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Parameter-Dependent Lyapunov Function Method Applied to Satellite Formation Keeping Control*

Yue Xincheng, Yang Ying, Duan Zhisheng, Geng Zhiyong
State Key Laboratory for Turbulence and Complex System, Department of Mechanics and Aerospace Technology,
College of Engineering, Peking University, Beijing 100871, P. R. China
E-mail: xy916@pku.edu.cn

Abstract: In this paper, a parameter-dependent Lyapunov function method is applied to design robust controller for a satellite formation flying system around the Earth based on an uncertainty model derived from a nonlinear relative position equation. In this model, nonzero eccentricity and varying semi-major axis are included as parametric uncertainties. Exogenous perturbations including $J2$ perturbation, atmospheric drag and actuation are bounded by functional uncertainties. The model can be regarded as a polytopic type uncertain system with 8 vertices, then the parameter-dependent Lyapunov function is proposed to design H_2 and H_∞ controllers respectively to maintain the relative position of the formation flying system. The simulation demonstrates that the proposed controller can realize the control the satellites formation-keeping.

Key Words: Parameter-dependent Lyapunov Function, Satellite Formation Keeping, Polytope, Linear Matrix Inequality (LMI)

1 INTRODUCTION

In recent years, there has been increasing interest in the research of formation flying system (FFS). Due to the potential advantages over the conventional large size monolithic satellite, such as low cost, mission flexibility, improved observation efficiency, increased reliability and enhanced survivability, numerous FFS are currently being proposed for both military and non-military missions. To date, many missions involving FFS are being developed by diverse agencies, such as NASA's New Millennium Program (DS1, DS2, EO-1, EO-3, ST-5, ST-6, ST-7, and ST-8) [1], MAXIM (Micro-Arcsecond X-ray Imaging Mission), TPF (Terrestrial Planet Finder) [2, 6].

In relative dynamic models for FFS, the Clohessy-Wiltshire (CW) or Hill Equation is most widely used[2–6]Hill equation is established based on two major assumptions:1) the reference object is in circular or near circular orbit and 2) the distance between the objects is small in comparison to their orbital radii to support simplifications [3]. In reference [3], formation configurations including in-plane, in-track, circular and projected circular are designed, and the formation and station keeping in the presence of perturbations are also discussed. Reference [4] gives two evaluated design to maintain the formation based on a linear approach using optimal control theory and on a nonlinear approach based on Lyapunov stability concepts. The mixed H_2/H_∞ output-feedback controller with regional pole placement constraints is also designed for satellite formation keeping based on linear Hills equations in reference [6].

However, The Hill equation is a linear approximation of the real system, and does not contain atmospheric drag, J_2 perturbation, and uncertainties. Furthermore, the application of the Hill equation is restricted by the above mentioned two assumptions. Therefore, mitigating the relative distance drift effects coming from $J2$ and elliptic reference orbits have been investigated by many researchers. Inalhan and How summarize the development for the relative motion equation with eccentricity [5]. In reference [2], an uncertainty model is derived from a nonlinear relative position model with eccentricity and semi-major axis variation considerations. In that model, the eccentricity uncertainty and semi-major axis variation are modeled as parametric uncertainties. Other perturbations, such as J2 effects and atmospheric drag, are bounded by functional uncertainties. And a μ robust controller is designed capable of achieving desired performance, robustness, and fuel consumption requirements [2].

In this paper, a parameter-dependent Lyapunov Function method is firstly applied to design robust controller for satellite formation-keeping based on the model proposed in [2]. There has been a considerable amount of works on extensions and applications of parameter-dependent Lyapunov function method to the analysis and design of robust filters and controllers [7-9]. We improve the results of the reference [7] to design state-feedback controller for our problems in this paper.

2 RELATIVE DYNAMICS MODEL [2]

The following presents the dynamics for the relative motion of a satellite with respect to a reference on an ellipse orbit. The satellites formation system can be simplified as a leader-follower pair, and the location of follower satellite within a formation is given by $\vec{R}_F = \vec{R}_L + \vec{\rho}$,where $\vec{R}_L$ and $\vec{R}_F$ are position vectors from the center of the earth to the leader and follower satellite respectively, and R_L and R_F are the corresponding position magnitude, $\vec{\rho}$ is the relative position vector from the leader to the follower. The J_2 and atmospheric drag perturbation are be considered as the exogenous disturbance, the relative position model under the inverse square gravitation field is presented in Eq.1, which is expressed in the local horizontal local vertical frame (LVLH) and include exogenous control inputs. In LVLH, the x axis points radially outward from the earth's center, y axis is in-track direction along increasing true anomaly, and z axis completes the

