Proceedings of the 29th Chinese Control Conference
July 29-31, 2010, Beijing, China

Modeling and Analysis of Quantized Feedback of Networked Control System*

WEN Kai, GENG Zhiyong
Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering,
Peking University, Beijing 100871, P. R. China
E-mail: kewin1983@126.com

Abstract: This paper studies the problem of stability for a class of networked control systems with the aperiodic transmission delays and the quantization errors. Under the assumptions that the system works under the Try-Once-Discard (TOD) protocol and the transmission delays are bounded, the entire system is modeled as a discrete system interconnected with a 2×2 block uncertainty, which is the typical structure in the robust control theory. With the approaches in robust control, three less conservative and easy to use results are derived by employing the small gain theorem, the Integral Quadratic Constraint (IQC) method and the μ tool, respectively. Finally, an illustrative example is presented to show the usefulness and effectiveness of the results.

Key Words: Networked Control Systems, Aperiodic Delay, Quantization Error, Robust Control

1 INTRODUCTION

With the enlarging scale of systems in industry, people have been searching for an effective management. The primary advantages make networked control system become a perfect choice. When a control loop is closed via network, we label it a networked control system (NCS)[1]. The defining feature of NCS is that the information (including the control related signals) is exchanged among control system components (sensors, controllers, actuators) using a band width limited serial communication channel[2]. This type of information transmission reduces system wiring, eases maintenance and diagnosis and increases system agility[3]. The introduction of the network also brings some new problems and challenges, such as network induced delay, packet dropout, finite word length effect, time-varying transmission period and multi-packet transmission[4]. One can see that NCS is convenient in practice but complex in theoretic analysis.

In all these problems, the network induced delay has received the most attention. In [5], a new method to obtain a maximum allowable delay is proposed. The results relating to the stabilization problem for a class of NCS with random delays have been reported in [6], in which sufficient conditions for the solvability of this problem are obtained and a Linear Matrix Inequality (LMI) approach is proposed. By decomposing time delay into the fixed and varying parts, the NCS with time-varying delays is modeled as a parameter-uncertain system and the controller design methods based on the delay dependent stability conditions are presented in [7]. There are a number of other stability criterions of NCS have been obtained directly from the constructed Lyapunov-Krasovskii functions[8–11], where NCS is treated as the time delay system.

It is obvious that network induced delay is mainly influenced by the network scheduling, which is decided by the communication protocol of the network. With scheduling, the controllers get the measurement one by one and send the control signals to the actuators. For the controllers, this process is much like sampling the output of the plant with the help of the communication channels. So, another effect of the scheduling is sampling. In [12], the author gives an interval model of NCS with time-varying sampling periods and time-varying network induced delays and studies the stability of the sampled-data networked control system using the Lyapunov-Krasovskii functional. Motivated by the widespread use of networked and embedded control systems, [13] has considered the stability of aperiodically sampled-data feedback control systems and proposed a stability analysis algorithm based on the small gain framework by showing robustness of sampled-data systems against perturbations caused by the variation of the sampling intervals. The most significant contribution of this paper is that it gives an algorithm to check the stability different from the so-called input delay approach[14]. The way it processes the aperiodic sampling is beautiful for the robust control.

Besides scheduling, digitization is another difference between NCS and the classical closed-loop system. The classical control theory, which is based on the standard assumption that data transmission required by the system can be performed with infinite precision, may not be valid in NCS. In NCS, the signals are digitized and due to the finite word length effects, only a finite number of bits can be transmitted over the channel during any transmission interval. The main issue in control (stabilization) of systems with such channels is that of quantization[15]. The network based control has strengthened the importance of the study on quantized feedback control[16]. [17] considers the effects of quantization in NCS. Sufficient stability conditions for both the static and dynamic quantization schemes are derived. For the feedback control with static quantizers, [18] revisits the sector bound approach to the quantized feedback control, gives a comprehensive study on the systems with logarithmic quantizers, and presents the complete results for both the SISO and MIMO linear discrete-time systems. Following this work, [19] studies the situation of the network induced delay together with the quantization nonlinearity. A new general framework based on the quantization dependent Lyapunov function is proposed, and less conservative results are obtained.

* This work is supported by National Nature Science Foundation under Grant 10832006.

As is known, the data generated by the sensors need not be immoderately accurate, otherwise they will be saturated and the longer delays will deteriorate the control performance. On the other hand, the coarse data will enlarge the upper bound of the uncertainty caused by quantization. So, the effectiveness of both the bandwidth and the quantization should be considered simultaneously in the analysis of NCS. Several results have been published regarding quantization issues and sampled data problems[20]. These results attempt to characterize the stability properties of NCS when the number of bits used by each network packet is finite and small. [15] brings forward a unified emulation-like approach to the analysis and design of the control systems with quantization and time scheduling.

