Problem 1: Stability of a thin film in an electric field. In a paper in *Nature* it was shown that a thin film subject to an electric field can create patterns on a surface. Here we will examine the stability of a thin film subject to a uniform electric field. Consider a thin film of viscous, non-conducting fluid residing on an electrode at potential ϕ_b b. In the absence of the electric field, the film has thickness h_0, and at a distance H above the bottom electrode is another electrode maintained at potential ϕ_t. It can be shown that the additional pressure acting in the film due to the imposed electric field is equal to

$$p_e = \frac{1}{2}\epsilon_f \frac{(\phi_b - \phi_t)^2}{h + (\epsilon_f/\epsilon)H}$$

where h is the local thickness of the film and ϵ_f and ϵ are the permittivities of the fluid film and air, respectively.

(a) Modify the evolution equation for the thickness of the film h or thin-film equation to include the additional pressure due to the electric field. Include the effects of surface tension and van der Waals forces as well.

(b) Perform a linear stability analysis assuming sinusoidal disturbances in only one direction. Under what conditions do the disturbances grow or decay?

(c) What is the fastest-growing wavelength?

(d) What is the effect of van der Waals forces on this fastest growing mode, i.e., does it increases or decrease the wavelength? What is the inverse time constant, s for the fastest-growing mode?

Problem 2: Capillary Waves in a fluid of finite depth. Consider a film of fluid with density ρ and surface tension γ. The film is subjected to some force that creates a wave on the surface of the form

$$\eta(x,t) = \hat{\eta}\exp[i(kx - \omega t)] + h$$

where k is the wavenumber, x is the horizontal coordinate, ω is the frequency and t is time. The mean depth of the fluid is h and the amplitude of the wave is $\hat{\eta}$. Assume the amplitude is sufficiently small and gravity and surface tension are important.

(a) What are the linearized equations that describe the dynamics of the fluid and the interface? What are the boundary conditions?

(b) Compute the wave-speed as a function of the parameters and the depth of the fluid. What is the capillary wave speed? What is the gravity wave speed?