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right-handed reference frame.

$$\ddot{\vec{\rho}} = -\ddot{\vec{\omega}} \times \vec{\rho} - 2\vec{\omega} \times \dot{\vec{\rho}} - \vec{\omega} \times (\vec{\omega} \times \vec{\rho}) + \frac{\mu}{R_L^3}\vec{R}_L - \frac{\mu}{R_F^3}\vec{R}_F + \vec{U} + \vec{a}_d \tag{1}$$

where μ is the gravitational constant of the earth, $\vec{U} = [u_x, u_y, u_z]^{\mathrm{T}}$ is the control acceleration acting on all three directions, and $\vec{a}_d$ is exogenous disturbance acceleration vectors. $\vec{\omega} = [0,0,\dot{f}_L]^{\mathrm{T}}$ and $\dot{\vec{\omega}} = [0,0,\ddot{f}_L]^{\mathrm{T}}$ are angular velocity of the leader with true anomaly rate $\dot{f}_L$ and true anomaly acceleration $\ddot{f}_L$. Letting $\vec{\rho} = [x,y,z]^{\mathrm{T}}$ and $\dot{\vec{\rho}} = [v_x, v_y, v_z]^{\mathrm{T}}$, the linear model considering the eccentricity reference orbit is derived in Eq.2 which neglect the exogenous part [2].

If the reference orbit is a circular or near circular, the model Eq.2 can be simplified as the CW (or Hill) equation shown in Eq.3,where $n = \sqrt{\mu / a_{L0}^3}$ is the natural frequency of the reference orbit and $a_{L0} = R_L$ is the nominal semi-major axis of the leader's orbit.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dot{f}_L^2 + 2\frac{\mu}{R_L^3} & \ddot{f}_L & 0 & 0 & 2\dot{f}_L & 0 \\ -\ddot{f}_L & \dot{f}_L^2 - \frac{\mu}{R_L^3} & 0 & -2\dot{f}_L & 0 & 0 \\ 0 & 0 & -\frac{\mu}{R_L^3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{3}$$

Using fundamental orbital mechanics describing planetary, the radius , the true anomaly rate and true acceleration can be written as

$$R = \frac{a(1-e^2)}{1+e\cos f} \tag{4}$$

$$\dot{f} = \frac{\sqrt{\mu a(1-e^2)}}{R^2} \tag{5}$$

$$\ddot{f} = \frac{-\mu e(1+e\cos f)^3 \sin f}{a^3(1-e^2)^3} \tag{6}$$

Substituting Eq.5 and 6 with subscript L into Eq.2, the linear dynamics model with eccentricity and semi-major axis variations can be rewritten as shown in Eq.7, where the exogenous disturbances are all included and regarded as functional uncertainties.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 \cdot A_{4,1} & A_{4,2} & 0 & 0 & 2n \cdot A_{4,5} & 0 \\ A_{5,1} & A_{5,2} & 0 & -2n \cdot A_{5,4} & 0 & 0 \\ 0 & 0 & -n^2 \cdot A_{6,3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} + \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix} \vec{a}_d \tag{7}$$

where

$$A_{4,1} = \frac{(1+e_L\cos f_L)^3(3+e_L\cos f_L)}{3(1-e_L^2)^3}(\frac{a_{L0}}{a_L})^3$$

$$A_{4,2} = \frac{2\mu e_L(1+e_L\cos f_L)^3 \sin f_L}{a_L^3(1-e_L^2)^3}$$

$$A_{4,5} = \frac{(1+e_L\cos f_L)^2}{(1-e_L^2)^{3/2}}(\frac{a_{L0}}{a_L})^{3/2}$$

$$A_{5,1} = -A_{4,2}$$

$$A_{5,2} = \frac{\mu e_L(1+e_L\cos f_L)^3 \cos f_L}{a_L^3(1-e_L^2)^3}$$

$$A_{5,4} = A_{4,5}$$

$$A_{6,3} = \frac{(1+e_{L\cos f_L})^3}{(1-e_L^2)^3}(\frac{a_{L0}}{a_L})^3$$

Comparing Eq. 7 with the CW model ($Eq.3$), the following statements can be made [2]. First, if the eccentricity is zero with a nominal semi-major axis, they are identical. Second, the eigenvalues of the CW equation are all on the imaginary axis.In this paper,according to the analysis of reference [2],eccentricity and semi-major axis variations are modeled parametrically as shown in Eq.8. The nominal model (CW equation) will be the equation without uncertainties $W_i\Delta, i \in 1,...,5$, where $\|\Delta\| \leqslant 1$ and W_i is the uncertainties boundary for the ith parameter. All exogenous perturbations are assumed to be bounded energy noise, and the components are expressed as $[w_x, w_y, w_z]^{\mathrm{T}}$.