Inspired by the results in [13, 18], the aim of this paper is to consider the stability analysis of NCS under the robust control framework. We will show that NCS under Try-Once-Discard (TOD) protocol can be treated as a time varying sampled data system with transmission delays and sector bounded uncertainties in the input channel. And the time varying sampling system can be treated as a periodic sampling system with bounded uncertainties. With these steps, NCS is modeled as a discrete time LTI plant with a 2×2 blocked uncertainty. Under the framework for robust control, an attempt is made to get rid of the familiar Lyapunov function and LMI pattern for the system analysis and give a less contractive result based on the Integral Quadratic Constraint (IQC) method and the convenience result for designing based on μ tool.

The rest part of the paper is organized as follows. In section II, the notations that will be used are given and the system that will be dealt with is modeled in a general form. In section III, the equivalent system model that is typical in robust control is got first, and then with the robust analysis methods, three theorems are obtained. In section IV, the simulation of a networked control system is given to show the effectiveness of our results.

2 NOTATIONS AND MODELING OF NETWORKED CONTROL SYSTEM

2.1 Notations

The set of real numbers is denoted by R and the positive integers by Z_+. L_∞ is the standard Banach space of essentially bounded and measurable functions on R, while H_∞ is the usual Banach space of bounded and analytic (scalar or matrix-valued depending on the context) functions in the open right half plane. $\|\cdot\|$ denotes the usual L_∞ (or H_∞) norm. Operator is a mapping from one normed space into another, which can be considered as a mathematical object representing an input output system. A causal operator $H : L_e \to L_e$ is bounded if the gain defined as

$$\|H\| = \sup_{f \in L, f \neq 0} \frac{\|H(f)\|}{\|f\|}$$

is finite.

Then we give the notion of Integral Quadratic Constraint (IQC), which makes it much easier to obtain general and flexible results when multipliers and loop transformations are used in the framework of the small gain theorem.

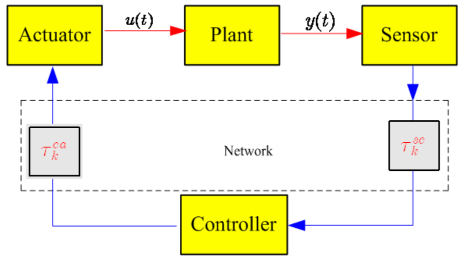

Fig. 1 Networked Control System

Definition (IQC)[21] 1 *Let $\Pi : jR \to C$ be a bounded and self-adjoint operator and $\Delta : L_e \to L_e$ be a causal bounded operator. Then Δ satisfies the IQC defined by Π if*

$$\sigma(v, \Delta(v)) = \left\langle \begin{bmatrix} v \\ \Delta(v) \end{bmatrix}, \Pi \begin{bmatrix} v \\ \Delta(v) \end{bmatrix} \right\rangle \geqslant 0.$$

We will use the shorthand notation $\Delta \in IQC(\Pi)$ to mean that Δ satisfies the IQC defined by Π.

2.2 Modeling of Networked Control System

In the communication network, one of the main indices is the quality of service (QoS). In order to guarantee QoS, buffers, priorities are allocated to each connection and the feedback congestion controls are used to allocate the available link capacities. It is required to be robust with respect to the unknown network parameters and the loss of control information. In contrast to the widely used computer network, NCS is concerned primarily with the quality of real-time reliable service. The control of the network is just one of the steps to the quality of performance (QoP), that is, the performance of the controlled plant is the aim. The network induced delay directly contributes to the delay in the control loop and may lead to the instability of the plant. Thus, in real-time control systems, the newest data is the best data. Following the simple TOD protocol first proposed in [1], we assume that the sampling rate at the output of the plant is fast enough to ensure that the sensor always has new data to transmit. Consider the typical networked control system setup as illustrated in Fig.1. The plant is a linear time invariant system described by

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where x and u denote the state and the input taking values in R^n and R^m respectively. Assume (A, B) is stabilizable and there exists K such that the eigenvalues of $A + BK$ have the negative real parts, which means K stabilizes the plant. So, let K be the feedback gain matrix, the closed-loop system is

$$G : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ u(t) = Kx(t) \end{cases}. \tag{2}$$

But the controller is connected to the plant via a network which should also be included in the system.

