

Distributed Control of Heterogeneous Systems with Nonlinear Interconnections

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Abstract: This paper studies the distributed stabilizing controller design for a class of heterogeneous systems with static nonlinear interconnections. The controllers work in a distributed manner that each of them locally stabilizes a subsystem and the entire system consisting of these subsystems then achieves global stability. Both the state feedback and output feedback control methods are derived. Finally, a comparative example is presented to show the usefulness and effectiveness of the results.

Key Words: Distributed control; Heterogeneous systems; Nonlinear interconnection; L_2 stability

1 Introduction

Over the past few years, there has been a rapidly growing interest in the system and control community in the study of coordination, communication and distributed control for networked dynamic systems of many components acting locally and interacting with each other under various interconnection constraints. Systems such as formation control [1, 2], synchronous control[3], and multi-agent cooperative control[4] are of this kind. In fact, most complex machinery can be divided into subsystems with relatively independent functionalities. Also, the advanced industrial systems are built up by different parts with their own technological processing. All these systems can be classified as distributed systems. The characteristics of such systems are its large scale, hierarchical structure, relatively independent subsystems, loose interconnection and cooperation in a distributed manner.

Many approaches have been put forward to solve the stability and stabilization problems for distributed systems. The distributed control algorithms are developed which can be used as protocols for different types of cooperation as an interconnected system than as aggregate whole[5]. Prathyush[6] studies the global stability of networked systems consisting of subsystems with sector bounded nonlinearities. Gu[7] considers the delays that exists in each communication channels. For systems with constant communication delays, Hirche[8] gives a two steps distributed controller design method. First, local controllers are designed without consideration of network delays. Then, the effectiveness of the controllers are verified by stabilizing analysis. By using the Kronecker product, the conditions for stability and performance can be obtained by using only information and possibly some spectral characteristics of the interconnection matrix. This is a useful property in the analysis of complex dynamical systems[9]. These approaches may not be completely suitable to the case of practical engineering systems because they are derived by the idealization of the subsystems

such as the first-order integrators and the consideration of only identical node dynamics or homogeneous systems.

The studies on the distributed control of large-scale systems are another aspect of the related research. No simplification of the original subsystems is the character of these results and they are more general. The control problems for a spatially invariant system are studied in[10]. The results are simple and convenient for LTI systems but have limitations in the nonlinear systems. One of the meaningful results is given by Qu[11]. In the study, matrix theory is used to develop the main frame of distributed control protocol. The systems are divided into two hierarchies, including local stabilizing controller design and inter-subsystem cooperation protocol design. The purpose of local controller is to reform the closed-loop subsystems working as high-order integrators. Thus, each subsystem will work well even the fault occurs in communication channels. And the protocol is to regulate the behaviors of the subsystems to achieve the cooperation.

In the above-mentioned related works, each one only emphasizes on some type of the phenomena that exist in distributed heterogeneous systems while ignoring the others. From the system point of view, it is not necessary to investigate from the device level, which removes the functionalities of the subsystems. And it is not reasonable to oversimplify the subsystems into simple nodes in graph theory. A report[12] points out that the research is needed to understand multiple interconnected systems over realistic channels that work together in a distributed fashion. At the same time, interconnections via realistic channels must be considered explicitly as they significantly affect the dynamic behavior of the control system[13]. In this paper, the imperfect channels, interconnection topologies and heterogeneous entities, the three aspects of problems in real systems, are considered together, as a first attempt but in a relatively coarse manner. Based on our previous results[14], we finally get the stabilizing controller synthesis method for systems that are interconnected via non-ideal channels.

The rest part of the paper is organized as follows. In section II, the notations that will be used are given and the pre-

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liminaries are listed in a general form. In section III, both the state-feedback and the output-feedback controller design methods are derived. In section IV, the simulation is given to show the effectiveness of our results.

