Energy shaping for coordinating internally actuated vehicles

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Abstract This paper considers the stable coordination problem of two vehicles equipping with internal moving mass actuators. The coordinating and stabilizing control are derived by energy shaping. The proposed method is physically motivated and avoids cancelation or domination of nonlinearities.
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Keywords moving mass actuator, under actuated, energy shaping, stabilization

Internally actuated systems are enjoying more and more attention from engineers as well as control theorists. For example, internal moving masses are used as actuators for a hypersonic re-entry vehicle, because the temperature and pressure outside are very high for external control devices.[1] Moving mass actuators have also been proposed for precise orbit control in spacecraft formations.[2] A fleet of underwater gliders equipped with buoyancy engines and internal mass redistribution systems are used to form underwater mobile sensor networks over long time periods,[3] because actuators housed internally are isolated from the environment and less prone to physical damage. However, internally actuated systems are often under actuated thus provide big challenges for control design.

Mechanical systems with internal moving mass actuators form an important class of internally actuated systems. The behavior of a mechanical system is closely related to its energy, moving mass actuators influence the system's behavior by changing the kinetic metric and/or the potential energy of the system. The actuated system remains mechanical, of which many structure properties can be exploited to help the control design. However, mechanical systems with only internal control are necessarily neither linearly controllable nor feedback linearizable since the system's momentum is conserved. Energy-based control offers a promising approach to design nonlinear control. First, energy based methods preserve the mechanical structure and do not rely on nonlinearity cancelation by linearization or domination by high gain. Second, such methods exploit structural properties of the mechanical system and obtain stability and control results that hold over a larger domain than can be obtained using linear design method.

Energy shaping is a energy based method that shapes the potential and/or kinetic energy of the system in order to make the desired behavior a stable solution of the "shaped" system. Feedback control implements the shaping procedure. Reference 4 initiates the energy shaping design for stabilizing middle axis rotations of a rigid spacecraft using a single internal rotor. Reference 5 employs kinetic energy shaping to stabilize steady motions of underwater vehicles using internal rotors. Reference 6 uses the method of controlled lagrangians, an algorithmic energy shaping approach, to stabilize steady motions of vehicle systems using moving mass actuators. Without external control, a stable steady motion, i.e. relative equilibrium, is the best one can hope for. However, stable relative equilibria are low-energy, natural motions of mechanical systems and provide attractive solutions to motion planning problems.[7]

This paper investigates the stable coordination of two vehicles using internal moving mass actuators. Specifically, we aim to asymptotically stabilize a relative equilibrium which corresponds to two vehicles synchronizing their rotations as well as moving mass positions. To coordinate, artificial potentials are introduced to couple individuals such that the group acts like one multi-body system. To stabilize, the energy of group is shaped such that it takes extremum at the desired relative equilibrium. Asymptotical stability is achieved after feedback damping injection. As far as we know, the only work solving the stabilization and synchronization problem of a class of under actuated mechanical systems using energy shaping is Ref. 8, wherein the control is applied externally. This paper employs energy shaping to stably coordinate internally actuated vehicles.

Consider a planar vehicle spinning about its geometry center O shown in Fig. 1.[6] A moving mass m moves, under the influence of a control force, along a slot which is displaced at distance a from the center. Let I be the moment of inertia of vehicle about its spin axis. Let y be the displacement of moving mass along the slot. The configuration space of the system is $Q = SO(2) \times \mathbb{R}$,

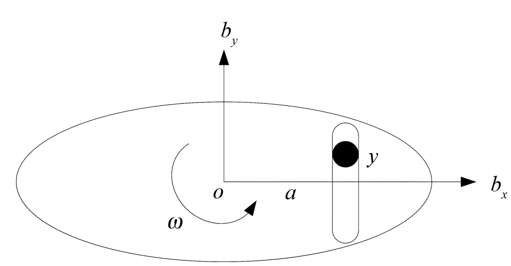

Fig. 1. Single vehicle with a moving mass along a slot

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where $SO(2)$ is the group of rotations in $\mathbb{R}^2$, and $\mathbb{R}$ is the one dimensional Euclidean space. The Lagrangian $L : TQ \to \mathbb{R}$ is invariant under the action of $SO(2)$. The reduced Lagrangian $l : TQ/SO(2) \to \mathbb{R}$ is the kinetic energy of the system (assumes no potential). Define the non-dimensional quantities[6]