Fig.1 shows a configuration of a single loop NCS whose feedback loop is closed via a network. In general, the sensor samples the output signals, and the quantizer converts the measurements into the digital format, puts them into a packet. Then, the packet is sent to the controller when it gets

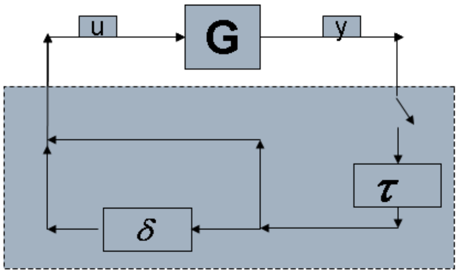

Fig. 2 the Identical System

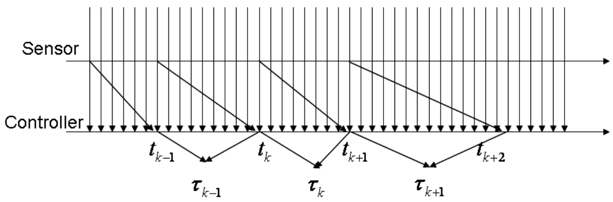

Fig. 3 the TOD protocol and Time Delay

the permission to access the network. The controller uses the received data to generate a control signal which is then sent to the actuator, where the zero-order holder keeps the signal constant until the next one comes. Considering the effects of the network, we pull all the components out and leave alone the plant. So the system in Fig.1 is equivalent to the one in Fig.2.

The first and foremost issue needed to deal with is the networked induced delay, which is unavoidable in NCS. There are two sources of delays from the network: sensor-to-controller delay τ_{sc} and controller-to-actuator delay τ_{ca}. The controller computational delay can be absorbed into either τ_{sc} or τ_{ca} without loss of generality. For the fixed control law, the sensor-to-controller delay and controller-to-actuator delay can be lumped together as $\tau_k = \tau_{sc} + \tau_{ca}$ for analysis purposes[22]. τ_k is not necessarily less than the sampling period, which means that there are packet dropouts during transmission. With the network induced delay, the control input is calculated based on the delayed state,

$$\begin{array}{l} \tilde{x}(t) = x(t-\tau_k) \\ u(t) = K\tilde{x}(t) \end{array} . \tag{3}$$

The delay is included in this expression and the next step is to consider the sampling effect of the network.

This paper takes the network employing TOD protocol into consideration. As indicated in [1], TOD protocol chooses the new sampling data and discards the old data that has not been transmitted. The fast sampling assumption enables us to focus on the major problem about the delays that are induced by the network. It is known that the transmission itself is not time-consuming and the delays are mainly caused by waiting the permission to access the network. The controller keeps using the information arrived at time t_{k-1} until the next one arrives at time t_k. And under TOD protocol, the data that arrives at the controller side at time t_k is right the state that the sensor samples from the plant at time t_{k-1}. This can be seen from Fig.3. So the signal for the controller is updated at time t_k with the successfully transmitted state value of the plant at time t_{k-1}. This can be expressed

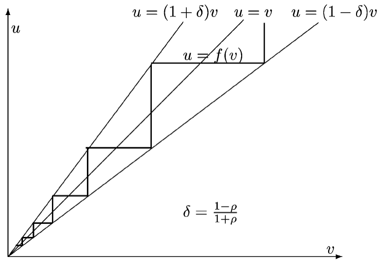

Fig. 4 Logarithmic quantizer

as

$$\begin{array}{l} \tilde{x}(t_k) = x(t_{k-1}) \\ u(t) = K\tilde{x}(t_k), \quad t \in [t_k, t_{k+1}). \end{array} \tag{4}$$

From equation 4, one can see that the network works as if a sensor that samples the state of the plant aperiodically and outputs the delayed data. All the successfully transmitting time $\{t_k\}$ is denoted as an uncertain set of discrete time instances satisfying $t_0 = 0$ and $0 < h_l \leqslant t_{k+1} - t_k \leqslant h_u < \infty$ for the given h_l and h_u. Let the sampling interval be $t_{k+1} - t_k = \tau_k$. If τ_k is constant for all sampling, the system is periodic [13]. Because of the scheduling of the network and the stochastic perturbations in environment, τ_k is time varying. But it is assumed to be bounded in $[h_l, h_u]$. Now the system can be viewed as an aperiodically sampled data control system, which is well studied in [23].

As mentioned at the beginning of the paper, the finite word length effect is another issue in NCS, which comes to quantization problem in [15]. Here, we consider the logarithmic static and time invariant quantizer. The static quantization policy presumes that the quantized result depends only on the input at that time. It has the following form

$$u(k) = f(v(k)), \tag{5}$$

where $f(\cdot)$ is the quantizer, which is assumed to be symmetric, i.e. $f(-v) = -f(v)$. A quantizer is called logarithmic if the set of quantized levels is characterized by

$$\mathcal{U} = \{\pm u_i, u_i = \rho^i u_0, i = \pm 1, \pm 2, \cdots\} \cup \{\pm u_0\} \cup \{0\}, \quad 0 < \rho < 1, u_0 > 0. \tag{6}$$

So the quantizer f is defined as follows[18],

$$f(v) = \begin{cases} u_i, & \frac{1}{1+\delta} u_i < v \leqslant \frac{1}{1-\delta} u_i, v > 0, \\ 0, & v = 0, \\ -f(-v), & v < 0, \end{cases} \tag{7}$$

where $\delta = \frac{1-\rho}{1+\rho}$, and it is illustrated in Fig.4. The quantization error is defined by

$$e(k) = u(k) - v(k) = f(Kx(k)) - Kx(k) = \Delta(k)Kx(k), \tag{8}$$

where $\Delta(k)$ is a scalar and belongs to $[-\delta, \delta]$. We see from Fig.4 that for a logarithmic static quantizer, the quantization error is bounded by a single parameter sector which is related to the quantization density via δ.