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{v}_x\\ \dot{v}_y\\ \dot{v}_z\end{bmatrix}=\begin{bmatrix}0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ 3n^2(1+W_1\Delta)&0+W_2\Delta&0&0&2n(1+W_5\Delta)&0\\ 0-W_2\Delta&0+W_3\Delta&0&-2n(1+W_5\Delta)&0&0\\ 0&0&-n^2(1+W_4\Delta)&0&0&0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ v_x\\ v_y\\ v_z\end{bmatrix}+\begin{bmatrix}0_{3\times3}\\ I_{3\times3}\end{bmatrix}\begin{bmatrix}u_x\\ u_y\\ u_z\end{bmatrix}+\begin{bmatrix}0_{3\times3}\\ I_{3\times3}\end{bmatrix}\begin{bmatrix}w_x\\ w_y\\ w_z\end{bmatrix} \tag{8}$$

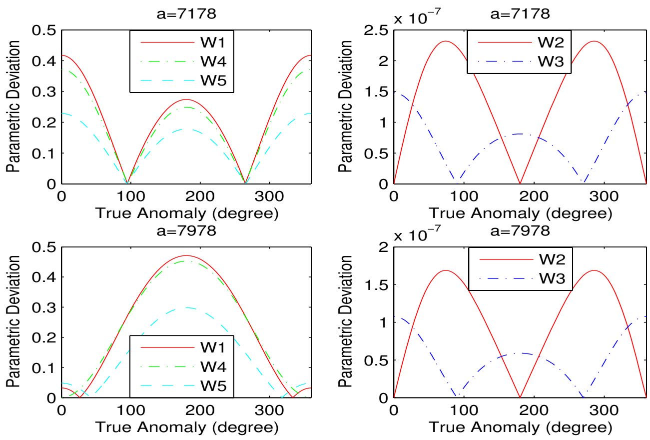

Fig. 1 Parametric absolute deviation due to nonzero eccentricity and different semi-major axis

As an example, for the eccentricity and semi-major axis parametric uncertainties, if the mission requires that the maximum eccentricity is bounded by 0.1 and the semi-major axis is varying between 7,178 and 7,978 km,two extreme cases are shown in Fig.1 From the figure, the uncertainty magnitudes are calculated as $W_1 = 0.47, W_2 = 2.3\times10^{-7}, W_3 = 1.5\times10^{-7}, W_4 = 0.44$, and $W_5 = 0.29$. The following controller design and simulation both are based on this example.

Remark 1 In this section, the relative dynamics model of satellite formation flying system mainly root in reference [2]. However, the reference [2] gives a improper formula for the Eq. 7, consequently the numerical values of W_i are incorrect for the same example.

3 RELATIVE POSITION KEEPING CONTROLLER DESIGN

The H_2 and H_∞ controllers for relative position keeping of the satellite formation flying system will be designed to achieve performance, robust with respect to the nonzero eccentricity and semi-major axis change. According to the example in the Section 2, we choose $W_1 = 0.47, W_4 = 0.44, W_5 = 0.29$, and $W_{2,3} = 0$ because the values are close to zero. When each Δ in the Eq.8 is set as $1, -1$ respectively, the system (8) can be regarded as a polytopic type uncertain system with 8 vertices, so parameter-dependent Lyapunov function method can be applied to the system to design robust H_2 and H_∞ controllers. Let $\boldsymbol{r} = [x, y, z]^{\mathrm{T}}, \boldsymbol{v} = [v_x, v_y, v_z]^{\mathrm{T}}$, the system 8 can be described by

$$\begin{aligned}\dot{\boldsymbol{r}} &= \boldsymbol{v}\\ \dot{\boldsymbol{v}} &= \tilde{A}_1\boldsymbol{r}+\tilde{A}_2\boldsymbol{v}+I_{3\times3}\boldsymbol{u}+I_{3\times3}\boldsymbol{w}\end{aligned} \tag{9}$$