$$\tilde{y} = \frac{y}{a}, \tilde{\omega} = \frac{\omega}{\omega_0}, \tilde{t} = \omega_0 t, \alpha = \frac{I}{ma^2} + 1, \tilde{l} = \frac{l}{ma^2\omega_0^2},$$

where $\omega_0 > 0$ is the equilibrium angular rate. Denote $\dot{\tilde{y}}$ the derivative with respect to $\tilde{t}$, and drop the tilde for convenience, the non-dimensional reduced Lagrangian is

$$l\left(\omega, y, \dot{y}\right) = \frac{1}{2}\begin{pmatrix} \omega \\ \dot{y} \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \alpha + y^2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \omega \\ \dot{y} \end{pmatrix}. \tag{1}$$

In the absence of physical dissipation, the controlled dynamics are given by the Euler-Lagrange (EL) equations

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial l}{\partial \omega}\right) = 0, \\ &\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial l}{\partial \dot{y}}\right) - \frac{\partial l}{\partial y} = u, \end{aligned} \tag{2}$$

where u is the control applied to the moving mass. Clearly, the total angular momentum $\pi_s = \partial l/\partial \omega = (\alpha + y^2)\omega + \dot{y}$ is conserved at all times. Thus, the dynamics evolve on a constant momentum surface $(\alpha + y^2)\omega + \dot{y} = \mathrm{const}$.

A relative equilibrium is an equilibrium for reduced dynamics (2). The relative equilibrium $e : (\omega, y, \dot{y}) = (1, 0, 0)$ corresponds to the vehicle spinning at a constant angular rate ω_0 and the moving mass being stationary at the center of the slot. However, the eigenvalues corresponding to the linearization of (2) at e have positive real parts, thus e is unstable.[6]

Now, we extend the framework and consider a group of two identical vehicles. The reduced Lagrangian l_g of the group is

$$l_g(\omega_1, y_1, \dot{y}_1, \omega_2, y_2, \dot{y}_2) = l_1(\omega_1, y_1, \dot{y}_1) + l_2(\omega_2, y_2, \dot{y}_2),$$

where l_1, l_2 is given by Eq. (1). The controlled dynamics are given by EL equations

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial l_g}{\partial \boldsymbol{\omega}}\right) = \boldsymbol{0}, \\ &\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial l_g}{\partial \dot{\boldsymbol{y}}}\right) - \frac{\partial l}{\partial \boldsymbol{y}} = \boldsymbol{u}, \end{aligned} \tag{3}$$

where $\boldsymbol{\omega} = (\omega_1, \omega_2), \boldsymbol{y} = (y_1, y_2), \boldsymbol{u} = (u_1, u_2)$. Let $\pi_{s1} = \partial l_1/\partial \omega_1, \pi_{s2} = \partial l_2/\partial \omega_2$, the angular momentum of the group $\pi_{sg} = \pi_{s1} + \pi_{s2}$ is also conserved at all times.

If two vehicles evolve on the same angular momentum surface, a family of relative equilibria (also unstable)

$$e : (\omega_1, y_1, \dot{y}_1, \omega_2, y_2, \dot{y}_2) = (1, \bar{y}, 0, 1, \bar{y}, 0) \tag{4}$$

corresponds to both vehicles spinning at a common angular rate, with two moving masses being stationary at position $\bar{y}$ in the respective slot. To stabilize Eq. (4), we make the following assumptions.

Assumption 1 Two vehicles have the same initial angular momentum π_0.

Assumption 2 Two vehicles communicate to each other about the relative position of their moving masses.

The objective is to design feedback control u_1, u_2 to stabilize Eq. (4) with respect to perturbations that lie on the surface $\pi_{s1} = \pi_{s2} = \pi_0$.