With the analysis above, the resultant feedback system, denoted by T, is described by

$$\dot{x}(t) = Ax(t) + BK(1+\Delta)x(t_{k-1}), t \in [t_k, t_{k+1}), \tag{9}$$

where Δ is the short term of $\Delta(k)$. The state of the system is evolving as follows,

$$x(t_{k+1}) = e^{A\tau_k}x(t_k) + \int_0^{\tau_k} e^{Ah}dhBK(1+\Delta)x(t_{k-1})$$
$$= A(\tau_k)x(t_k) + B(\tau_k)K(1+\Delta)x(t_{k-1}), \tag{10}$$

where $A(\tau_k) = e^{A\tau_k}$ and $B(\tau_k) = \int_0^{\tau_k} e^{Ah}dhB$ are varying according to τ_k. If all the time intervals between two successful transmitted signals are the same to h_0, $A(\tau_k)$ and $B(\tau_k)$ are constant matrices and the system is periodic. The quantized feedback sampled data system is well studied in [23]. Actually, even a well-designed network can fail to perform the equal time interval transmission. But the assumption that the interval is centered around τ_0 is reasonable. In order to express this property, we define

$$\tau_k = \tau_0 + \theta_k, \tag{11}$$

where θ_k is the perturbation of the network and belongs to $[h_l - \tau_0, h_u - \tau_0]$. Let[13]

$$\Delta(\theta_k) = \int_0^{\theta_k} e^{A\eta}d\eta, \tag{12}$$

it follows that

$$A(\tau_k) = e^{A\tau_k} = e^{A(\tau_0+\theta_k)} = (I + \Delta(\theta_k)A)A(\tau_0),$$
$$B(\tau_k) = \int_0^{\tau_k} e^{A(\tau_k-\eta)}d\eta B$$
$$= \int_0^{\tau_0} e^{A(\tau_0+\theta_k-\eta)}d\eta B + \int_{\tau_0}^{\tau_0+\theta_k} e^{A(\tau_0+\theta_k-\eta)}d\eta B$$
$$= (I + \Delta(\theta_k)A)B(\tau_0) + \Delta(\theta_k)B. \tag{13}$$

For only the state of the updating time, the discrete time system is obtained,

$$x(k+1) = [(I + \Delta(\theta_k)A)A(\tau_0)]x(k)$$
$$+[(I + \Delta(\theta_k)A)B(\tau_0) + \Delta(\theta_k)B]K(1+\Delta)x(k-1). \tag{14}$$

It should be noticed that $\Delta(\theta_k)$ and Δ are two different terms.

3 MAIN RESULTS

This section is to derive the condition under which the system T in (14) is robust stable for all the delays in (11) and the quantization satisfying (7).

For the system with uncertainties, the basic principle is often referred to as "pulling out the Δ's"[24]. System in Fig.2 can be redrawn as a standard one via pulling out the Δ's. Define $z(k)^T = [x(k)^T \quad x(k-1)^T]^T$ as the augmented state vector, the matrices of the augmented closed-loop system can be obtained by computing the corresponding transfer matrices in the original system, where

$$\tilde{G} = \begin{bmatrix} AA(\tau_0) \\ AB(\tau_0) + B \end{bmatrix} \left(zI - \begin{bmatrix} A(\tau_0) & B(\tau_0)K \\ I & 0 \end{bmatrix} \right)^{-1}, \tag{15}$$

and all the Δ's are lumped into a block matrix

$$\tilde{\Delta} = \begin{bmatrix} \Delta(\theta_k) & B(\tau_0)K\Delta \\ & \Delta(\theta_k) \end{bmatrix}. \tag{16}$$

Now, the system is transformed into a standard block structure as illustrated in Fig.5.