where $\tilde{A}_1$ and $\tilde{A}_2$ are defined in Eq.8. In order to realize satellite formation station-keeping control, we set desire relative position $\boldsymbol{r}_c$ as constant. Considering the relative position error $\boldsymbol{e} = \boldsymbol{r} - \boldsymbol{r}_c$, the expanded system is given as

$$\begin{aligned}\begin{bmatrix}\dot{\boldsymbol{r}}_c\\ \dot{\boldsymbol{e}}\\ \dot{\boldsymbol{v}}\end{bmatrix} &= \begin{bmatrix}0&0&0\\ 0&0&I\\ \tilde{A}_1&\tilde{A}_1&\tilde{A}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{r}_c\\ \boldsymbol{e}\\ \boldsymbol{v}\end{bmatrix}+\begin{bmatrix}0\\ 0\\ I\end{bmatrix}\boldsymbol{u}+\begin{bmatrix}0\\ 0\\ I\end{bmatrix}\boldsymbol{w}\\ &\triangleq A\boldsymbol{x}+B_1\boldsymbol{u}+B_2\boldsymbol{w}\end{aligned} \tag{10}$$

The output of the system is defined as $\boldsymbol{y} = [y_1, y_2, y_3]^{\mathrm{T}} = \boldsymbol{e}$. In the following, we propose the theorem for designing a controller to the polytopic system. Considering the following continuous-time linear system

$$\begin{aligned}\dot{x} &= Ax+B_1u+B_2w\\ z &= Cx\end{aligned} \tag{11}$$

where $x\in\mathbb{R}^n$ is the system state vector, $z\in\mathbb{R}^m$ is the system output, $u\in\mathbb{R}^p$ is the exogenous disturbance signal, $w\in\mathbb{R}^q$ is the control input. Assuming that the system matrices lie within the polytope with N vertices

$$\begin{aligned}\Omega \triangleq \{(A,B_1,B_2,C)|&(A,B_1,B_2,C)\\ &=\sum_{i=1}^{N}\tau_i(A_i,B_{1,i},B_{2,i},C_i)\\ &\tau_i\geqslant 0,\sum_{i=1}^{N}\tau_i=1\}\end{aligned} \tag{12}$$

the state-feedback control problem is then to find, for prescribed scalars $0<\gamma_2,\gamma_\infty$, the state-feedback gain K such that the control law of

$$u = Kx \tag{13}$$

guarantees that the closed system is stable and the transfer function matrix satisfies

$$\|G(s)_{wz}\|_2 < \gamma_2 \tag{14}$$

$$\|G(s)_{wz}\|_\infty < \gamma_\infty \tag{15}$$

where $G(s)_{wz} = C[sI-(A+B_1K)]^{-1}B_2$. The lemmas for the controller design are given by the following. For the following continuous time linear closed system

$$\begin{aligned} \dot{x}_{cl} &= A_{cl}x_{cl} + B_{cl}w_{cl} \\ z_{cl} &= C_{cl}x_{cl} \end{aligned} \tag{16}$$

Lemma 1 [7] For a given $\gamma_2 > 0, \|G(s) = C_{cl}(sI - A_{cl})^{-1}B_{cl}\|_2 < \gamma_2$ is satisfied if and only if there exist matrices (F,G,P,Z) with $P = P^{\mathrm{T}} > 0$ such that

$$\begin{gathered} \begin{bmatrix} -G-G^{\mathrm{T}} & \Phi_1 & GB_{cl} \\ \Phi_1^{\mathrm{T}} & A_{cl}F^{\mathrm{T}}+FA_{cl} & FB_{cl} \\ B_i^{\mathrm{T}}G^{\mathrm{T}} & B_{cl}^{\mathrm{T}}F^{\mathrm{T}} & -I \end{bmatrix} < 0 \\ \begin{bmatrix} P & C_{cl}^{\mathrm{T}} \\ C_{cl} & Z \end{bmatrix} < 0 \\ trace(Z) < \gamma_2 \end{gathered} \tag{17}$$

where $\Phi_1 = P - F^{\mathrm{T}} + GA_{cl}$

Lemma 2 [7] For a given $\gamma_\infty > 0$ and for all nonzero $w \in \boldsymbol{L}_2[0,\infty]$, $\|z_{cl}\|_2 < \gamma_\infty\|w_{cl}\|_2$ if and only if there exist matrices (F,G,P) with $P = P^{\mathrm{T}} > 0$ such that