2 Notations and Preliminaries

2.1 Notations

The notations are standard. The set of real numbers is denoted by $\mathbf{R}$; $\mathbf{R}_+$ and $\bar{\mathbf{R}}$ denote the non-negative subset and closure of $\mathbf{R}$, respectively. The set of complex numbers is denoted by $\mathbf{C}$. The group of matrices in the real and complex fields is denoted by $\mathbf{R}^{m \times n}$ and $\mathbf{C}^{m \times n}$. The set of m by n stable rational transfer functions is denoted by $\mathbf{RH}_\infty^{m \times n}$. For notational convenience, the dimensions will not be given unless pertinent to the discussion.

The space $\mathbf{L}_2$ is the set of functions defined on $\mathbf{R}_+$ with values in $\mathbf{C}^n$ for which the following quantity is finite:

$$\int_0^\infty \|u(t)\|^2 dt,$$

where $\|\cdot\|$ is the Euclid norm. The inner product on $\mathbf{L}_2$ is defined as

$$\langle u, v \rangle = \int_0^\infty u(t)^* v(t) dt, u, v \in \mathbf{L}_2$$

and the corresponding $\mathbf{L}_2$ norm is $\|u\|_2 := \sqrt{\langle u, u \rangle}$.

$\mathbf{L}_{2e}$ is the space of functions for which the following quantity is finite for every $T \geq 0$:

$$\int_0^T \|u(t)\|^2 dt.$$

It is obvious that $\mathbf{L}_2$ is a subspace of $\mathbf{L}_{2e}$.

The notation of operator is important in this paper. An operator is a mapping from one normed space to another, which can be considered as a mathematical object that represents an input-output system. The induced norm of a linear operator is defined by

$$\|H\|_{\mathbf{L}_2 \rightarrow \mathbf{L}_2} = \sup_{v \in \mathbf{L}_2, v \neq 0} \frac{\|Hv\|_2}{\|v\|_2}.$$

A linear operator H is positive definite if $H = H^*$, and there exists $\alpha > 0$, such that

$$\langle u, Hu \rangle = \int_0^\infty u^*(t)(Hu)(t) dt \geq \alpha^2 \|u\|_2^2, \forall u \in \mathbf{L}_2.$$

2.2 Preliminaries

Now, consider a distributed heterogeneous system composed of n ($n \geq 2$) interconnected subsystems as illustrated in Fig.1. The state space equation of subsystem G_i is

$$G_i : \begin{cases} \dot{x}_i = A_i x_i + B_i v_i \\ y_i = C_i x_i \end{cases} \quad i = 1, 2, \dots, n. \quad (1)$$

The subsystems are interconnected via the time-varying static nonlinear links. Here, “static” means the output of the links at time t only depends on the input at time t . The nonlinearity is time-varying because of the non-idealization of the links. If the output of subsystem j feeds into subsystem

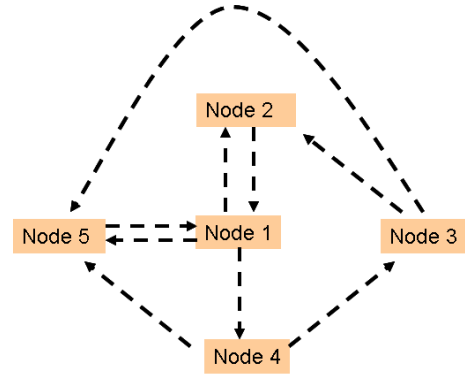


Fig. 1: The Heterogeneous Subsystems Interconnected via Nonlinear Links

i , then there is a link between them. Denote the directed link by $\varphi_{ij} : \mathbf{R}_+ \times Y_j \rightarrow U_i, \varphi_{ij}(t, 0) = 0, \forall t \in \mathbf{R}_+$, where φ_{ij} is continuous and satisfies the Lipschitz condition about the second variable. Assume that the output of the link satisfies the sector condition:

$$\underline{k}_{ij} y_j(t) \leq \varphi_{ij}(t, y_j(t)) \leq \bar{k}_{ij} y_j(t), \forall y_j \in Y_j, \forall t \in \mathbf{R}_+, \quad (2)$$

where $\underline{k}_{ij}, \bar{k}_{ij}$ are constant scalars, y_j is the output of subsystem j . The expression of the link implies that the link is time-varying. This general constraint is supposed to cover the majority of the nonlinearities in the link. Denote $A = \text{diag}(A_i), B = \text{diag}(B_i), C = \text{diag}(C_i), \underline{K} = [\underline{k}_{ij}]_{n \times n}, \bar{K} = [\bar{k}_{ij}]_{n \times n}$. It is proved[14] that

Theorem 1 The distributed heterogeneous system is finite-gain $\mathbf{L}_2$ stable if there exists $P > 0$ such that

$$M + \begin{bmatrix} A^T P + P A & P B \\ B^T P & 0 \end{bmatrix} \leq 0,$$

where

$$M = \begin{bmatrix} -C^*(\underline{K}^T \bar{K} + \bar{K}^T \underline{K})C & C^*(\underline{K}^T + \bar{K}^T) \\ (\underline{K} + \bar{K})C & -2I \end{bmatrix}.$$

This stability certification can be used to analyze a network with nonlinear subsystems and non-ideal links.

3 Controller Design for Distributed Heterogeneous Systems

If feasible and deemed to be necessary, a centralized control with global information can be designed to achieve the optimal performance. For robustness, it is better to have each of the plants stabilized by its local control, while the network is to provide information sharing toward synthesizing or adjusting reference inputs to the plants. Nonetheless, the presence of the networked loop could cause problems for stability and performance of the overall system. Indeed, for large-scale interconnected systems, decentralized control (feedback control individually installed at each of the plants) is often desired. The distributed controllers must be designed to satisfy the following two requirements:

1. Each subsystem is stabilized by its local controller using its own information;

2. The entire system is stabilized naturally without any other improvement, i.e. the entire system is stabilized by these local controllers.

For subsystem G_i given in (1), denote its input as $v_i = u_i + w_i + e_i$ where u_i , w_i and e_i are the local control input, information from other subsystems and external signal, respectively. Let subsystem G_i is locally stabilized by

$$u_i = \Gamma_i(y_i), \quad (3)$$

where $\Gamma(\cdot)$ is the control law and has specific formations in different situations. Input information from the other subsystems via the nonlinear interconnections is denoted by $w_i = \sum_{j=1}^{j=n} \varphi_{ij}(y_j)$. Thus, the controlled subsystem G_i is described by

$$G_i : \begin{cases} \dot{x}_i = A_i x_i + B_i(u_i + w_i + e_i) \\ y_i = C_i x_i \\ w_i = \sum_{j=1}^{j=n} \varphi_{ij}(y_j) \\ u_i = \Gamma_i(y_i) \end{cases} \quad (4)$$

All the subsystems together can be represented by

$$y = Gv, \quad (5)$$

where $G = \text{diag}(G_i)$ is called the system operator. The expressions of $\varphi(\cdot) = [\varphi_{ij}(\cdot)]$ and $\Gamma(\cdot) = \text{diag}(\Gamma_1(\cdot), \Gamma_2(\cdot), \dots, \Gamma_n(\cdot))$ are called the interconnection operator and the control operator, respectively. For u_i, y_i , the stabilized subsystem G_i can be considered as an operator mapping from the signal space $L_{2e}(U_i)$ to $L_{2e}(Y_i)$. With these three operators, the entire system is modeled as

$$G : \begin{cases} \dot{x} = Ax + B(u + w + e) \\ y = Cx \\ w = \varphi(y) \\ u = \Gamma(y), \end{cases} \quad (6)$$

and illustrated in Fig.2[11].