The method of controlled lagrangians (CL)[9] starts with a system with lagrangian

$$L(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - V(\boldsymbol{q}),$$

where $\boldsymbol{M}(\boldsymbol{q})$ is the kinetic metric, $V(\boldsymbol{q})$ is the potential energy. Assume that the generalized coordinates $\boldsymbol{q}$ is chosen such that the controlled EL equations can be written as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \frac{\partial V}{\partial \boldsymbol{q}} = \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{u} \end{pmatrix}, \tag{5}$$

where $\boldsymbol{C}$ is the Coriolis matrix associate with $\boldsymbol{M}$.

The CL method provides a feedback control $\boldsymbol{u}(\boldsymbol{q}, \dot{\boldsymbol{q}})$, and a modified lagrangian L_c such that the close-loop equations are free EL equations

$$\boldsymbol{M}_c(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_c(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \frac{\partial V_c}{\partial \boldsymbol{q}} = \boldsymbol{0}, \tag{6}$$

where $\boldsymbol{M}_c$ is a control modified kinetic metric, V_c is a control modified potential energy. The conditions under which such a feedback control exists are called matching conditions.

Solving Eq. (6) for $\ddot{\boldsymbol{q}}$ and substituting into Eq. (5) relate $\boldsymbol{M}$, V to the control modified $\boldsymbol{M}_c$, V_c,

$$\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{u} \end{pmatrix} = \boldsymbol{M}\boldsymbol{M}_c^{-1}\left(-\boldsymbol{C}_c\dot{\boldsymbol{q}} - \frac{\partial V_c}{\partial \boldsymbol{q}}\right) + \boldsymbol{C}\dot{\boldsymbol{q}} + \frac{\partial V}{\partial \boldsymbol{q}}. \tag{7}$$

The matching conditions are given by the upper part of Eq. (7). They are a set of nonlinear partial differential equations (PDEs) in $\boldsymbol{M}_c$ and V_c. Solving PDEs and using the lower part of Eq. (7) give the feedback control $\boldsymbol{u}$. Usually, matchable $\boldsymbol{M}_c$ and V_c are parameterized functions, wherein the parameters in turn appear in the feedback control as control parameters.

After matching, the close-loop system is a free lagrangian system, of which the energy

$$E_c(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{M}_c(\boldsymbol{q})\dot{\boldsymbol{q}} + V_c(\boldsymbol{q})$$

is conserved, thus qualifies as a Lyapunov function for stability analysis and stabilization design. The control parameters are chosen such that the desired equilibrium is a minimum (maximum) of E_c, thus achieve Lyapunov stability. One may add feedback dissipation to make

$\dot{E}_c \le 0$ ($\dot{E}_c \ge 0$) and assess asymptotic stability using LaSalle's principle.

Open-loop Eq. (3) can be written collectively as

$$\boldsymbol{M}\dot{\boldsymbol{\psi}} + \boldsymbol{C}\boldsymbol{\psi} = \boldsymbol{G}\boldsymbol{u}, \tag{8}$$

and $\boldsymbol{\psi} = (\boldsymbol{\psi}_1, \boldsymbol{\psi}_2)$, $\boldsymbol{M} = \mathrm{diag}(\boldsymbol{M}_1, \boldsymbol{M}_2)$, $\boldsymbol{C} = \mathrm{diag}(\boldsymbol{C}_1, \boldsymbol{C}_2)$, $\boldsymbol{G} = \mathrm{diag}(\boldsymbol{G}_1, \boldsymbol{G}_1)$, where $\boldsymbol{G}_1 = (0, 1)$, for $i = 1, 2$, $\boldsymbol{\psi}_i = (\omega_i, \dot{y}_i)$,

$$\boldsymbol{M}_i = \begin{pmatrix} \alpha + y_i^2 & 1 \\ 1 & 1 \end{pmatrix}, \boldsymbol{C}_i = \begin{pmatrix} y_i \dot{y}_i & y_i \omega_i \\ -y_i \omega_i & 0 \end{pmatrix}.$$