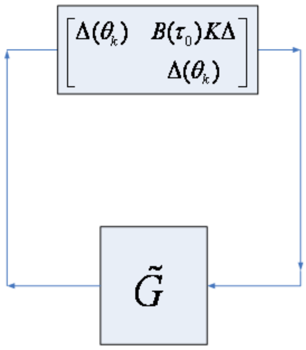

Fig. 5 the Equivalent System

The familiar approach to check the stability of the following system

$$x(k+1) = Mx(k) + Nx(k-1) \tag{17}$$

is using the Lyapunov function as follows

$$V(k) = x(k)^T Px(k) + x(k-1)^T Rx(k-1). \tag{18}$$

The difference of $V(k)$ is

$$V(k+1) - V(k) = \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}^T \Omega \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}, \tag{19}$$

where

$$\Omega = \begin{bmatrix} M^TPM - P + R & M^TPN \\ * & N^TPN - R \end{bmatrix}.$$

Thus, the stability criterions can be formulated as the feasibility of

$$\Omega < 0. \tag{20}$$

A powerful and efficient toolbox in MATLAB has been available for solving the LMI feasible problems. The results are easy and ready to get, but not that convictive in practice. Because it hides most of the information for practical usage and transfers the system analysis problem to a meaningless mathematical problem. In some situations, it is hard to find a common matrix $P = P^T > 0$ for inequality (20) with all values in $[h_l, h_u]$ and the time varying matrices M, N. If the LMI is infeasible, it gives null information and that is little help for the analysis and synthesis. The method in [25] can be used to reduce the conservativeness of this common Lyapunov function method. But we are not going to expand the research in this way.

Fig.5 is the typical block diagram in robust control, and in the robust control framework, there are many powerful tools to deal with this kind of systems, which will be illustrated in the following.

Regarding system T as a feedback connection of the LTI system $\tilde{G}$ and the time varying matrix $\tilde{\Delta}$. We obtain a criterion by simply applying the small gain theorem.

Theorem 1 *Let the system in Fig.5 is well-posed. Let the time interval of the sampling and the boundary of the quantizers be given. There exists a common matrix P for inequality (20) if $\rho(A+BK) < 1$, and*

$$\gamma\|\tilde{\Delta}\| \leqslant 1, \tag{21}$$

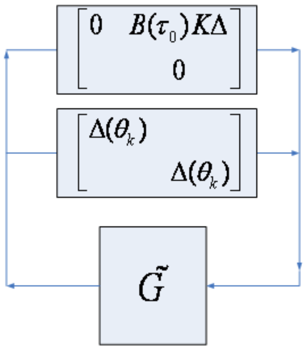

Fig. 6 the Equivalent System

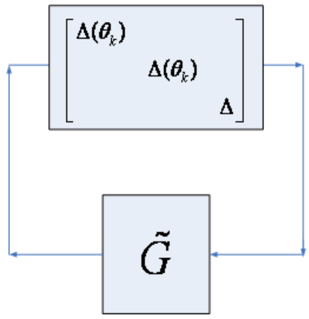

Fig. 7 the Equivalent System

where γ is an upper bound of $\|\tilde{G}\|$:

$$\gamma > \|\tilde{G}\|. \tag{22}$$

Remark This result is obvious, so, the proof is omitted. The purpose of this theorem is to find out 'how big' $\tilde{\Delta}$ can be before instability occurs. Note that $\tilde{\Delta}$ is a 2×2 block matrix which has structure information, so, it is conservative to deal with it as a full block uncertainty in theorem 1.

The difficulty in dealing with this uncertainty is the upright part of $\tilde{\Delta}$. Following the process in [21], we split $\tilde{\Delta}$ into two blocks as in Fig.6 and get theorem 2.

Theorem 2 *Let the system in Fig.6 is well-posed. If*
(i) $\Delta(\theta_k) \in IQC(\Pi)$
(ii) $\|\Delta\| \leqslant \gamma$
(iii) there exists $\varepsilon > 0$ such that

$$\varepsilon > \left(\frac{2b}{a} + \frac{\gamma\|\tilde{G}\|^2}{1-\gamma\|\tilde{G}\|}\right) \cdot \frac{\gamma\|\tilde{G}\|^2}{1-\gamma\|\tilde{G}\|} \tag{23}$$

and

$$\begin{bmatrix} \tilde{G} \\ I \end{bmatrix}^* \Pi \begin{bmatrix} \tilde{G} \\ I \end{bmatrix} \leqslant -\varepsilon I, \tag{24}$$

where $a = \|\Pi_{11}\|$, $b = \|\Pi_{11}\| \cdot \|\tilde{G}\| + \|\Pi_{12}\|$, then the system is stable.