$$\begin{bmatrix} -G-G^{\mathrm{T}} & \Phi_2 & GB_{cl} & 0 \\ \Phi_2^{\mathrm{T}} & A_{cl}^{\mathrm{T}}F^{\mathrm{T}}+FA_{cl} & FB_{cl} & C_{cl}^{\mathrm{T}} \\ B_{cl}G^{\mathrm{T}} & B_{cl}F^{\mathrm{T}} & -\gamma_\infty I & 0 \\ 0 & C_{cl}^{\mathrm{T}} & 0 & -\gamma_\infty I \end{bmatrix} < 0 \tag{18}$$

where $\Phi_2 = P - F^{\mathrm{T}} + GA_{cl}$.

Base on the lemma 1 and 2, we can establish the following solutions for the robust H_2 and H_∞ control problems.

Theorem 1 **(H_2 controller)** Considering the system (11) on the polytope (12), a state-feedback controller of the form (13) that gives a suboptimal guaranteed minimizing $G(s)_{wz}$ can be derived from the following optimization:

$$\min trace(Z)$$

$s.t.$

$$\begin{gathered} \begin{bmatrix} -G-G^{\mathrm{T}} & \Psi_1^{\mathrm{T}} & G^{\mathrm{T}}C_i^{\mathrm{T}} \\ \Psi_1 & \Omega_1 & \alpha G^{\mathrm{T}}C_i^{\mathrm{T}} \\ C_iG & \alpha C_iG & -I \end{bmatrix} < 0 \\ \begin{bmatrix} P_i & B_{2,i} \\ B_{2,i}^{\mathrm{T}} & Z \end{bmatrix} < 0 \end{gathered} \tag{19}$$

where matrices (G,Q,Z,P_i) have proper dimension and $P_i = P_i^{\mathrm{T}} > 0$, $\Psi_1 = P_i - \alpha G^{\mathrm{T}} + A_iG + B_{1,i}Q$, $\Omega_1 = \alpha(A_iG + B_{1,i}Q) + \alpha(A_iG + B_{1,i}Q)^{\mathrm{T}}$ $i = 1,\cdots,N$, α is constant. When such a pair of matrices Q and G are found, a solution to the problem cab be given as

$$K = QG^{-1}$$

Theorem 2 **(H_∞ controller)** Considering the system (11) on the polytope (12), a state-feedback controller of the form (13) that guarantee (15) for a given $\gamma_\infty > 0$ can be derived if and only if there exist matrices (F,G,Q) and $P_i = P_i^{\mathrm{T}} > 0$ such that

$$\begin{bmatrix} -G-G^{\mathrm{T}} & \Psi_2^{\mathrm{T}} & G^{\mathrm{T}}C_i^{\mathrm{T}} & 0 \\ \Psi_2 & \Omega_2 & \alpha G^{\mathrm{T}}C_i^{\mathrm{T}} & B_{2,i} \\ C_iG & \alpha C_iG & -\gamma_\infty^2 I & 0 \\ 0 & B_{2,i}^{\mathrm{T}} & 0 & -\gamma_\infty^2 I \end{bmatrix} < 0 \tag{20}$$

where matrices (G,Q,P_i) have proper dimension, $\Psi_2 = P_i - \alpha G^{\mathrm{T}} + A_iG + B_{1,i}Q$, $\Omega_2 = \alpha(A_iG + B_{1,i}Q) + \alpha(A_iG + B_{1,i}Q)^{\mathrm{T}}$, $i = 1,\cdots,N$, α is constant. If the existence is affirmative, the state-feedback gain K is given by

$$K = QG^{-1}$$

Remark 2 The Lemma 1 and 2 can not utilized to design state feedback controller directly, because the corresponding matrix inequalities are not the standard LMIs. In Theorem 1 and 2, the forms of the LMIs are mended. Additionally, in the two lemmas, the matrix F and G are both related with A_{cl}, for simplifying our problem, the relationship of matrix F and G is set as $F = \alpha G$ in the above two theorems , $\alpha > 0$ is constant.

The theorem 1 and 2 can be applied to the system (10) to design robust H_2 and H_∞ controllers for the relative position keeping of the satellite formation. The problems are convex optimization problems involved LMIs which can be solved by mincx command in LMI Control Toolbox in MATLAB [10, 11].