Let the close-loop equations be $\boldsymbol{M}_c\dot{\boldsymbol{\psi}} + \boldsymbol{C}_c\boldsymbol{\psi} + \partial V_c/\partial \boldsymbol{y} = \boldsymbol{0}$, and let $\boldsymbol{G}_1^{\perp} = (1, 0)$, Eq. (7) gives the kinetic and potential matching conditions

$$\begin{pmatrix} \boldsymbol{G}_1^{\perp} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_1^{\perp} \end{pmatrix} (-\boldsymbol{M}\boldsymbol{M}_c^{-1}\boldsymbol{C}_c\boldsymbol{\psi} + \boldsymbol{C}\boldsymbol{\psi}) = \boldsymbol{0},$$
$$\begin{pmatrix} \boldsymbol{G}_1^{\perp} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_1^{\perp} \end{pmatrix} \boldsymbol{M}\boldsymbol{M}_c^{-1}\frac{\partial V_c}{\partial \boldsymbol{y}} = \boldsymbol{0}. \tag{9}$$

To stabilize relative equilibrium (4), an artificial potential V_c is introduced as

$$V_c = \frac{k_1}{2}(y_1 - y_2)^2 + \frac{k_2}{2}(y_1 - \bar{y})^2, \tag{10}$$

where parameters k_1, k_2 are to be designed in the stability analysis later. Physically speaking, this artificial potential acts like adding a linear spring connecting moving masses in the pair of vehicles.

To make $\boldsymbol{M}_c$ matchable with the limited control, a solution to Eq. (9) can be constructed by simple linear algebraic operations[6]

$$\boldsymbol{M}_c = \begin{pmatrix} \boldsymbol{M}_{c1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_{c2} \end{pmatrix},$$
$$\boldsymbol{M}_{ci} = \begin{pmatrix} \alpha + y_i^2 & 1 \\ 1 & \rho/(\alpha + y_i^2) \end{pmatrix}, \tag{11}$$

where ρ is a parameter. To make $\boldsymbol{M}_c$ nonsingular, it needs $\mathrm{Det}(\boldsymbol{M}_c) \neq 0 \Rightarrow \rho \neq 1$.

However, Ref. 6 observes that matching solution (11) does not lead to a proof of nonlinear stability. Moreover, seeking other matching solutions involves solving PDEs and do not ensure the existence of a stable solution. Different from a spectral stability analysis,[6] we apply matching solution (11) to the following system

$$\bar{\boldsymbol{M}}\dot{\boldsymbol{\psi}} + \bar{\boldsymbol{C}}\boldsymbol{\psi} = \boldsymbol{G}\bar{\boldsymbol{u}}, \tag{12}$$

where $\bar{\boldsymbol{M}}$ and $\bar{\boldsymbol{C}}$ have the same form in Eq. (8), wherein

$$\bar{\boldsymbol{M}}_i = \begin{pmatrix} \alpha + (y_i - \bar{y})^2 & 1 \\ 1 & 1 \end{pmatrix},$$
$$\bar{\boldsymbol{C}}_i = \begin{pmatrix} (y_i - \bar{y})\dot{y} & (y_i - \bar{y})\omega_i \\ -(y_i - \bar{y})\omega_i & \frac{-\omega_i(y_i - \bar{y})}{[\alpha + (y_i - \bar{y})^2]^2} \end{pmatrix}.$$

Lemma 1 Two systems (8) and (12) produce the same state solutions in coordinates $(y_1, y_2, \boldsymbol{\psi})$, if and only if two controls satisfy

$$\boldsymbol{u} = \boldsymbol{G}^{\mathrm{T}}\left[\boldsymbol{M}\bar{\boldsymbol{M}}^{-1}\left(\boldsymbol{G}\bar{\boldsymbol{u}} - \bar{\boldsymbol{C}}\boldsymbol{\psi}\right) + \boldsymbol{C}\boldsymbol{\psi}\right]. \tag{13}$$

Proof Solve for $\dot{\boldsymbol{\psi}}$ in Eq. (8) and Eq. (12), respectively

$$\dot{\boldsymbol{\psi}} = \boldsymbol{M}^{-1}(\boldsymbol{G}\boldsymbol{u} - \boldsymbol{C}\boldsymbol{\psi}),$$
$$\dot{\boldsymbol{\psi}}_{\mathrm{bar}} = \bar{\boldsymbol{M}}^{-1}(\boldsymbol{G}\bar{\boldsymbol{u}} - \bar{\boldsymbol{C}}\boldsymbol{\psi}_{\mathrm{bar}}),$$

where the solution with subscript "bar" is to Eq. (12). Straightly, $\dot{\boldsymbol{\psi}} = \dot{\boldsymbol{\psi}}_{\mathrm{bar}}$ if and only if Eq. (13) holds.