Proof. Let $w = \Delta(\theta_k)v$ and all the signals are bounded. For $\Delta(\theta_k) \in IQC(\Pi)$, we have

$$\begin{aligned}
0 &\leqslant \left\langle \begin{bmatrix} v \\ w \end{bmatrix}, \Pi \begin{bmatrix} v \\ w \end{bmatrix} \right\rangle \\
&= \left\langle \begin{bmatrix} v - \tilde{G}w + \tilde{G}w \\ w \end{bmatrix}, \Pi \begin{bmatrix} v - \tilde{G}w + \tilde{G}w \\ w \end{bmatrix} \right\rangle \\
&= \left\langle \begin{bmatrix} v - \tilde{G}w \\ 0 \end{bmatrix}, \Pi \begin{bmatrix} v - \tilde{G}w \\ 0 \end{bmatrix} \right\rangle \\
&\quad + 2\left\langle \begin{bmatrix} v - \tilde{G}w \\ w \end{bmatrix}, \Pi \begin{bmatrix} \tilde{G}w \\ w \end{bmatrix} \right\rangle + \left\langle \begin{bmatrix} \tilde{G}w \\ w \end{bmatrix}, \Pi \begin{bmatrix} \tilde{G}w \\ w \end{bmatrix} \right\rangle \\
&\leqslant \|\Pi_{11}\| \|(I - \tilde{G}\Delta(\theta_k))(v)\|^2 \\
&\quad + 2\left(\|\Pi_{11}\| \cdot \|\tilde{G}\| + \|\Pi_{12}\right) \cdot \|(I - \tilde{G}\Delta(\theta_k))(v)\| \cdot \|w\| \\
&\quad - \varepsilon\|w\|^2,
\end{aligned} \tag{25}$$

where the first inequality follows since $\Delta(\theta_k) \in IQC(\Pi)$ and the last inequality follows from standard use of Cauchys inequality and the stability condition (24). Then we get

$$\|v\| \leqslant c_0 \|(I - \tilde{G}\Delta(\theta_k))(v)\|, \tag{26}$$

i.e.

$$\|(I - \tilde{G}\Delta(\theta_k))^{-1}\| \leqslant c_0, \tag{27}$$

where

$$c_0 = (1 + \|\tilde{G}\|/c_1),$$

$$c_1 = -\frac{b}{a} + \sqrt{\frac{b^2}{a^2} + \frac{\varepsilon}{a}},$$

$$a = \|\Pi_{11}\|, b = \|\Pi_{11}\| \cdot \|\tilde{G}\| + \|\Pi_{12}\|.$$

Then consider

$$[I - \tilde{G}(\Delta(\theta_k) + \Delta)] = (I - \tilde{G}\Delta(\theta_k))[I - (I - \tilde{G}\Delta(\theta_k))^{-1}\tilde{G}\Delta], \tag{28}$$

from inequality (27), equation (28) is reversible if

$$\|(I - \tilde{G}\Delta(\theta_k))^{-1}\tilde{G}\Delta\| = \|\tilde{G}\Delta\| \cdot \|(I - \tilde{G}\Delta(\theta_k))^{-1}\| < 1, \tag{29}$$

i.e.

$$\|\Delta\| \leqslant \gamma < \frac{1}{c_0\|\tilde{G}\|}. \tag{30}$$

The theorem is proven.

Remark In this paper, the nominal system is assumed to be stable then equation (24) is equivalent to the condition

$$\begin{bmatrix} \tilde{G}(e^{jw}) \\ I \end{bmatrix}^* \Pi(e^{jw}) \begin{bmatrix} \tilde{G}(e^{jw}) \\ I \end{bmatrix} \leqslant -\varepsilon I, \forall w \in [-\pi, \pi]. \tag{31}$$

Using Kalman-Yakubovich-Popov (KYP) lemma[26], this frequency domain criterion is equivalent to a number of conditions on the system matrices in the realization of $\tilde{G}$ and Π and it is numerical solvable in a LMI form.

Another powerful tool to deal with the structured uncertainties is μ theory. The uncertain block $\tilde{\Delta}$ in theorem 1 is not a full matrix nor a diagonal one. It has certain structure but not in a diagonal form that can be treated directly by μ theory. For equation (14), if we define $z(k)^T = [x(k)^T \quad x(k-1)^T \quad x(k-1)^T]^T$ as the augmented state vector, the augmented closed-loop system is rearranged in a standard configuration of μ theory as in Fig.7, where

$$\tilde{G} = \begin{bmatrix} AA(\tau_0) \\ AB(\tau_0) + B \\ B(\tau_0)K \end{bmatrix} \left(zI - \begin{bmatrix} A(\tau_0) & B(\tau_0)K & 0 \\ I & 0 & 0 \\ I & 0 & 0 \end{bmatrix} \right)^{-1}, \tag{32}$$

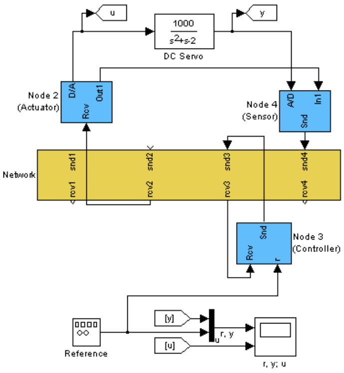

Fig. 8 the Simulation of System

and the uncertain block would then be in the following general form

$$\bar{\Delta} = \begin{bmatrix} \Delta(\theta_k) & & \\ & \Delta(\theta_k) & \\ & & \Delta \end{bmatrix}. \tag{33}$$

Fig.7 is the typical block diagram in μ theory.