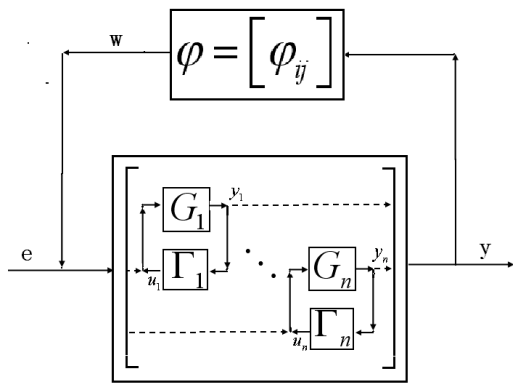


Fig. 2: Distributed Control of Heterogeneous Systems

3.1 State Feedback Controller Design

If all the states of each controlled plant are detectable, state feedback control is feasible and direct. Let the control law of subsystem i be

$$u_i = F_i x_i, i = 1, 2, \dots, n. \quad (7)$$

Denote $F = \text{diag}(F_1, F_2, \dots, F_n)$. Let the external signals be zero, the equation of the entire system with locally closed loop control is

$$\begin{cases} \dot{x} = Ax + BFx + w \\ w = \varphi(y). \end{cases} \quad (8)$$

And we have the following theorem,

Theorem 2 For system (8), if there exists matrix $W = \text{diag}(W_1, W_2, \dots, W_n)$ and positive define matrix $X = \text{diag}(X_1, X_2, \dots, X_n)$ satisfying

1.

$$AX + BW + (AX + BW)^T < 0;$$

2.

$$\begin{bmatrix} \Theta & X(\bar{K} - \underline{K})^T + B \\ ((\bar{K} - \underline{K})X + B^T & -2I \end{bmatrix} \leq 0,$$

where

$$\Theta = (A + B\underline{K})X + BW + ((A + B\underline{K})X + BW)^T,$$

W, X are blocked accordingly to the orders of the subsystems. Then, there exists distributed state feedback controllers with $F = WX^{-1}$ and each diagonal block F_i , $i = 1, 2, \dots, n$ is the corresponding local stabilizing controller.

Proof. Condition (2) is first proved here:

Since it is the state feedback, $C = I$, stable conditions in Theorem 1 becomes

$$\begin{bmatrix} -\underline{K}^T \bar{K} - \bar{K}^T \underline{K} & \underline{K}^T + \bar{K}^T \\ \underline{K} + \bar{K} & -2I \end{bmatrix} + \begin{bmatrix} (A + BF)^T P + P(A + BF) & PB \\ B^T P & 0 \end{bmatrix} \leq 0. \quad (9)$$

There are two variables in nonlinear Inequality (9), that are F and P . In order to solve this inequality numerically, an equivalent linear matrix inequality(LMI) is got by variable transformation method. A preparation is need to avoid new quadratic items. Rewrite inequality (9) as

$$\begin{bmatrix} \Omega & \underline{K}^T + \bar{K}^T + PB \\ \underline{K} + \bar{K} + B^T P & -2I \end{bmatrix} \leq 0, \quad (10)$$

where

$$\Omega = -\underline{K}^T \bar{K} - \bar{K}^T \underline{K} + (A + BF)^T P + P(A + BF).$$

Since $-2I < 0$, by the Schur complement, we obtain

$$-\underline{K}^T \bar{K} - \bar{K}^T \underline{K} + (A + BF)^T P + P(A + BF) + \frac{1}{2}(\underline{K}^T + \bar{K}^T + PB)(\underline{K} + \bar{K} + B^T P) \leq 0. \quad (11)$$

Then we can get

$$(A + BF)^T P + P(A + BF) + \underline{K}^T B^T P + PB\underline{K} + \frac{1}{2}[(\bar{K}^T - \underline{K}^T + PB)(\bar{K} - \underline{K} + B^T P)] \leq 0. \quad (12)$$

Using the Schur complement again to (12) and multiply left and right sides by $\text{diag}(P^{-1}, I)$, we get

$$\begin{bmatrix} \Lambda & P^{-1}(\bar{K} - \underline{K})^T + B \\ (\bar{K} - \underline{K})P^{-1} + B^T & -2I \end{bmatrix} \leq 0, \quad (13)$$

where

$$\Lambda = P^{-1}(A + BF + B\underline{K})^T + (A + BF + B\underline{K})P^{-1}.$$

Substituting $X = P^{-1}$ and $W = FX$ into inequality (13), condition(2) is proved.

