Preface

Control theory has been well developed in the last 80 years ever since the establishment of the Nyquist criterion in the 1930s. A central issue in control science is to analyze the stability of specific motions and to design controllers to realize those motions even when the systems are subject to uncertainties or perturbations. Using a simple algebraic technique, the aforementioned problem can be transformed into the stability problem of a system at its equilibrium, and the design of a controller is to make the corresponding close-loop system stable. This approach has come to be well-known today as the stabilization problem, and therefore it is believed that the most fundamental problem in control science is the stability analysis of the system equilibrium and how to design a controller to stabilize it.

The concept of motion or equilibrium stability was formulated and carefully studied by Lyapunov more than a century ago. It originated from the continuous dependence of the solutions of an ordinary differential equation (ODE) on its initial values, and was then developed by extending the time-domain to the infinite interval. This concept and various corresponding analytic methods stimulated the rapid and fruitful developments of stability theory to the benefit of control science. As a result, much of modern control theory focuses on stability and stabilization issues.

There are two methods to describe a control system. One is formulated in the time-domain, often referred to as the state-space method, which considers time as an independent variable and uses differential or other appropriate equations to describe the system dynamics. In the state-space setting, both the input and the output signals are expressed as time-dependent functions. The relationship between the input and the output is expressed via system state equations indirectly, and the characteristics of the system are usually not represented explicitly by the equations. Even so, this way of description is widely adopted by control scientists and engineers, since there are a lot of mature mathematical tools that can be utilized, especially for LTI systems where many powerful mathematical methods, such as linear algebra and numerical mathematics, can be applied. The other one is formulated in the frequency-domain, which is very popular in the study of LTI systems. Theoretically, it can be obtained from integral transformation (Fourier transform) of time-domain equations because of its time-invariance. Essentially, it gives the input/output relationship in the frequency-domain based on the fact that for LTI systems a harmonic input will generate a harmonic steady output of the same frequency. This relationship can also be interpreted as that the system steady output is the product of the system frequency characteristic and the system input. This model uses the frequency of harmonic wave as the independent variable and has clear physical meaning. Therefore, it has been widely used in control engineering. For a practical system, usually only the input and the output information can be obtained. The system states and the state equations are determined only in the sense of mathematical equivalence. For instance, when there is a revertible linear transformation on the system states in an LTI system, although the equations that describe the system have been changed, the basic input-output relationship through the equations remain unchanged. Obviously, the approach based on frequency characteristics or transformation functions has the superiority for describing the input-output relationship with more explicit physical interpretation. Moreover, frequency-domain descriptions can be obtained via experiments, and there are also a handful of approximate but effective engineering methods for systems analysis and design. Therefore, frequency methods are welcomed by researchers and practitioners in control system applications. With the rapid development of computer science, many effective computing methods have been applied to solving complicated and large-scale problems in science and engineering, e.g., using algorithms related to numerical linear algebra, which makes the state-space approach even more efficient and effective. Using these methods, it is possible to design controllers without considering their physical meanings, then verify and improve the original ideas through simulations. In this way, both the frequency-domain methods and state-space methods can be further developed swiftly.

In the history of control science development, there were several successful encounters between frequency-domain methods and state-space methods. These encounters have led to a spurt in the evolvement of control science, and have established a strong link between frequency-domain methods, which possess physical and practical characteristics, and the state-space methods, that have the powerful support of mathematical tools, thus being able to handle time-varying and even nonlinear systems. This important link has actually become a new growing point of control science.

As early as the time when Wiener proposed the filtering and prediction problems of stationary stochastic processes, the solutions of the integral equations describing the process could be archived via Fourier transform in the frequency-domain framework. At that time, this method was just an interesting experiment. The real encounter between time-domain and frequency-domain methods took place through the important research on absolute stability of control systems.

The notion of absolute stability was first proposed by Lur'e and Postnikov in the 1940s. Since then, a large number of papers and monographs have appeared which investigate the problem of absolute stability. The basic system considered is composed of a linear time-invariant feed-forward part and a nonlinear memoryless feedback part, subject to a sector-bounded constraint. The system is said to be absolute stable if for any nonlinear function satisfying the sector-bounded constraint the system is globally asymptotically stable in the sense of Lyapunov. The fundamental problem in the study of absolute stability is to establish conditions of absolute stability for the system. The conditions should consist of parameters of the sector-bounded constraint and the information provided by the linear part of the given system.