Theorem 3 *Let the system as in Fig.7 is well-posed. The system is stable if*
(i) $\|\Delta(\theta_k)\| < \gamma$
(ii) $\|\Delta\| \leqslant \delta$
(iii)

$$\mu_{\bar{\Delta}}(\bar{G}) < \frac{1}{max\{\gamma, \delta\}}. \tag{34}$$

Remark The reciprocal of the structure singular value denotes a frequency dependent stability margin. Follow this stability result, we can figure out which part of the system that needs improvement. It is easy to understand and ready to use in practice.

4 EXAMPLE AND SIMULATION

To illustrate the validity of the results obtained, let us consider the following example. In this example, Truetime Toolbox[27] is used to build up the network environment including the sensor, communication bus, and the actuator. System in Fig.8 simulates the general NCS structure in Fig.1. In the control loop, the sensor, the actuator, and the controller are spatially distributed in network. The controller and the actuator nodes are event driven. The sensor node works under TOD protocol, which means the sensor samples the process and sends the newest measurement to the controller node whenever it gets the permission to access the network. The controller computes the control input signal based on the predesigned control law and sends it via the network to the actuator node. The significant difference to the periodic sampling is that it may center around the designed period and vary according to the network environment. The plant is a DC servo and the transfer function is[27]

$$G(s) = \frac{1000}{s^2 + s - 2}, \tag{35}$$

and the controller is

$$K(s) = 1.5(1 + 0.035s), \tag{36}$$

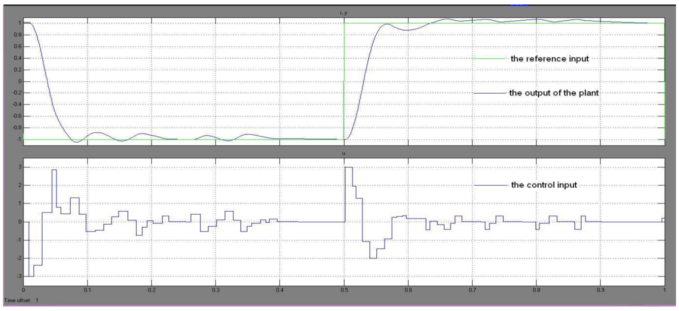

Fig. 9 the Simulation Result

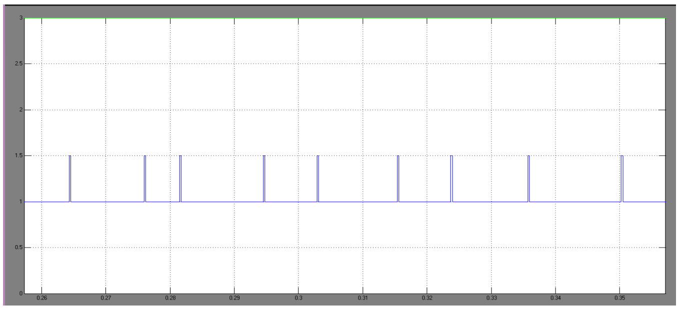

Fig. 10 the Transmission Time of the Data

which is a typical PD controller.

In the simulation, the nominal transmission period is $h_0 = 0.01s$ with random perturbations taking values in $[-0.005, 0.005]$s as illustrated in Fig.10. From equation (37), the quantization is expressed as follows,

$$u = (1 + \Delta)v, \|\Delta\| < 0.0255. \tag{37}$$

The simulation result is shown in Fig.9 and the entire system is stable. For $\theta_k \in [-0.005, 0.005]$, $\|\Delta(\theta_k)\|$ is in $[0.995, 1.005]$. Because the H_∞ norm of $\tilde{G}$ in theorem is 0.6676, theorem 1 is verified. Using KYP lemma to get the LMI result of theorem 2, where $\Pi = \begin{bmatrix} -0.995 \times 1.005 & 0.995 \\ 1.005 & -1 \end{bmatrix}$, it is also hold for this system. As to theorem 3, it holds if theorem 1 holds. These results demonstrate the effectiveness of the obtained criterions.

5 CONCLUSION

In this paper, we model the NCS as a system with the time varying transmission intervals and the quantization errors, which is intrinsically different from some existing results. Inspired by the work in [13, 18], the NCS is described by a perturbed discrete time system in the robust control framework. Three stability criterions have been proposed that are less conservative and not difficult for application in practical analysis.

Although both the uncertain transmission intervals and the quantization errors are considered in this paper, there are still issues such as scheduling of the network, multi-packet transmission in NCS. The modeling and analysis of the networked control system in a more precise manner is worthy of being further studied.