As to condition (1), it is easy to see that

$$(A + BF)^T P + P(A + BF) < 0,$$

since the positiveness of matrix P . Because of the diagonal formation and dimension consistency of matrix F with the subsystems, the controlled subsystems are stable. The theorem is proved.

Remark 1 In Theorem 1, the stable conditions are LMIs about matrix X and W . It can be solved by the feasp solver in LMI toolbox in Matlab. The diagonal structure requirement is to get the distributed controllers after variable transformation.

Remark 2 With all states observable, each subsystem is stabilized by its own information. They share their output information to achieve the minimum error to the expected value.

3.2 Output Feedback Controller Design

The output feedback control here is realized by the dynamic control structure. Let the plant equation and controller equation be

$$\begin{aligned} G_i : & \begin{cases} \dot{x}_i = A_i x_i + B_{1i} u_i + B_{2i} w_i \\ y_i = C_i x_i \end{cases}, \\ K_i : & \begin{cases} \dot{\hat{x}}_i = A_{ki} \hat{x}_i + B_{ki} y_i \\ y_i = C_{ki} \hat{x}_i + D_{ki} y_i \end{cases}. \end{aligned} \quad (14)$$

Let each subsystem is stabilizable and detectable, and the plant dynamics with local stabilizing controllers are described by

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{x}} \end{bmatrix} &= \begin{bmatrix} A + B_1 D_k C & B_1 C_k \\ B_k C & A_k \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} w, \\ y &= [C \quad 0] \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \end{aligned} \quad (15)$$

where $A = \text{diag}(A_i)$, $B_1 = \text{diag}(B_{1i})$,

$B_2 = \text{diag}(B_{2i})$, $C = \text{diag}(C_i)$, $x = [x_1^T, \dots, x_n^T]^T$,

$\hat{x} = [\hat{x}_1^T, \dots, \hat{x}_n^T]^T$, $y = [y_1^T, \dots, y_n^T]^T$.

Let

$$K = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix}$$

and it contains all the parameters needed for the dynamic output feedback control. Also, the structure of matrix K is in accordance with the subsystems.

Introducing the matrices notations as the following

$$\begin{aligned} A_0 &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0 & B_1 \\ I & 0 \end{bmatrix}, C_0 = \begin{bmatrix} 0 & C \\ I & 0 \end{bmatrix}, \\ A_{cl} &= A_0 + B_0 K C_0, B_{cl} = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}, C_{cl} = [C \quad 0], \end{aligned}$$

the entire system consisting of closed loop subsystem interconnecting via non-ideal channel is stable, if the conditions in Theorem 1 are satisfied:

$$\begin{aligned} & \begin{bmatrix} -C_{cl}^* (\underline{K}^T \bar{K} + \bar{K}^T \underline{K}) C_{cl} & C_{cl}^* (\underline{K}^T + \bar{K}^T) \\ (\underline{K} + \bar{K}) C_{cl} & -2I \end{bmatrix} \\ & + \begin{bmatrix} A_{cl}^T P + P A_{cl} & P B_{cl} \\ B_{cl}^T P & 0 \end{bmatrix} \leq 0. \end{aligned} \quad (16)$$