From the very beginning, Lur'e studied this problem in the state-space framework. Therefore, a natural idea is to construct a quadratic Lyapunov function containing the states and the nonlinear characteristics of the system to determine the asymptotic stability of the system. However, Lyapunov equations and inequalities had not been fully studied at that time. Most of the research was carried out based on the Jordan canonical form through linear transformations, and the conditions on absolute stability was reduced to the existence of solutions to some algebraic equations. For more than ten years after the problem was proposed, it had been widely believed that Lyapunov method is the most appropriate and perhaps the only effective way to solve the problem of absolute stability.

In 1960, the First International Federation Automatic Control (IFAC) World Congress was held in Moscow. It symbolized the globalization of control science in the scientific world. In that conference, V. M. Popov presented an amazing result on the frequency-domain criterion of absolute stability derived by using only Fourier transformation. Thereafter, this result was developed and finally formulated as the Popov criterion and Circle criterion. This is a "strong stimulus" which motivated many people to try to find out the basic relationship between the frequency-domain criteria and the time-domain Lyapunov methods. A naturally occurring problem is, which of these two methods is more effective? If one method can be used to test the system stability, can the other one do the same? With a long-time effort made by many scientists from different countries worldwide, this intrinsic relationship was gradually revealed. It is now known that the frequency-domain criterion for absolute stability is formulated in terms of some frequency-domain inequalities about the frequency response of the linear part of the system and the parameters of the nonlinear sector-bounded constraints; while in the Lyapunov function method, the problem of absolute stability is reduced to the existence of a positive-definite matrix P to a matrix inequality with system matrices and parameters of the sector-bounded constraints as coefficients. The feasibility of the LMI (Linear Matrix Inequalities) condition ensures the negative definiteness of the total derivative of the Lyapunov function $x^T Px$. Now, it is also known that the Kalman-Yakubovic-Popov (KYP) Lemma bridges the gap between these two methods and establishes the equivalence relationship between the frequency-domain and time-domain inequalities. The well-known positive real lemma and the bounded real lemma can be regarded as special forms of the KYP lemma.

The dynamics of a system should be reflected by the global nature of the direction of the trajectory flow in the system state space. To describe the global nature, a Lyapunov function in terms of the system states is usually considered. The value of the function determines a hypersurface in the space. If the time derivative of the function along all the trajectories has a fixed sign (for instance, a negative sign), then one knows the global flow direction of all points on the hypersurface along the moving direction of the system trajectories. Due to this geometric view, the Lyapunov method has become an effective tool for determining the asymptotic stability or the instability of a system. For linear systems, when the quadratic Lyapunov function is positive definite, the corresponding hypersurface is an ellipsoid and the system is asymptotically stable. If the function has a negative value, then the system is unstable. In fact, whether the total derivative (along all of the system trajectories) has a fixed sign or not is the essence of the Lyapunov method, which can be used to discuss not only the asymptotic stability or instability of a single equilibrium, but also other system properties such as boundedness of trajectories.

Linear system models are comparatively simple, since the principle of linear superposition holds for linear models. The dynamic characteristics of a linear system in the whole state space can be obtained by investigating that in the neighborhood of the equilibrium, the origin of coordinates. For time-invariant linear systems, when a quadratic Lyapunov function is adopted, an important result on the system dynamics is described in the following theorem.

Theorem. For an n-dimensional time-invariant linear system

$$\dot{x} = Ax, \quad x \in \mathbb{R}^n, \quad A \in \mathbb{R}^{n\times n}$$

where A is a constant real matrix, suppose that A has k eigenvalues with negative real parts and $n-k$ eigenvalues with positive real parts. If there exists a symmetric matrix P such that

$$PA + A^T P = -Q, \quad Q > 0,$$

then P has $n-k$ negative eigenvalues and k positive eigenvalues.

This theorem is an extension of a corresponding classical Lyapunov result. It can also be used to discuss some other global properties of nonlinear systems.

When a global property of a system is considered, it should describe the global nature of the system rather than just some peculiar properties of a specific solution of the system; for instance, the existence of multiple equilibrium states in the system, if the system is just with a single equilibrium state and it is asymptotically stable, the boundedness of all solutions, the existence of auto-oscillations, and the nonexistence of chaos, etc. Meanwhile, such a property should be operational, or in other words mathematically provable or computationally tractable. In this book, we mainly study global properties by means of the Lyapunov function method, dynamic system analysis and ordinary differential equations theory, where the time-domain results will be interpreted in the frequency-domain framework via the KYP Lemma.