The energy shaping control implementing Eqs. (10) and (11) for system (12) is

$$\bar{\boldsymbol{u}}_{\mathrm{es}} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G})^{-1}\boldsymbol{G}^{\mathrm{T}} \times \left[\bar{\boldsymbol{C}}\boldsymbol{\psi} - \bar{\boldsymbol{M}}\bar{\boldsymbol{M}}_c^{-1}\left(\bar{\boldsymbol{C}}_c\boldsymbol{\psi} + \frac{\partial V_c}{\partial \boldsymbol{y}}\right)\right], \tag{14}$$

where $\bar{\boldsymbol{M}}_c$ has the same form of $\boldsymbol{M}_c$ with y_i replaced by $z_i := (y_i - \bar{y})$, and

$$\bar{\boldsymbol{C}}_c = \mathrm{diag}(\bar{\boldsymbol{C}}_{c1}, \bar{\boldsymbol{C}}_{c2}),$$
$$\bar{\boldsymbol{C}}_{ci} = \begin{pmatrix} z_i \dot{y}_i & z_i \omega_i \\ -z_i \omega_i & -\rho\omega_i z_i/(\alpha + z_i^2)^2 \end{pmatrix}.$$

We analyze the Lyapunov stability of relative equilibrium $\bar{e} : (\omega_1, z_1, \dot{y}_1, \omega_2, z_2, \dot{y}_2) = (1, 0, 0, 1, 0, 0)$ of system (12), which corresponds to relative equilibrium (4) of system (8).

Proposition 1 The energy shaping control $\bar{\boldsymbol{u}}_{\mathrm{es}}$ stabilizes relative equilibrium $\bar{e}$ with $\rho < 1$, $k_1 < 0$, and $k_2 < 0$.

Proof Notice that the control-modified energy $E_c = \frac{1}{2}\boldsymbol{\psi}^{\mathrm{T}}\bar{\boldsymbol{M}}_c\boldsymbol{\psi} + V_c$ is conserved. A candidate Lyapunov function for $\bar{e}$ can be constructed as $E_{\phi} = E_c + \phi(\pi_{sg})$ where

$$\phi(\pi_{sg}) = \frac{\sigma}{2}(\pi_{s1}^2 + \pi_{s2}^2) - (1 + \sigma\alpha)\pi_{s1} - (1 + \sigma\alpha)\pi_{s2}.$$

One can check that $\mathrm{grad}(E_{\phi})|_{\bar{e}} = \boldsymbol{0}$, and $\mathrm{Hess}(E_{\phi})|_{\bar{e}}$ is negative definite provided

$$1 + \alpha\sigma < 0, \rho - 1 < 0, k_1 < 0, k_2 < 0.$$

Therefore, $\bar{e}$ can be made a maximum of E_{ϕ}, thus proves Lyapunov stability by Energy-Casimir method.

Adding the feedback dissipation[6] as $\bar{\boldsymbol{u}} = \bar{\boldsymbol{u}}_{\mathrm{es}} + \bar{\boldsymbol{u}}_{\mathrm{diss}}$, the close-loop equations become

$$\bar{\boldsymbol{M}}_c\dot{\boldsymbol{\psi}} + \bar{\boldsymbol{C}}_c\boldsymbol{\psi} + \frac{\partial V_c}{\partial \boldsymbol{y}} = \bar{\boldsymbol{M}}_c\bar{\boldsymbol{M}}^{-1}\boldsymbol{G}\bar{\boldsymbol{u}}_{\mathrm{diss}}.$$

