of the fluid, g is the gravitational constant and γ is the interfacial tension. Make the plots for $x/\ell_c = 0.1, 0.5, 1, 5, 10$ and $\theta_1 = \pi/3$.

(e) How small must d be for the solution in part (a) to be valid?

(f) Again assuming $\theta_1 + \theta_2 = \pi$, what is the force between the particles as a function of d? Plot it.

(g) For what values of θ_1 and θ_2 do the particles repel one another?

Problem 2: Lateral capillary force between two cylinders. Consider two cylinders of radii a floating on an air-liquid interface with their axes of rotation perpendicular to the interface. The cylinders are very near each other with *a minimal separation distance of* d *with* $d \ll a$. The fluid has a density ρ and wets with the cylinders with a contact angle θ.

(a) In this limit, what is the height of the liquid $h(y)$ in the narrow gap between the cylinders?

(b) What is the formula for computing the force acting on a cylinder, again in the limit of $d \ll a$?

(c) Using the formula in part (b), what is the force acting on a cylinder?