The main difficulties of nonlinear problems come from two aspects. One is the dimensional difficulty. For a one-dimensional system, $\dot{\xi} = \xi^3$, for instance, although the equation is nonlinear, it can be easily checked as if its solution is asymptotically stable without using advanced mathematics. For a one-dimensional time-variant system, $\dot{\xi} = a(t)\xi$, for instance with $a(t) \le -\beta < 0$, the asymptotic stability of the solution can be obtained immediately. It also indicates that the frozen-coefficient method is applicable in this case. When the dimension is greater than one, the solution of the system may generate "rotation", and the above discussion may not be valid. In the case when the dimension is only two, there still exist some theoretical results based on the qualitative theory of planar dynamical systems. When the dimension is greater than two, however, difficulties encountered in analysis are far more than one can imagine. The other aspect is the essential nonlinearity in the system, which is the main difficulty in nonlinear analysis which typically cause many complicated dynamic behaviors that would never happen in linear systems. A convenient method for a nonlinear analysis is to use the same framework for linear systems, but it cannot solve the essential nonlinearity problem in nonlinear systems. Similar to the most fundamental non-convexity difficulty in nonlinear programming, there are no available mathematical tools for effectively handling the essential nonlinearity in the study of general nonlinear control systems.

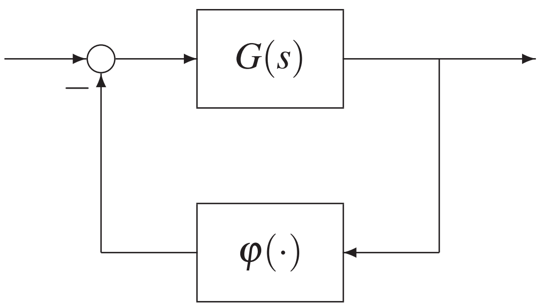
Nonlinear systems theory originated from research on nonlinear oscillations, since auto-oscillation is the most common nonlinear phenomenon found in nonlinear mechanical systems, such as the escapements frequently used in clocks and watches, or such as found in the van der Pol circuit. This phenomenon corresponds to an isolated periodic solution of the system which cannot exist in linear systems. The research on auto-oscillations was the first hot topic in the field of nonlinear systems research, from the 1940s to the 1960s. But the work was restrained to second-order systems due to the lack of powerful computational tools and mathematical analysis methods.

The second phase in the development of nonlinear science starts from the finding of chaos, which is far more complicated than auto-oscillations. The occurrence of auto-oscillations did not go beyond common imaginations, because this phenomenon frequently emerges, in both natural and artificial systems such as the beating of the heart and the swinging of pendulums in mechanical clocks. In geometry, it is just a circle repeating itself constantly, therefore a kind of regular motion as compared to chaotic dynamics. What is needed to study is why this periodic dynamic behavior can be produced and how to produce it in a specific nonlinear system without external periodic excitement. However, this is not the case for chaos, which demonstrates some so-called fantastic nonlinear properties and completely altered people's conventional view from several aspects. First of all, the existence of chaos indicates that a deterministic system can produce stochastic-like dynamical behaviors with the ergodicity property. Secondly, it shows high sensitivity to initial conditions that linear and general nonlinear systems do not have. Thirdly, it typically demonstrates a strange attractor of the system, which is not the usual fixed point (zero-dimensional) and limit cycle (one-dimensional) but a set of points with a very complicated structure having a fractional dimension. Such a special dynamic process has attracted a lot of interest from almost all scientific communities in the past half a century. However, for the same reason as of lacking powerful mathematical tools, although there are many qualitative results for low-dimensional systems, quantitative analysis of higher-dimensional chaotic systems basically relies on numerical computations and approximate analytic approaches today.

From the control theoretic point of view, the main research interest on auto-oscillations or chaos is focused on how to design a controller to affect the dynamic behavior; that is, to produce or eliminate such a behavior in the given system by means of control. Both auto-oscillations and chaos are non-convergent but bounded evolutionary processes. Auto-oscillations also have the property of isolation. Neither of these two phenomena can exist in linear systems, but exist only in two kinds of bounded evolutionary processes: one is the trivial equilibrium point, which is either a unique point or a subspace; the other is a compound oscillation composing of one or several simple harmonic oscillations. A constant multiple of a same class of compound oscillations is still a possible oscillation and all of them together still compose a subspace. Apart from these two kinds of bounded dynamical behaviors, the solutions of a linear system can be divided into two parts, one is the convergent solutions and the other is unbounded solutions. When the system does not have eigenvalues located on the imaginary axis, all its bounded solutions will converge. This property is known as the property of dichotomy. For a linear system, dichotomy is not a crucial property, but for a nonlinear system this property can be used to exclude all processes that are bounded but not convergent, such as auto-oscillations and chaos. When a system does not have eigenvalues located on the imaginary axis, from the premier theorem the distribution of the eigenvalues of the solution matrix P to the Lyapunov equation is converted to that of the system relative to the imaginary axis. This fact provides a fundamental principle for, and indeed facilitates, the study of dichotomy and the construction of the existing area of limit cycles.