E_c is no longer conserved and its changing rate $\dot{E}_c = \boldsymbol{\psi}^{\mathrm{T}}\bar{\boldsymbol{M}}_c\bar{\boldsymbol{M}}^{-1}\boldsymbol{G}\bar{\boldsymbol{u}}_{\mathrm{diss}}$. Choosing $\boldsymbol{G}\bar{\boldsymbol{u}}_{\mathrm{diss}} = \bar{\boldsymbol{M}}\bar{\boldsymbol{M}}_c^{-1}\boldsymbol{R}\boldsymbol{\psi}$

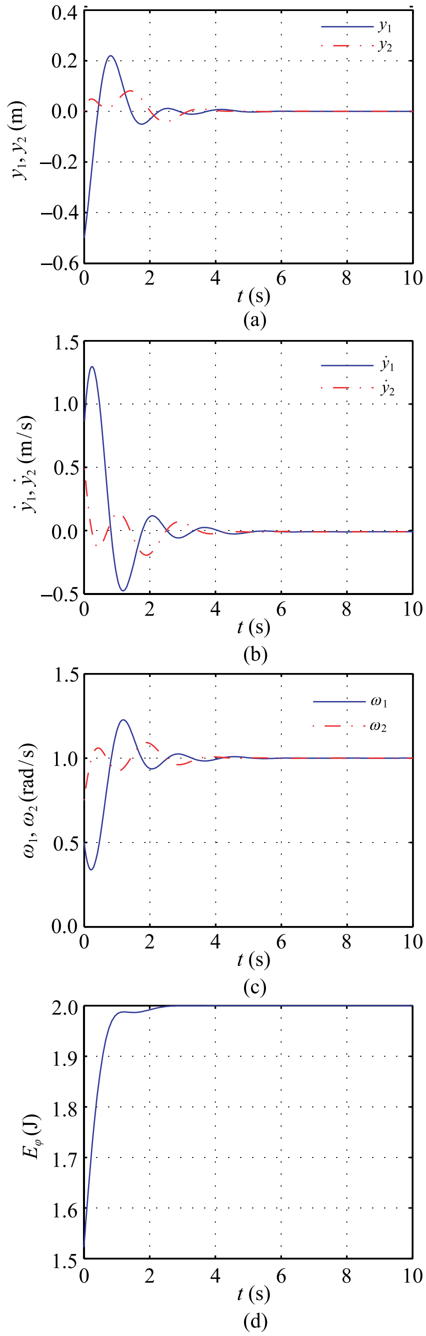

Fig. 2. Time history for the states (a) y_1, y_2, (b) $\dot{y}_1, \dot{y}_2$, (c) ω_1, ω_2, and the controlled energy (d) E_ϕ