REFERENCES

[1] G. C. Walsh, H. Ye, and L. Bushnell. Stability analysis of networked control systems[C]// Proceedings of the American Control Conference. 1999: 2876-2880.

[2] N. H. El-Farra. Failure analysis in networked process control systems with control and communication constraints[C]// http://www.nt.ntnu.no/users/skoge/prost/proceedings/aiche-2005/non-topical/Non%20topical/papers/454g.pdf

[3] T. C. Yang. Networked control system: a brief survey[C]// IEE Proceedings of Control Theory Application. 2006: 403-412.

[4] X. L. Zhu, G. H. Yang. State feedback controller design of networked control systems with multiplepacket transmission[J]. International Journal of Control, 2009, 82(1): 86-94.

[5] D. Kim, Y. S. Lee, W. H. Kwon. Maximum allowable delay bounds of networked control systems[J]. Control Engineering Practice, 2003, 11(11): 1301 - 1313.

[6] L. Zhang, Y. Shi, T. Chen. A new method for stabilization of networked control systems with random delays[J]. IEEE Transactions on Automatic Control, 2005, 50(8): 1177 - 1181.

[7] Y. J. Pan, H. J. Marquez, and T. W. Chen. Remote stabilization of networked control systems with unknown time varying delays by LMI techniques[C]// Proceedings of the 44th IEEE Conference on Decision and Control. 2005: 1589-1594.

[8] D. Yue, Q. Han, and C. Peng. State feedback controller design of networked control systems[J]. IEEE Transactions on Circuits and Systems II, 2004, 51: 640-644.

[9] H. Gao, T. Chen and J. Lam. A new delay system approach to network-based control[J]. Automatica, 2008, 44(1): 39-52.

[10] P. Seiler, and R. Sengupta. An H_∞ approach to networked control[J]. IEEE Transactions on Automatic Control, 2005, 50(3): 356-364.

[11] D. Yue, Q. Han, and J. Lam. Network-based robust H_∞ control of systems with uncertainty[J]. Automatica, 2005, 41(6): 999-1007.

[12] C. C. Chen, H. Sandra and B. Martin. Sampled-Data networked control systems with random time delay[C]// IFAC 2008, 2008.

[13] H. Fujioka. Stability analysis for a class of networked/embedded control systems: a discrete-time approach[C]// Proceedings of the American Control Conference. 2008: 11-13.

[14] E. Fridman, A. Seuret, and J. Richard. Robust sampled-data stabilization of linear systems: an input delay approach[J]. Automatica, 2004, 40(8): 1441-1446.

[15] D. Nesic and D. Liberzon. A unified framework for design and analysis of networked and quantized control systems[C]// Chinese Control and Decision Conference. 2009: 43-52.

[16] G. C. Goodwin, H. Haimovich, D. E. Quevedo, and J. S. Welsh. A moving horizon approach to networked control system design[J]. IEEE Transactions on Automatic Control, 2004, 49(9): 1427-1445.

[17] L. A. Montestruque and P. J. Antsklis. Static and dynamic quantization in model-based networked control systems[J]. International Journal of Control, 2007, 80(1): 87-101.

[18] M. Y. Fu and L. H. Xie. The sector bound approach to quantized feedback control[J]. IEEE Transactions on Automatic Control, 2005, 50(11): 1698-1711.

[19] H. J. Gao and T. W. Chen. A new approach to quantized feedback control systems[J]. Automatica, 2008, 44(2): 534-542.

[20] Q. Ling and M.D. Lemmon. Stability of quantized control systems under dynamic bit assignment[C]// Procedings of the 2004 American Control Conference. 2004: 4915-4920.

[21] U. Jonsson. Lecture Note on Integral Quadratic Constraints[M]. Department of Mathematics, Royal Institute of Technology, 2001.

[22] G. Xie and L. Wang. Stabilization of networked control systems with time-varying network-induced delay[C]// Proceedings of the 43rd IEEE Conference on Decision and Control. 2004: 3551-3556.

[23] H. Ishii and B.A. Francis. Limited data rate in control systems with networks[M]. Berlin: Springer-Verlag, Chapter 4, 2002.

[24] K. Zhou and J.C. Doyle. Essential of robust control[M]. Prentice-Hall, Chapter 9, 1999.

[25] M. C. de Oliveira, J. Bernussou and J. C. Geromel. A new discrete time robust stability conditon[J]. Systems & Control Letters, 1999, 37(4): 261-265.

[26] A. Rantzer. On the Kalman-Yakubovich-Popov lemma[J]. System & Control Letters, 1996, 28(1): 7-10.

[27] A. Cervin, D. Henriksson, B. Lincoln and K. Arzen. Jitterbug and trueTime: analysis tools for realtime control systems[C]// 2nd Workshop on RealTime Tools. 2002.