The demands on control systems are numerous and in various forms, many of their problems cannot be reduced to the stability of a single equilibrium. Sometimes, demanding stability is hard to satisfy, at other times, it may be unnecessary. Since some nonlinear characteristics of the system are uncertain, the equilibrium position may move constantly. In this case, it is unreasonable to design a controller to stabilize such an equilibrium state, and the primary demand would be that all the solutions of the system are bounded, i.e., there are no divergent solutions in the system. Furthermore, one may require every solution be convergent to a certain equilibrium. This property is known as a gradient-like behavior in systems with multiple isolated equilibria. An interesting fact for a time-invariant nonlinear system is that when the system is gradient-like, there must exist at least one equilibrium that is not asymptotically stable in the sense of Lyapunov. This conclusion is drawn from the following contradiction: if all the equilibria are asymptotically stable, then each equilibrium has an open set of domain of attraction, while denumerable open sets cannot cover the whole space. It is also known that even though a system is gradient-like, the characteristics of the trajectories and the equilibria of the system in the phase space are considerably complicated. Such a complex situation is very common in electric power systems, for instance, and is a key to further understand the modern power systems.

With so many and so complex non-conventional dynamic characteristics, it is hard to make progress if one addresses the problems based on a very general model of nonlinear systems, since the available mathematical theory can only provide general conclusions rather than concrete details. Therefore, the theme of this book will be focused on a commonly used model in control science and engineering — the Lur'e-type systems, which is composed of a linear feed-forward part and a nonlinear feedback part, as depicted by the following block diagram:



This model was first proposed by Lur'e, with the background that a hydraulic valve was used as the actuator in a driving system at that time. Since the characteristic curve of the hydraulic valve is nonlinear with uncertainty, $\varphi(\sigma)$ is an uncertain function with constraints. The feature that the nonlinearity can be separated from the linear part brought convenience to the understanding and study of many physical systems. In fact, many typical nonlinear systems, such as Chua's circuit and the Lorenz system, can be classified into this type. As a matter of fact, Lur'e system has become a preferable model for investigating nonlinear dynamics such as chaos today. During the first two decades since the proposal of the Lur'e system framework, the research focus was on the absolute stability. Afterwards, Leonov and some other researchers further extended the research scope to other properties of the Lur'e systems, including dichotomy, gradient-like behaviors and auto-oscillations. These efforts made it possible to design controllers guaranteeing or eliminating some dynamical properties of the given system. The main controller design methods adopted in this book is to embed the controller parameters into some matrix inequalities by means of the KYP lemma and to reduce the problem to the feasibility of solving such matrix inequalities. By taking advantage of the peculiar merits of matrix inequality methods, this book will further extend the frequency-domain results on global properties, developed by Leonov *et al.*, to controllers design and robustness analysis, which will provide a basic theoretical and methodological framework for future investigations and applications.

This book is organized as follows: Chapters 1–2 are two introductory chapters, presenting basic definitions and the major analytical tools that will be used to study systems with stationary equilibrium sets. More specifically, Chapter 1 reviews some basic system concepts and formulas related to linear matrix inequalities; Chapter 2 introduces some useful tools in control systems synthesis and the linear matrix inequality approach to the standard optimal and suboptimal H_∞ control theory. Chapter 3 discusses analysis and control problems for positive realness. In Chapter 4, a unified framework is proposed for analyzing the absolute stability and dichotomy of Lur'e systems. Chapter 5 introduces two kinds of special forms of pendulum-like feedback systems and gives both time-domain and frequency-domain conditions on global properties of such systems. Chapter 6 is devoted to controllers design for a class of pendulum-like systems, which can ensure some global properties and preserve physical and dynamical phenomena of the pendulum-like systems simultaneously. Chapter 7 studies control problems for a class of systems with input nonlinearities. In Chapter 8, a time-domain approach to robust analysis and control for uncertain feedback nonlinear systems is presented and discussed. Chapter 9 is devoted to robust analysis and synthesis on the nonexistence of periodic oscillations in nonlinear Lur'e systems. Chapter 10 considers interconnected systems and discusses the effects of interconnections on system stability and performances. Chapter 11 demonstrates some applications of the theories established in the previous chapters using Chua's circuit as the main example. Finally, a bibliography and index are provided to complete the presentation of the entire book.

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