Similar to the process in Theorem 2, it is not difficult to get

$$H + Q^T K^T R + R^T K Q \leq 0, \quad (17)$$

where

$$H = \begin{bmatrix} \Xi & C_{cl}^* (\bar{K} - \underline{K})^T + P B_{cl} \\ B_{cl}^T P + (\bar{K} - \underline{K}) C_{cl} & -2I \end{bmatrix},$$

$$\Xi = (A_0 + B_{cl} \underline{K} C_{cl})^T P + P (A_0 + B_{cl} \underline{K} C_{cl}),$$

$$Q = [C_0 \quad 0], R = [B_0^T P \quad 0].$$

Equation (17) is the standard formalization of output feedback controller design in robust control theory(see [15]). The Projection Lemma is used to eliminate variables in inequality (17) and the result is as follows:

Theorem 3 For system (15), if there exist positive symmetrical matrices X and Y satisfying

1. $\begin{bmatrix} N_C \\ I \end{bmatrix}^T \Pi_C \begin{bmatrix} N_C \\ I \end{bmatrix} \leq 0,$
2. $\begin{bmatrix} N_{B_2} \\ I \end{bmatrix}^T \Pi_{B_2} \begin{bmatrix} N_{B_2} \\ I \end{bmatrix} \leq 0,$
3. $\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0,$

where

$$\begin{aligned} \Pi_C &= \begin{bmatrix} (A + B_2 \underline{K} C)^T X + X(A + B_2 \underline{K} C) & C^* (\bar{K} - \underline{K})^T + X B_2 \\ B_2^T X + (\bar{K} - \underline{K}) C & -2I \end{bmatrix}, \\ \Pi_{B_2} &= \begin{bmatrix} (A + B_2 \underline{K} C) Y + Y(A + B_2 \underline{K} C)^T & C^* (\bar{K} - \underline{K})^T Y + B_2 \\ B_2^T + Y(\bar{K} - \underline{K}) C & -2I \end{bmatrix}, \end{aligned}$$

N_C and N_{B_2} are constructed by any set of the basis vectors from the null-spaces of C and B_2 .

Matrix K is calculated by the following two steps:

1. Obtain matrices X and Y from conditions in Theorem 3;
2. Use the singular value decomposition (SVD) approach to get the value of $X_2 = X - Y^{-1}$. Let

$$P = \begin{bmatrix} X & X_2 \\ X_2^T & I \end{bmatrix};$$

3. Substitute P into the following inequalities and get matrix K .

$$\begin{cases} A_{cl}^T P + P A_{cl} < 0 \\ H + Q^T K^T R + R^T K Q \leq 0 \end{cases} \quad (18)$$

$K_i (i = 1, 2, \dots, n)$ in K is the parameters of stabilizing controllers for each subsystem.

Remark 3 There is no diagonal requirement of X, Y, X_2, P except K . The two inequalities in Theorem 3 guarantee the stability of each subsystem and the entire system.

Remark 4 In both the state feedback and the output feedback control design process, the dimension of LMIs increases with the number of subsystems as well as their dimension. The result is not scale free because of the heterogeneity.

4 Example and Simulation

In this section, a simulation example of Fig.1 is presented to demonstrate the validity of the results obtained. We consider the group of five heterogeneous nodes with the directed interconnection relationship predefined by Fig.1. The nodes are all unstable LTI systems:

$$G_1 = \frac{1}{s-1}; G_2 = \frac{1}{s-2}; G_3 = \frac{1}{s-3};$$

$$G_4 = \frac{1}{s-4}; G_5 = \frac{1}{s-5}.$$

For each node, the outputs of all the nodes in the network are available and only the signal from the right sources according to the interconnection are allowed in. Instead of $v_i = y_j$ as given in most papers, the selected outputs are bounded by the sector conditions. They feed into the destination nodes via the static nonlinear links together with the external inputs.