4 NUMERICAL SIMULATION

In this section, numerical simulation is presented to test the proposed controller for satellite formation keeping. The initial state are assumed as

$$\begin{array}{lll} e_x = 10\mathrm{Km} & e_y = 20\mathrm{Km} & e_z = -10\mathrm{Km} \\ v_x = 1\mathrm{m/s} & v_y = 0.5\mathrm{m/s} & v_z = -0.5\mathrm{m/s} \end{array}$$

In computer simulation, the minimum problems are solved by MINCX solver in LMI Control Toolbox in Matlab, we can obtain $\gamma_2 = 0.0094$ and $\gamma_\infty = 0.9746$. However, the optimal problems have very high gains in state feedback matrix, consequently the controllers have serious fuel consumption. From the simulation results, we also find that the optimal controllers are not capable of improving the performance of the closed system markedly, and the computer CPU time is much long to solve the optimal problems. Therefore, we set $\gamma_2 = \gamma_\infty = 1$, so the minimum problems are transferred to the suboptimal feasible problems, and the results of the 8 vertices are shown in Fig. 2 and 3. As seen from the results, the proposed approach has accomplished the control of the satellites formation relative position keeping.

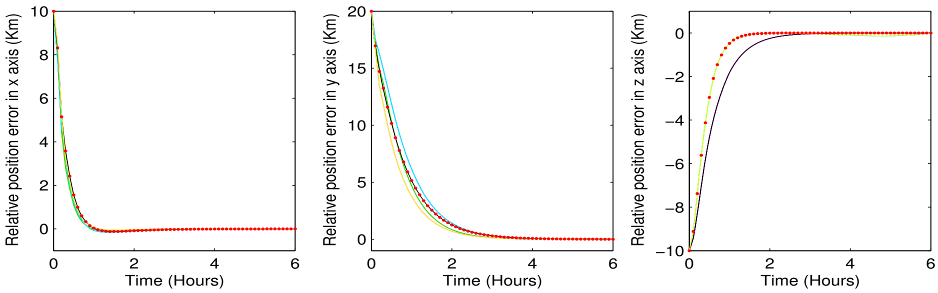

Fig. 2 Relative position error in three axes : H_2 control case

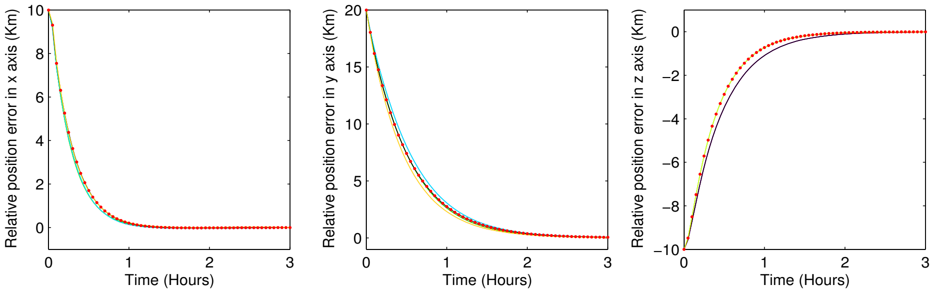

Fig. 3 Relative position error in there axes : H_∞ control case

5 CONCLUSION

In this paper, an uncertainty model with the zero eccentricity and varying semi-major axis consideration is derived based on the nonlinear relative position model for the leader-follower satellite formation. In this model, the nominal system is Hill equation, and the eccentricity and semi-major axis are incorporated into the model as parametric uncertainties, and the exogenous disturbances such as J_2 effect, atmosphere drag and sensor noise are bounded by functional uncertainties. Then a parameter-dependent Lyapunov function method is applied to the polytopic type system to design robust H_2 and H_∞ controllers for the relative position keeping of the formation flying system. The feasible simulation results demonstrate the validity of the above controller.

The main contributions of this paper have two areas: 1) we amend the relative dynamics model of the satellite formation proposed in the reference [2], and 2) we propose the theorems for design robust sate-feedback controller which also can be applied to other polytopic type uncertain system. However, the discussion of this paper is just a primary research, and future works will include the following topics: Firstly, in this paper, the exogenous disturbances are just bounded by functional uncertainties, further research will consider the effect of the J_2 effect, atmosphere drag and sensor noise etc; Secondly, we don't consider the constraint on the fuel consumption in this paper, which should be taken into account in the engineering practice.

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