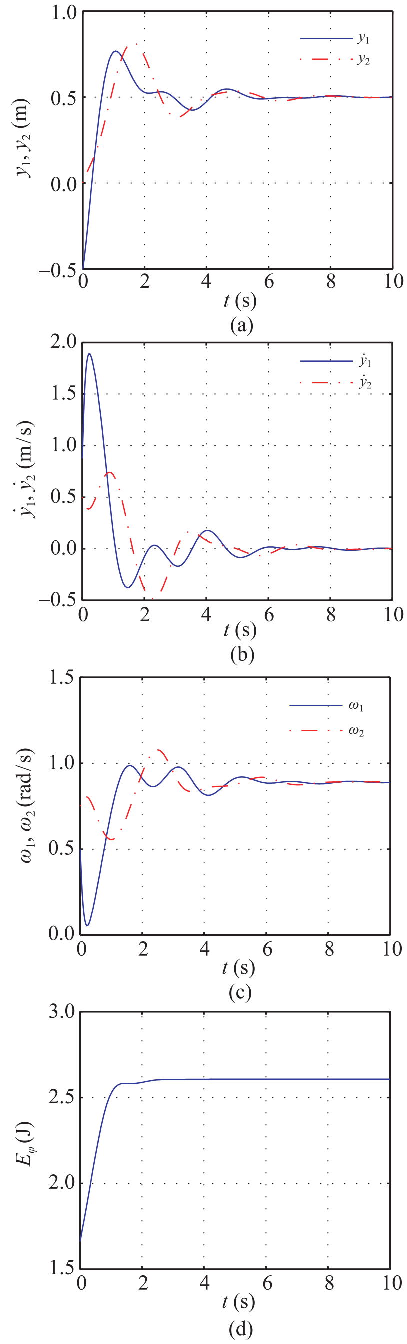

Fig. 3. Time history for the states (a) y_1, y_2, (b) $\dot{y}_1, \dot{y}_2$, (c) ω_1, ω_2, and the controlled energy (d) E_ϕ

gives $\dot{E}_c = \boldsymbol{\psi}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\psi}$. Let $\boldsymbol{R} = \mathrm{diag}(\boldsymbol{R}_1, \boldsymbol{R}_1)$, $\boldsymbol{R}_1 = \mathrm{diag}(0, k_{\mathrm{diss}})$, the feedback dissipation is

$$\bar{\boldsymbol{u}}_{\mathrm{diss}} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G})^{-1}\boldsymbol{G}^{\mathrm{T}}\bar{\boldsymbol{M}}\bar{\boldsymbol{M}}_c^{-1}\boldsymbol{R}\boldsymbol{\psi}. \tag{15}$$

Proposition 2 The feedback control $\bar{\boldsymbol{u}} = \bar{\boldsymbol{u}}_{\mathrm{es}} + \bar{\boldsymbol{u}}_{\mathrm{diss}}$ asymptotically stabilizes relative equilibrium $\bar{e}$ for $k_{\mathrm{diss}} > 0$.

The asymptotical stability can be directly proved by LaSalles invariance principle. Invoking Lemma 1, we immediately have

Corollary 1 The feedback control

$$\boldsymbol{u} = \boldsymbol{G}^{\mathrm{T}}\left\{\boldsymbol{M}\bar{\boldsymbol{M}}^{-1}\left[\boldsymbol{G}(\bar{\boldsymbol{u}}_{\mathrm{es}} + \bar{\boldsymbol{u}}_{\mathrm{diss}}) - \bar{\boldsymbol{C}}\boldsymbol{\psi}\right] + \boldsymbol{C}\boldsymbol{\psi}\right\}$$

asymptotically stabilizes relative equilibrium (4) of system (8) with $\rho < 1$, $k_1 < 0$, $k_2 < 0$, and $k_{\mathrm{diss}} > 0$.

We illustrate the above control design with a numerical simulation. Let $(\omega_1, y_1, \dot{y}_1, \omega_2, y_2, \dot{y}_2) = (1, \bar{y}, 0, 1, \bar{y}, 0)$ be the desired relative equilibrium. The physical parameters of vehicle are chosen such that $\alpha = 2$. The control parameters are $k_1 = k_2 = -1$, $\rho = 0.5$, and $k_{\mathrm{diss}} = 0.5$.

The initial conditions are $(\omega_1, y_1, \dot{y}_1, \omega_2, y_2, \dot{y}_2)(0) = (0.5, -0.5, 0.875, 0.75, 0, 0.5)$. The controlled dynamics evolve on a conserved angular momentum surface $\pi_{s1} = \pi_{s1}(0) = 2$, $\pi_{s2} = \pi_{s2}(0) = 2$.

Let $\bar{y} = 0$, Fig. 2 shows the evolution of the states and controlled energy of the two-vehicle group.

On the same constant momentum surface, let $\bar{y} = 0.5$. Figure 3 shows the asymptotical stabilization with a nonzero equilibrium position for the moving mass.

In this paper, we have considered the stable coordination problem of two vehicles equipping with internal moving mass actuators. The energy based control is designed in the context of mechanics, and does not rely on cancelation or domination of nonlinearities.

However, the academic model employed in this paper dose not consider the effect of gravity. To include forces and moments due to the gravity, and use energy shaping to solve the coordination problem is our ongoing work. It is also of interest to consider the coordination problem in a group of $N > 2$ individuals with limited communications.

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