The interconnection structure F is $v(t) = \varphi(t, y(t))$. The output of the nonlinear interconnection is varying with time t but bounded by the sector $[\underline{k}_{i,j}, \bar{k}_{i,j}]$. And F is in sector

$$[\underline{K}, \bar{K}] = \left[\begin{bmatrix} 0 & 2 & 0 & 1.5 & 1.5 \\ -2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1.5 \\ 0 & 0 & -5 & 0 & 1.5 \\ 1.5 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 3 & 0 & 2 & 2 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 2 \\ 0 & 0 & -4 & 0 & 2 \\ 3 & 0 & 0 & 0 & 0 \end{bmatrix} \right]$$

Because this simulation is only used to demonstrate the usage of the results, the boundary may not have any practical meanings.

The original entire system is illustrated in Fig.3 and the locally controlled system is illustrated in Fig.4.

Fig.5(a) is the output of the original system and it is obviously unstable. With controller gains of $-2, -3, -4, -5, -6$, each subsystem is stable. But controllers that only stabilize the subsystems are not able to guarantee the property of the entire system. The graph in Fig.5(b) shows that a group of stable subsystems may not be stable with an arbitrary interconnection.

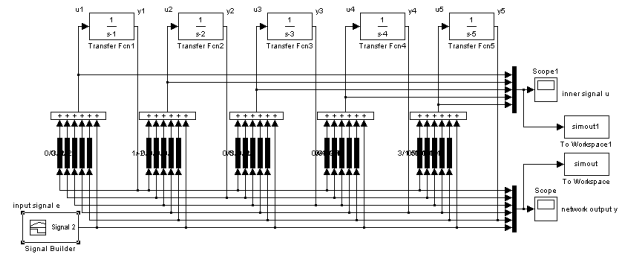


Fig. 3: the Original System

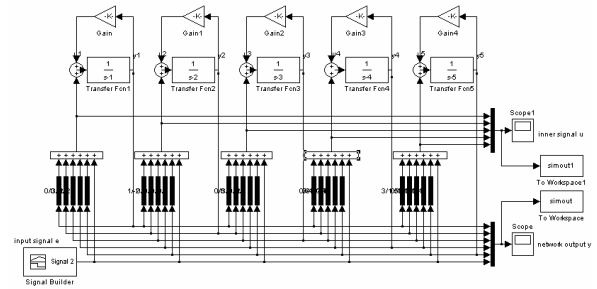


Fig. 4: System with Distributed Controllers

From Theorem 2, we have

$$F = \begin{bmatrix} -7.8235 & 0 & 0 & 0 & 0 \\ 0 & -4.683 & 0 & 0 & 0 \\ 0 & 0 & -7.0904 & 0 & 0 \\ 0 & 0 & 0 & -7.3199 & 0 \\ 0 & 0 & 0 & 0 & -8.8791 \end{bmatrix} \quad (19)$$

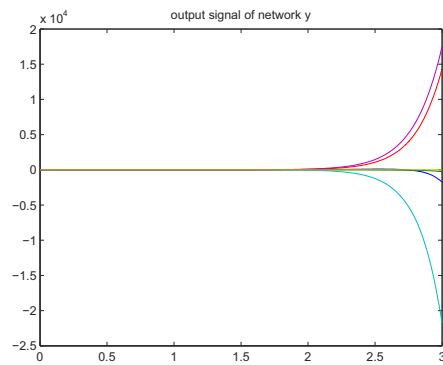
The corresponding distributed controllers stabilize the entire system as well as the subsystems, which is illustrated in Fig.6. The comparative results demonstrate the effectiveness of the obtained criteria.

5 Conclusion

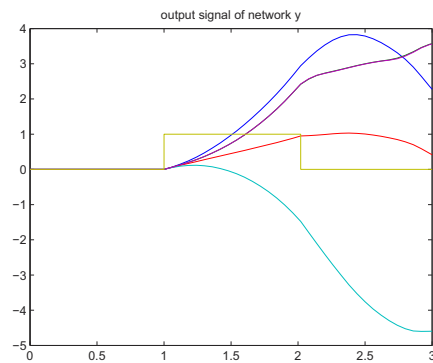
In this paper, we have studied the distributed control of the heterogeneous system with static nonlinear interconnections. Following the previous analysis results[14], the distributed controller design approach is given, including both the state feedback and output feedback. The control laws given in this paper have three advantages:

- It suits to the complex system behaviors. The heterogeneity of the subsystems and the non-idealization of the interconnections are considered in the controller design process;
- Each subsystem is stabilized by its local controllers using its own information, which enables each subsystem work well even faults occur in communication channels.;
- The entire system is stabilizing without any other modification, i.e. the entire system is stabilized by these local controllers.

Finally, a comparative simulation is used to demonstrate the convenience and effectiveness of the proposed methods.



(a) Output of the Original System



(b) Output of the System with General Local Controllers

Fig. 5: the Entire System is Unstable

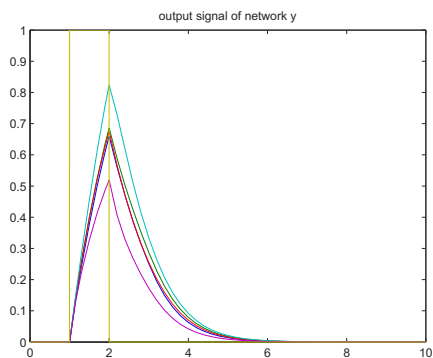


Fig. 6: Output of the System with Distributed Controllers Derived from Theorem 2

REFERENCES

- [1] E.W. Justha, P.S. Krishnaprasad, Equilibria and steering laws for planar formations, *Systems Control Lett.*, 52: 25-38, 2004.
- [2] B. Triplett, D. Klein, K. Morgansen, Discrete time kuramoto models with delay, in: *Lecture Notes in Control and Information Sciences*, vol. 331, Springer, Berlin, 2006.
- [3] X. Wang, X. Li, G. Chen, *Theory and application of complex network*, Tsinghua Publishing House, Beijing, 2006.
- [4] M. Khatir, E.J. Davison, Cooperative control of large systems, in: *Lecture Notes in Control and Information Sciences*, vol. 309, Springer, Berlin, 2006.
- [5] H. Fang, *Control of distributed systems with interconnection constraints*, Thesis of Phd, University of Notre Dame, 2007.
- [6] P. Prathyush, C. Edwards, Decentralised static output feedback stabilisation and synchronisation of networks, *Automatica*, 45(12):2910C2916, 2009.

- [7] K. Gu. Stability problem of systems with multiple delay channels, *Automatica*, 46(4):743-751, 2010.
- [8] S. Hirche, T. Matiakis, M. Buss, A distributed controller approach for delay-independent stability of networked control systems, *Automatica*, 45(8):1828-1836, 2009.
- [9] Z. Duan, J. Wang, G. Chen, L. Huang, Stability analysis and decentralized control of a class of complex dynamical networks, *Automatica*, 44:1028-1035, 2008.
- [10] B. Bamieh, F. Paganini, M. Dahleh, Distributed control of spatially invariant systems, *IEEE Trans. Automat. Control*, 47: 1091-1118, 2002.
- [11] Z. Qu. *Cooperative control of dynamical systems*. London: Springer-Verlag, 2009.
- [12] J. Baillieul, P.J. Antsaklis, Control and communication challenges in networked real-time systems, *Proc. IEEE*, 95(1):9-28, 2007.
- [13] P. Antsaklis, J. Baillieul, Special issue on technology of networked control systems, *Proc. of IEEE*, 95: 5-8, 2007.
- [14] K. Wen, Z. Geng, Stability of distributed heterogenous systems with static non-linear interconnections, *Syst. Control Lett.*, 59(11): 680-686, 2010.
- [15] K. Zhou, J.C. Doyle. *Essential of robust control*, Prentice-Hall, Chapter 9, 1999.